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Stochastic Reduced Order Modelling of a Fluid Structure Interaction System

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Abstract

Solution of large order fluid structure interaction (FSI) systems with parametric uncertainties demand intensive computational resources. The present study focusses on developing and implementing a novel stochastic reduced order model to resolve pressure induced oscillations of a disc-like structure as a generic fluid structure interaction system. Such models have applications in various heavy engineering systems, like turbo machinery industries. It is important to resolve the coupled dynamics, in order to avoid large oscillations and instabilities. The stochastic reduced order model uses a modal reduction approach together with sparse grid based polynomial chaos expansion (PCE) to truncate both the system degrees of freedom (dof) as well as the random modes. Further reduction in computational time is achieved by parallelization. Interfacing algorithms have also been developed that enable finite element (FE) modelling of the FSI system using commercial softwares and in-house developed codes.

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Keywords: Stochastic reduced order modelling; fluid structure interaction; PCE.

1. Introduction

The dynamic analysis of structural systems which have significant interactions with the surrounding fluid are modelled as problems in fluid structure interaction (FSI). Depending on the mode of interactions, FSI problems can be broadly classified in several categories. The present study focuses on the acoustic interaction effects, generally referred to in the literature as acousto-elasticity. For systems with simple geometries, analytical solutions have been developed in the literature for acousto-elastic problems [1–3]. However, systems with complicated geometries are not conducive for analytical modelling, and one has to resort to numerical techniques for their analysis. Accurate numerical modelling of the FSI systems not only require high fidelity finite element (FE) models for the structure, but also for the fluid, and hence, the numerical models are characterised by a large number of degrees-of-freedom (dofs). As a result, the dimension of the global system matrices obtained from discretization of the governing differential equations become very large and the analysis of these FSI systems, even if they are linear, is computationally intensive.

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Here, it is important to note that the models for the fluid-structure interacting system are developed based on specific assumptions about the structure, the fluid and their interactions. These assumptions though enable a simplification of the model, do not reflect the reality and result in errors in predicting the system behavior. The effects of these unknown errors in modelling can be incorporated into the analysis by treating some of these parameters as random variables and investigating how the uncertainties in modelling these parameters propagate through the system into the response. Unfortunately, treating the parameters as random variables increases the dimensionality of the FSI problem further and significantly increases the computational complexities.

The focus of the present study is to develop reduced order models for FSI systems with parameter uncertainties. The present study considers the interaction between structure and acoustic systems [1]. The interaction between the structure and the acoustic systems arises either from the flow induced vibrations of the structure or from the structure induced acoustic oscillations. It is essential to perform a coupled eigenvalue analysis because, this interaction, in turn could change the dynamics of both the structure and the acoustic system [3]. The coupled eigenvalue analysis should be carried out in the generalized coordinates. The parameter uncertainties are incorporated into the numerical analysis by modelling them as random variables and the uncoupled system matrices are modelled using FE. Reduction in the dimension of the FE matrices is obtained by reducing the size of the FE global uncoupled system matrices using System Equivalent Reduction Expansion Process (SEREP) [5] by retaining the dominant dynamical modes and the master dofs. The solution for the stochastic coupled eigenvalue problem can be obtained within the PCE framework by projecting the uncertainties into the Hilbert space [6,7]. Reduction in the stochastic dimension is achieved by retaining only the first few dominant stochastic modes. A sparse grid based stochastic collocation is used in PCE implementation to estimate the projections along the stochastic basis functions. Further computational reduction is achieved by parallelizing the developed codes using PETSc [8,9] in C platform. PETSc is an acronym for Portable Extensible toolkit for Scientific Computation and is a package of data structures and routines to address mathematical problems arising out of partial differential equations, can solve them in parallel over millions of nodes and produce graphical outputs for users. The basic parallel communication routines for PETSc are derived from MPI. The linear algebra routines are derived from the parent libraries such as BLAS and LAPACK. PETSc's tailor made routines are very useful in formulating problems of partial differential equations, time stepping, linear equations, matrix and vector operations, eigenvalue problem solvers etc [10]. The problems under consideration in this project include linear algebra operations, eigenvalue computations, numerical integration and interfacing with commercial software like MATLAB. Interfacing algorithms are developed that enable FE modelling using commercial FE softwares and importing the structure FE matrices and integrating them with the developed algorithms.

2. Methodology

In the present study, the FSI system that has been considered consists of an annular plate with cavities on both sides. The dynamical model consists of a thin plate immersed in a compressible fluid filled enclosure. The geometry and the corresponding system parameters are given in Fig. 1 and Table. 1. respectively. The theoretical formulation follows the analytical theory developed by Dowell *et al.*[1], which combines the rigid wall acoustic cavity modes and the *in vacuo* structural modes by means of suitable boundary conditions. The two field equations are combined and converted into a set of gyroscopically coupled ODEs by using the discretization technique given in Dowell *et al.*[1]. The discretized field equations governing the coupled behaviour of the stochastic FSI system with the random parameter (θ) can be expressed as [4],

$$V^a M^a_m(\theta)[\ddot{a}_m(t,\theta) + (\Omega^a_m)^2 a_m(t,\theta)] = -A_d c_0^2 \sum_k L^a_{mk}(\theta)\dot{q}_k(t,\theta), \tag{1}$$

$$V^b M^b_m(\theta)[\ddot{b}_m(t,\theta) + (\Omega^b_m)^2 b_m(t,\theta)] = A_d c_0^2 \sum_k L^b_{mk}(\theta)\dot{q}_k(t,\theta), \tag{2}$$

$$M_k(\theta)[\ddot{q}_k(t,\theta) + \omega_k^2 q_k(t,\theta)] = \rho_f A_d \Big(\sum_m L^a_{mk}(\theta)\dot{a}_m(t,\theta) - \sum_m L^b_{mk}(\theta)\dot{b}_m(t,\theta) \Big), \tag{3}$$

where, $\rho_f(\theta)$ is the density of fluid, $M_m(\theta)$, $M_k(\theta)$ and $\Omega_m(\theta)$, $\omega_k(\theta)$ are the modal masses and uncoupled natural frequencies of the m^{th} acoustic normal mode, and the k^{th} structural mode respectively. The plate-cavity coupling

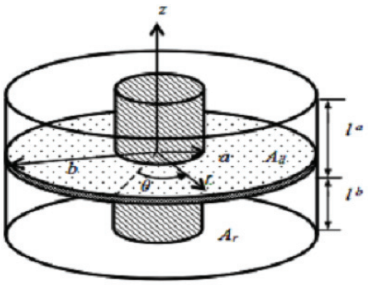


Fig. 1. A schematic of the considered FSI model.

Table 1. System parameters

Outer radius, b	205 mm
Inner radius, a	70 mm
Disc thickness, h	22 mm
Disc density, ρ_d	7700 kg/m^3
Fluid density, ρ_f	1.2 kg/m^3
Young's modulus, E	190 GPa
Poisson's ratio, ν	0.33
Speed of sound, c	$343m/s$
Cavity depth, $l^a = l^b$	10 mm

coefficient sub-matrix L_{mk} is given by, $L_{mk} = \frac{1}{A_d}\int_{A_d} F_m\psi_k \, \mathrm{d}A$, where, F_m and ψ_k are the rigid wall normal modes and *in vacuo* plate modes respectively. In matrix form, the discretized field equations for m cavity modes and k structural modes can be written as,

$$\mathbf{M}(\theta)\begin{Bmatrix}\ddot{a}_m(t,\theta)\\ \ddot{b}_m(t,\theta)\\ \ddot{q}_k(t,\theta)\end{Bmatrix} + \mathbf{L}(\theta)\begin{Bmatrix}\dot{a}_m(t,\theta)\\ \dot{b}_m(t,\theta)\\ \dot{q}_k(t,\theta)\end{Bmatrix} + \mathbf{K}(\theta)\begin{Bmatrix}a_m(t,\theta)\\ b_m(t,\theta)\\ q_k(t,\theta)\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix}. \tag{4}$$

The structure and the cavity have been modelled in COMSOL [11] and the uncoupled system matrices have been exported from the software platform. First, the uncoupled random eigenvalue analysis has been carried out for the structure and the fluid to get the dominant structural and acoustic modes. Subsequently, coupling effects have been introduced by considering the dominant acoustic and the structural modes, through the boundary degrees of freedom. In order to make the calculation of the coupling coefficients simple, the meshing for the structure and the fluid have been carried out in such a way that the nodes of the fluid region coincide with those of the structure at the interface. Next, the coupled random eigenvalue problem is solved to estimate the coupled frequencies for the full FSI system. Dimension reduction in the FSI system in Equation (4) has been achieved using PCE based SEREP with the retained dofs and dominant structural and acoustic modes. SEREP is used to find a low dimension subspace $\mathbf{T}(x_j) \in \Re^{N\times m}$ with $m << N$ in order to approximate the state vector $\mathbf{X}(t, x_j)$ in a reduced subspace as,

$$\mathbf{X}(t,x_j) = \begin{Bmatrix}\mathbf{X}_a(t,x_j)\\ \mathbf{X}_d(t,x_j)\end{Bmatrix} = \begin{Bmatrix}\mathbf{\Phi}_a(t,x_j)\\ \mathbf{\Phi}_d(t,x_j)\end{Bmatrix}\mathbf{\Phi}_a^g(t,x_j)\mathbf{X}_a(t,x_j) = \mathbf{T}(x_j)\mathbf{X}_a(t,x_j) \tag{5}$$

Here, $\mathbf{X}_a$ is the reduced state vector, $\mathbf{\Phi}$ is the eigenmatrix consists of the uncoupled acoustic and structural modes, $\mathbf{\Phi}_a^g$ is the generalized inverse of $\mathbf{\Phi}_a$ and $\mathbf{T}$ is the tranformation matrix. The subscript a and b denote the active and deleted dofs. The reduced subspace $\mathbf{X_a}(t, x_j)$ is chosen such that it contains all the master dofs and the dominant modes. The state vector consists of the structural response and the acoustic wave oscillations in the time domain. Here, x_j's represent the collocation points. Projecting Equation (4) on this subspace leads to a set of reduced order differential equations of the form,

$$\mathbf{M_a}(\theta)\begin{Bmatrix}\ddot{a}^a_m(t,\theta)\\ \ddot{b}^a_m(t,\theta)\\ \ddot{q}^a_k(t,\theta)\end{Bmatrix} + \mathbf{L_a}(\theta)\begin{Bmatrix}\dot{a}^a_m(t,\theta)\\ \dot{b}^a_m(t,\theta)\\ \dot{q}^a_k(t,\theta)\end{Bmatrix} + \mathbf{K_a}(\theta)\begin{Bmatrix}a^a_m(t,\theta)\\ b^a_m(t,\theta)\\ q^a_k(t,\theta)\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix}. \tag{6}$$

The parameter uncertainties are represented by PCE using deterministic coefficients and random basis functions. For example, the structural response in the modal domain, $q_k(t, \theta)$, can be expressed in PCE as,

$$q_k(t,\theta) = \sum_{i=0}^{k} q_k^i(t)\psi^i(\theta), \tag{7}$$

where, $q_k^i(t)$ are the deterministic coefficients and $\psi^i(\theta)$ are polynomials of random variable θ and constitute the random orthogonal basis functions. The deterministic coefficients are evaluated using a stochastic collocation approach [12,

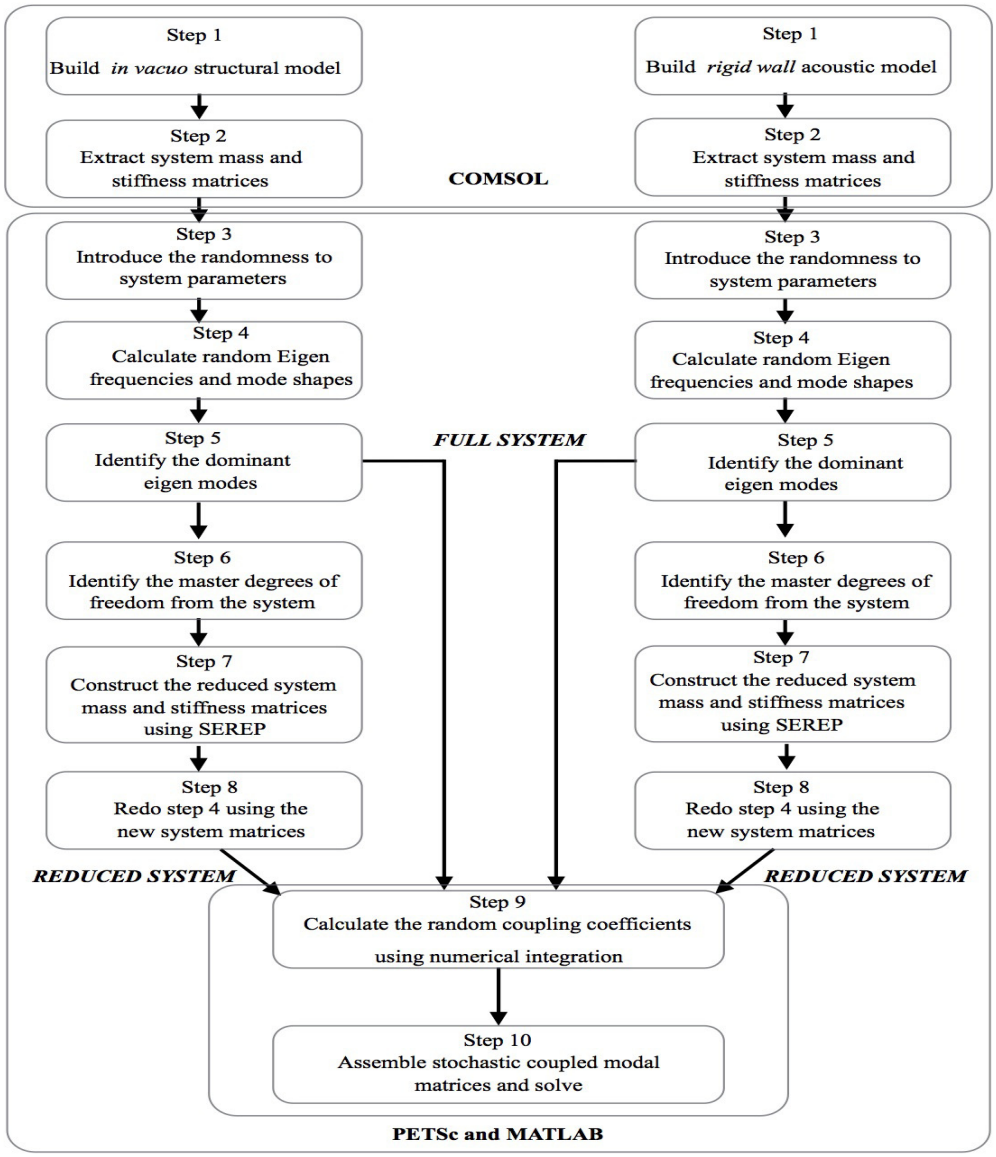


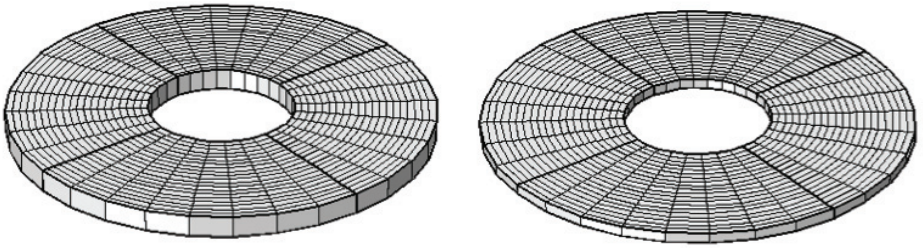
Fig. 2. A flowchart outlining the overall methodology for stochastic coupled eigenvalue problem.

13]. This involves solving the forward problem a large number of times corresponding to the tensorial grid points used in collocation. The number of such evaluations is reduced by adopting a sparse grid based approach [14]. The computational efficiency is further enhanced, by parallelizing the numerical codes using PETSc [8,9] formulation in *C*. The flowchart shown in the Fig. 2 explains the implementation of above described methodology.

3. Results and discussions

Preliminary numerical results are presented in order to validate the proposed methodology with the full system. In this preliminary study, the uncoupled eigenvalue problem for both disc and cavity are first considered. The un-

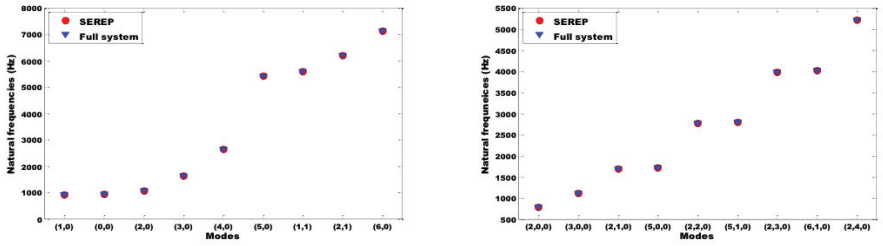
coupled disc and cavity are modelled in COMSOL with appropriate boundary conditions. The disc and the cavity are discretized into 296 edge elements, 640 hexahedral (bricks) elements, 1,424 quadrilateral elements and 16 vertex elements with total dof of 23,616 and 7,872 respectively. For the solid element, the dof at each node correspond to the displacements along the Cartesian coordinates and denoted by u, v, w, while for the cavity, modelled as fluid medium, the dof at each node represents the pressure and is denoted by p. The meshed geometries of the cavity and the disc are shown in Fig. 3.



(a) Disc (b) Cavity

Fig. 3. Meshed geometries of disc and cavity: The nodes at the interface coincide for the cavity and the disc. The radial, the circumferential and the axial direction are consisting of 41, 64 and 3 nodes respectively.

The system matrices of the full model (deterministic) are extracted separately for the disc and the cavity from COMSOL using interfacing algorithms and are used to perform the uncoupled eigenvalue analysis. Fig. 4 compares the uncoupled natural frequencies of the reduced model with the full model for the chosen modes. An inspection of these figures reveals an excellent agreement between the reduced model and the full model.

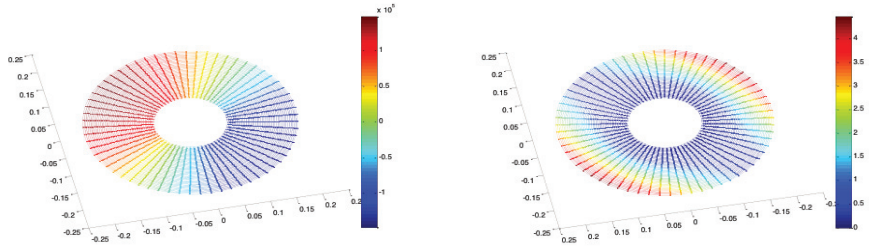


(a) Disc modes (b) Cavity modes

Fig. 4. Comparison of uncoupled natural frequencies of SEREP model with full model.

The cavity and disc mode shapes are in general expressed as (k_1, k_2, k_3) and (k_4, k_5) respectively. Here, k_1, k_4 represent the number of nodal diameters, k_2, k_5 represent the number of nodal circles and k_3 represents the z directional node number. Fig. 5 shows the uncoupled mode shapes of the $(1, 0, 0)$ cavity mode and the $(1, 0)$ disc mode. The uncoupled disc mode will couple only with uncoupled fluid mode of same nodal diameter [3]. As a consequence of this coupling rule, the full coupled eigenvalue problem can be recast into a large number of individual nodal diameter eigenvalue problems. This reduces the computational time significantly. SEREP technique is applied to this problem in order to reduce the computational time further.

In the uncoupled disc eigenvalue problem, the size of the global system matrices of the full system by excluding the boundary dofs is $23,040 \times 23,040$. At the interface, for each θ along the radial direction there are 40 dofs in the z direction for both the disc and the cavity and they are chosen as the master dofs in the SEREP model. In this study, the number of dominant modes are chosen same as that of the master dofs. Hence the size of the reduced system matrices become 40×40. On a desktop computer, the computational time taken to extract the 40 dominant modes in the full

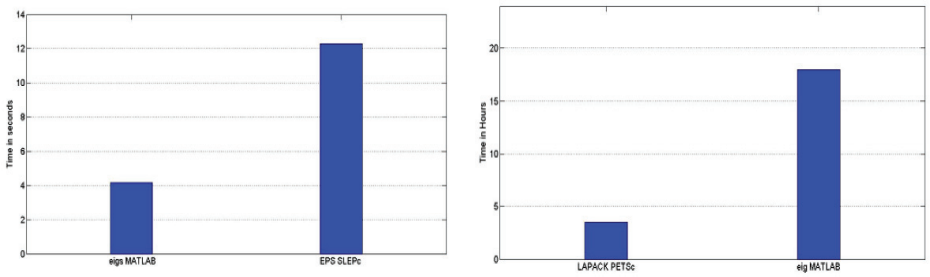


(a) Full model (1,0,0) mode shape of the cavity (b) Full model (1,0) mode shape of the disc

Fig. 5. Uncoupled mode shapes of the cavity and the disc.

model is around 13.47 seconds. The computational time for the SEREP problem comes down to just 1.283 seconds compared to the full model, without compromising on the accuracy.

In order to identify the dominant modes, it is essential to carry out an eigenvalue analysis of the full problem, which can be computationally expensive. By parallelizing the solution of the full eigenvalue problem using PETSc, the computational time is significantly reduced. The computational time taken by PETSc has been compared with that of MATLAB. Depending on whether one is interested only in a few eigenvalues or all the eigenvalues of the system, one uses different solvers. The EPS solver from SLEPc and eigs solver of MATLAB is efficient when only few eigenvalues are required for the analysis. On the other hand, if one is interested in computing the full spectrum of eigenvalues, one needs to use the LAPACK solver from PETSc or the eig solver from MATLAB. A comparison of the computational costs in estimating the eigenvalues using these solvers is shown in Fig. 6. Fig. 6(a) shows that MATLAB's eigs routine solves the system faster than EPS. This is expected as parallelization of the eigensolver used by SLEPc shows performance improvements when the size of the matrices is of the order 10^5 or higher. From Fig. 6(b), we see a huge difference between time consumed by PETSc solver and the time consumed by MATLAB solver for full spectrum eigenvalues. Though both the computational individual times are quite high, we see 80% enhancement in the performance by using PETSc based LAPACK solver for full spectrum eigenvalues.



(a) Few eigenvalues (b) Full spectrum of eigenvalues

Fig. 6. Comparison of computational time taken by MATLAB and PETSc.

4. Conclusion

A PCE based SEREP reduction strategy has been used to develop a reduced order model for the analysis of FSI system with parametric uncertainties. By means of the interfacing algorithms, the FE modelling of the uncoupled systems has been carried out using COMSOL. Subsequently, the FE system matrices of the system have been imported from the commercial software environment to carry out the stochastic coupled eigenvalue analysis. The computational complexities associated with the full eigenvalue problem have been reduced by parallelising the codes using PETSc.

There is a significant reduction in computational costs using SEREP model when compared to the full system model. Such a reduction in computational complexities is of significance in case of real FSI systems, and is necessary for carrying out further numerical investigations.

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