Strong Enhancement of Superconducting T_c in Ferromagnetic Phases

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It is shown that the critical temperature for spin-triplet, p-wave superconductivity mediated by spin fluctuations is generically much higher in a Heisenberg ferromagnetic phase than in a paramagnetic one, due to the coupling of the magnons to the longitudinal magnetic susceptibility. Together with the tendency of the low-temperature ferromagnetic transition in very clean Heisenberg magnets to be of first order, this qualitatively explains the phase diagram recently observed in UGe_2.

It has long been known that, in principle, the exchange of magnetic fluctuations between electrons can induce superconductivity [1]. Magnetic fluctuations become large in the vicinity of continuous magnetic phase transitions, which makes nearly ferromagnetic materials, or ferromagnets with a low Curie temperature, natural candidates for this phenomenon. In contrast to the much more common phonon-exchange case, which usually leads to electron pairing of spin-singlet, s-wave nature, the magnetically mediated pairing is strongest in the spin-triplet, p-wave channel. p-wave superconductivity is very sensitive to nonmagnetic impurities and therefore can be expected only in extremely pure samples. The combined requirements of high purity, low temperatures, and vicinity to a ferromagnetic transition severely restrict the number of promising materials. Indeed, until recently there were no convincing simple examples of magnetically induced superconductivity, and the paramagnon interpretation of superfluid ^{3}He [1,2] was considered the best example of pairing by exchange of magnetic fluctuations.

This situation has recently changed, due to the observation of the coexistence of ferromagnetism and superconductivity in UGe_2 [3]. In contrast to other uranium compounds, UGe_2 has more in common with classic d-electron ferromagnets, like Fe, Co, and Ni, than with heavy-fermion systems. The persistence of ferromagnetic order within the superconducting phase has been ascertained by means of neutron scattering, and the itinerant ferromagnetism and the superconductivity are believed to arise from the same electrons [3]. Since superconductivity in the presence of ferromagnetism must be of spin-triplet type, magnetically induced pairing is an obvious candidate for the observed superconductivity, although a phonon mechanism has also been proposed [4].

The nature of the phase diagram reported in Ref. [3] is, however, not obviously consistent with existing models of spin fluctuation induced superconductivity, see Fig. 1. Fay and Appel [5] have calculated the superconducting

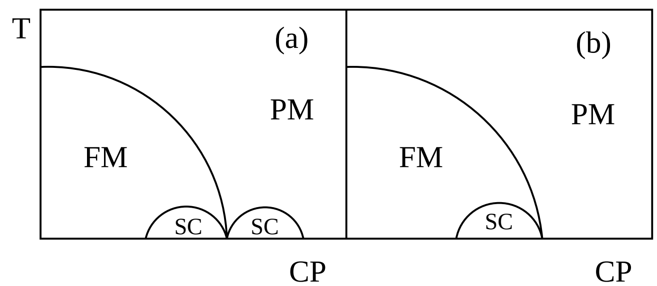

FIG. 1. Schematic phase diagram showing the paramagnetic (PM), ferromagnetic (FM), and superconducting phases (SC) in a temperature (T) - control parameter (CP) plane. (a) shows the qualitative prediction of paramagnon theory [5], and (b) qualitatively shows the phase diagram as observed in UGe_2 [3] and explained by the theory presented here. In Ref. [3], hydrostatic pressure serves as CP.

T_c for a p-wave, equal-spin pairing state in both the paramagnetic (PM) and ferromagnetic (FM) phases close to a continuous magnetic transition. Using a McMillan-type formula, they found values of T_c on either side of the transition that are within 20% of one another. Their shape of T_c as a function of the distance t from the magnetic transition is very similar to that obtained by Levin and Valls [2], who solved the Eliashberg equations numerically in the PM phase, although the absolute values of T_c are smaller in the McMillan approximation. More recently, Roussev and Millis [6] have obtained similar results in the PM phase. Contrary to this theoretical expectation of a superconducting phase diagram that is roughly symmetrical with respect to the magnetic phase boundary, Fig. 1(a), the authors of Ref. [3] observed superconductivity at temperatures up to about 500 mK within the FM phase only, Fig. 1(b). Qualitatively the same phase diagram has very recently been observed in $ZrZn_2$ [7]. Since the spin fluctuations become large on either side of the magnetic transition, it seems hard to reconcile this experimental result with paramagnon theory [8].

In this Letter we show that the observed phase diagram can nevertheless be understood in these terms. The key lies in the existence of spin waves or magnons in the FM phase, which couple to the longitudinal susceptibility and contribute a mode-mode coupling term to the latter that has no analog in the PM phase. We will see that this produces a superconducting transition temperature which under reasonable assumptions can easily be 50 times larger than in the PM phase.

We have included this effect in a McMillan-type T_c calculation similar to the one in Ref. [5] A representative result of our analysis is shown in Fig. 2. The solid curve (left-hand scale) represents the superconducting T_c as a function of the dimensionless distance t from the FM critical point. Also shown is the magnetization M in the FM phase, in units of the saturation magnetization $\mu_B\, n$, with n the electron number density and μ_B the Bohr magneton. T_c is measured in units of a characteristic temperature T_0 that is given by either the Fermi temperature or a band width, depending on the model considered. The dashed curve shows the result in the PM phase scaled by a factor of 50 (right-hand scale), and the dotted curve in the FM phase (also scaled by a factor of 50, right-hand scale) represents the result that is obtained upon neglecting the mode-mode coupling effect. Notice that the maximum T_c in the FM phase is more than 50 times higher than in the PM phase. This *relative* difference between T_c in the two phases is the important result of our analysis. The absolute values should not be taken very seriously, as calculating T_c is notoriously difficult and our simple mean-field treatment is certainly not adequate for this purpose. However, the relative comparison we expect to be reliable. It predicts a pronounced asymmetry between the PM and FM phases, which in the case of UGe_2 means that superconductivity in the PM phase should not be expected at temperatures above at most 10 mK, in agreement with the experiment. Also of interest is the fact that at low temperatures the FM transition in very clean itinerant Heisenberg systems is generically of first order, as has been predicted theoretically [9] and is indeed observed in UGe_2 [3] as well as in MnSi [10]. For the purpose of our discussion this simply means that values of $|t|$ smaller than some minimum value are not experimentally accessible.

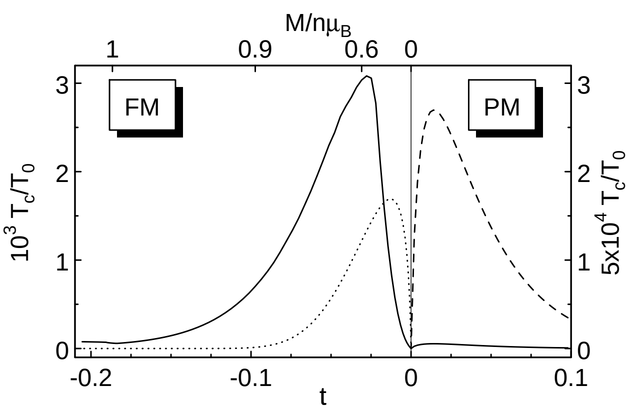

FIG. 2. Superconducting T_c (solid curve, left scale) as a function of the distance from the critical point t, and the magnetization M. The dashed line (right scale) shows T_c in the PM phase scaled by a factor of 50, and the dotted curve (right scale) is the result in the FM phase without the mode-mode coupling effect. See the text for further explanation.

In the remainder of this Letter we sketch the theoretical analysis that has led to these results. For an order parameter (OP) field, we choose $\mathcal{F}(x,y) = \psi_\uparrow(x)\,\psi_\uparrow(y)$, with $\psi_\sigma(x)$ an electronic field with spin index σ and space-time index x [11]. The OP, i.e. the expectation value $\langle \mathcal{F}(x,y)\rangle = F(x-y)$, is the anomalous Green function. The orbital symmetry of the OP is still unspecified, we will later choose the p-wave case.

We have derived coupled equations of motion for F and the normal Green function, G, that lead to a loop expansion for the equation of state [12]. Our model is a microscopic action S with a free-electron part, S_0, and spin-singlet and spin-triplet interaction terms,

$$S^t_{\rm int} = \frac{\Gamma_t}{2}\int dx\big[\mathbf{n}_s(x)\big]^2,\ S^s_{\rm int} = \frac{-\Gamma_s}{2}\int dx\big[n_c(x)\big]^2. \quad (1)$$

Here $\mathbf{n}_s(x)$ and $n_c(x)$ are the electronic spin and charge density fields, respectively, and Γ_t and Γ_s are the spin-triplet and spin-singlet interaction amplitudes. We assume that screening has been built into the starting action, so the interaction amplitudes are simply numbers. By putting $\Gamma_s = \Gamma_t$ one obtains the Hubbard model considered in Ref. [5]. The magnetic equation of state we treat in zero-loop approximation. The superconducting equation of state needs to be calculated in one-loop approximation in order to capture the spin-fluctuation induced pairing. It takes the form of linearized strong-coupling equations that are similar to those in Ref. [5]. These equations can be rewritten as an eigenvalue problem, which can then be solved numerically, using some theory for the (para)magnon propagators as input. This is the established procedure to calculate the critical temperature for phonon-mediated superconductivity [13], and it has been employed in the case of magnetically induced superconductivity or superfluidity in Refs. [2,6].

Even with a complete numerical solution of the strong-coupling equations, the superconducting T_c is notoriously hard to predict. This holds *a fortiori* in the case of magnetically mediated superconductivity since (1) there is much less experimental information about the paramagnon propagator, that could be used as input, than about phonon spectra, and (2) there is no analog of Migdal's theorem. Our ambition here is therefore *not* to calculate T_c, but rather to perform a *relative* comparison of T_c values in the PM and FM phases, respectively. For this purpose, a simple McMillan-type approximation for T_c [13] suffices. We obtain

$$T_c = T_0(t)\,\exp\left[-(1+d_0^L+2d_0^T)/d_1^L\right] \quad . \tag{2}$$

Here $T_0(t)$ is a temperature scale that will be specified below. Specializing to the p-wave case, the $d_{0,1}^{L,T}$ read

$$d_1^L = \frac{\Gamma_t N_F^\uparrow}{(k_F^\uparrow)^2}\int_0^{2k_F^\uparrow} dk\,k\,\left(1-\frac{k^2}{2(k_F^\uparrow)^2}\right) D_L(k,i0)\ , \tag{3a}$$

$$d_0^L = \frac{\Gamma_t N_F^\uparrow}{(k_F^\uparrow)^2}\int_0^{2k_F^\uparrow} dk\,k\,D_L(k,i0) \quad , \tag{3b}$$

$$d_0^T = \frac{\Gamma_t N_F^\uparrow}{(k_F^\uparrow)^2}\int_{k_F^\uparrow-k_F^\downarrow}^{k_F^\uparrow+k_F^\downarrow} dk\,k\,D_T(k,i0) \quad . \tag{3c}$$

$k_F^\uparrow(k_F^\downarrow)$ are the Fermi wavenumbers for the up (down)-spin Fermi surface, and $N_F^\uparrow$ is the density of states at the up-spin Fermi surface. In the PM phase, $k_F^\uparrow = k_F^\downarrow \equiv k_F$. $D_{L,T}(q)$ are the longitudinal and transverse (para)magnon propagators. They are related to the electronic spin susceptibility χ via $D_{L,T}(q) = \chi_{L,T}(q)/2N_F$, with N_F the density of states at the Fermi level in the PM phase. We use the expressions that were derived in Ref. [14], with one crucial modification that we will discuss below. From that paper we obtain in the PM phase, and in the limit of small wavenumbers,

$$D_{L,T}(q,i0) = 1/[t + b_{L,T}(q/2k_F)^2] \quad , \tag{4}$$

In the Gaussian approximation of Ref. [14], $b_L = b_T = 1/3$. However, there is no reason to prefer this Gaussian approximation over any other approximation scheme. The functional form of the long-wavelength expression, Eq. (4), on the other hand, is generic. We therefore adopt Eq. (4) with $b_{L,T}$ arbitrary coefficients of $O(1)$. By the same reasoning, we have in the FM phase, in the limit of long wavelengths and small frequencies,

$$D_L(q,i0) = 1/[5|t|/4 + b_L(q/2k_F)^2] \quad , \tag{5a}$$

$$D_T(q,i\Omega) = \frac{\Delta/4\epsilon_F}{(1-t)^2}\left(\frac{1}{i\Omega/4\epsilon_F + (\Delta/2\epsilon_F)b_T(q/2k_F)^2} - \frac{1}{i\Omega/4\epsilon_F - (\Delta/2\epsilon_F)b_T(q/2k_F)^2}\right) \quad , \tag{5b}$$

with Δ the Stoner band splitting. For $0 < \Delta < n\Gamma_t$, Δ is related to the magnetization M by $M = \mu_{\rm B}\Delta/\Gamma_t$.

Two comments: (1) In a strict long-wavelength expansion of the propagators from Ref. [14] the b_L and b_T in Eqs. (5a,5b) become magnetization dependent. We ignore this effect and use the same values as in the PM phase. We have compared this approximation against using the full propagators from Ref. [14], see below. (2) The factor of 5/4 in Eq. (5a) arises since we keep the particle number density fixed, as is the case experimentally, rather than the chemical potential, see Ref. [5].

We now consider the longitudinal magnetic propagator in the FM phase in more detail. In a Heisenberg ferromagnet, the transverse spin waves or magnons couple to the longitudinal susceptibility χ_L. This effect is most easily demonstrated within a nonlinear sigma-model description of the ferromagnet, which treats the order parameter $\mathbf{M}$ as a vector of fixed length M, and parameterizes it as $\mathbf{M} = M\,(\sigma(x), \pi_1(x), \pi_2(x))$ with $\sigma^2 + \pi_1^2 + \pi_2^2 = 1$, M the magnetization, and x a space-time index [15,16]. The diagonal part of the π_i-π_j propagator, $\langle \pi_i \pi_i\rangle = (M^2/2N_F)\,D_T$, is proportional to the transverse propagator D_T, and the off-diagonal part has been calculated in Ref. [16]. The longitudinal propagator, $D_L = (M^2/2N_F)\langle\sigma(x)\sigma(y)\rangle$, can be expanded in a series of π-correlation functions,

$$\langle\sigma(x)\sigma(y)\rangle = 1 - 2\langle\pi_i(x)\pi^i(x)\rangle + \langle\pi_i(x)\pi^i(x)\pi_j(y)\pi^j(y)\rangle + \ldots \tag{6}$$

where repeated indices are summed over. At one-loop order, the term of order π^4 yields the diagram shown in Fig. 3. Notice that the sigma model, which neglects all longitudinal fluctuations, replaces the external legs by constants. Power counting shows that at nonzero temperature, and for dimensions $d < 4$, this contribution causes the homogeneous longitudinal susceptibility to diverge everywhere in the FM phase [17]. More generally, this one-loop contribution, together with the zero-loop one, Eq. (5a), yields a functional form for D_L in the FM phase that is exact at small wavenumbers. This diagram has no analog in the PM phase, while all other renormalizations of the propagators will give comparable contributions in both the PM and FM phases. It is therefore reasonable to calculate T_c based on a one-loop approximation in the FM phase, and compare it to a zero-loop calculation in the PM phase. We have used Eq. (5b) for the internal propagators in Fig. 3. Since the coupling constants involve a wavenumber integral, Eqs. (3), we also need to go beyond the sigma model and keep the wavenumber dependence of the external ones. For computational simplicity, we have modeled the external legs by replacing Eq. (5a) with a step function that cuts off the momentum integral at $k/2k_F = \sqrt{5|t|/4b_L}$. With these approximations, the momentum integral in Fig. 3 can be done analytically, leaving the frequency sum to be performed numerically. The result, and the resulting contribution to d_1^L and d_0^L, depend on the temperature, so Eq. (2) now needs to be solved self-consistently for T_c.

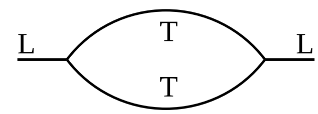

FIG. 3. Mode-mode coupling contribution to the longitudinal (L) propagator D_L from the transverse (T) ones.

We still need to specify the temperature scale $T_0(t)$. Following Ref. [5], we use the prefactor of $|t|$ in Eqs. (4,5a) as a rough measure of the magnetic excitation energy,

$$T_0(t) = T_0\,[\Theta(t)\,t + \Theta(-t)\,5|t|/4] \quad , \qquad (7)$$

with T_0 a microscopic temperature scale that is related to the Fermi temperature (for free electrons) or a band width (for band electrons). This qualitatively reflects the suppression of the superconducting T_c near the FM transition due to effective mass effects [2,5,6].

We are now in a position to choose parameters and calculate explicit results. We put $\Gamma_s = \Gamma_t$ [5]; other reasonable choices yield similar results. Let us first ignore the mode-mode coupling contribution to d_1^L and d_0^L. We have performed the calculation both with the full propagators from Ref. [14] and with the schematic Landau propagators, Eqs. (4) and (5a). With $b_L = 0.23$, $b_T = 0.4$ the two results are within 10% of one another, and also very similar to those obtained by Fay and Appel [5]. We then use these values of $b_{L,T}$ to calculate the mode-mode coupling contribution, and solve the T_c-equation. The result is shown in Fig. 1 and has been discussed above. We have also explored the effect of varying the parameters $b_{L,T}$. With $b_L = b_T = 1$ we obtain the result shown in Fig. 4. The (unphysical) zero-loop result in the FM phase is very sensitive to the parameters, while the enhancement of the (physical) one-loop result over the T_c in the PM phase is rather robust. However, the position of the maximum of T_c changes compared to Fig. 1; it now occurs at the point where the magnetization reaches its saturation value. The reason is as follows. As one approaches the magnetization saturation point from low magnetization values, the transverse coupling constant d_0^T vanishes, and remains zero in the saturated region. Effectively, the Heisenberg system turns into the Ising model discussed in Ref. [6]. If the longitudinal coupling constant d_1^L still has a substantial value at that point, then this leads to an increase in T_c. This is a very strong effect in the zero-loop contribution, see Fig. 4, and the effect qualitatively survives in the one-loop result. If, however, d_1^L is already very small, then d_0^T going to zero has only a small effect on T_c, as is the case in Fig. 1. Which of these two cases is realized depends on the parameter values. We finally mention that the first order nature of the magnetic transition [9,3] adds another mechanism for suppressing T_c in the PM phase: For a sufficiently strong first order transition, and if the case shown in Fig. 4 is realized, then the effective t may be large enough everywhere for the system to miss the maximum of T_c in the PM phase, but not in the FM phase.

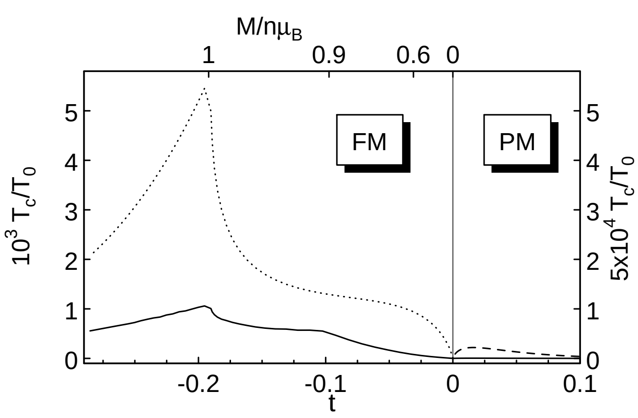

FIG. 4. Same as Fig. 2, but for different parameter values (see the text).

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