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Simple method of calculating dynamic set-point weighting parameters for time delayed unstable processes

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Abstract: A simple and optimal method is suggested for the calculation of the set point weighting parameter in PID controllers for unstable FOPTD systems. This method requires equating the coefficients of s and s^3 both in numerator and denominator of closed loop transfer function for a servo problem. This method gives an uncomplicated equation for the set point weighting parameters (β) and (**γ**). The performance of the proposed set point weighted controller that uses the β and **γ** is then compared with that of a method in which the β and **γ** values are obtained by two degrees of freedom controller design technique. The proposed method provides significantly improved closed loop performances when compared to the methods in the literature.

Keywords: Set point weighting, Delay systems, Process control and instrumentation, unstable systems, H_2 minimization.

1. INTRODUCTION

Because of its basic structure, the proportional-integral-derivative controller is commonly used in chemical processes. Nevertheless, time delay is inevitable in most of the chemical processes due to recycle loops and transportation delays. It is intractable for traditional PID controllers to guarantee the stability of time delay processes. In addition, it's more difficult to design the PID controller for a process that exhibits a time delay, which is open-loop unstable. Larger overshoots are obtained if the system is unstable. In such cases, set-point filter or set point weighting for PID controllers should be used to minimize the overshoot.

The IMC-PID tuning rule has only one modifiable parameter. Quite a few tuning methods have been proposed based on the direct synthesis method and the IMC method for stable and unstable processes with time delay. Garcia and Morari (1982) IMC structure yields an apparent analysis of controller design and provides a basic parametrization of all stabilizing controllers for stable or unstable processes. The development of PID tuning rules based on IMC was derived from Rivera et al. (1986). PID tuning rules improved by a first-order filter in series are derived for several classes of stable process with the aim at zero- and first-order pade approximation for time delay. Rotstein and Lewin's (1991) work can be considered as the initial investigation of an IMC-PID tuning rule for an unstable process. Time delay was ignored in their work. Lee et al. (2000) demonstrated an extension to the unstable process with time delay. They made use of Maclaurin series approximation as an alternative of a time delay approximation to the one originally derived from IMC to give an ideal controller. The partial internal model control, which is capable of controlling both stable and unstable processes was proposed by Wang et al. (2001). Enhanced design of a PID filter (i.e., a PID cascade with a lead-lag element) of cascade control systems for unstable processes with time delay was instigated by Dasari et al. (2016).

Chen et al. (2008) suggested the tuning rule for set-point weighting on the basis of the three-element control structure. In Prashanti and Chidambaram (2000), equations are proposed to calculate the set point weighting parameters for both proportional and derivative modes for unstable first order processes plus time delay systems. Sree and Chidambaram (2004) have suggested the calculation of the set point weighting parameter for unstable systems with a zero. Rao and Chidambaram (2006) further extended the work of Sree and Chidambaram (2004) to a PID controller integrated with a lead-lag compensator.

In the previously mentioned works, the set point weighting on the derivative action is not taken into consideration (i.e., the derivative weighting parameter is set to unity). Nasution et al. (2011) considered derivative mode and made use of Optimal H_2 IMC-PID controller with set point weighting for time delayed unstable processes. On the other hand, it can be demonstrated that the set-point tracking performance will be enhanced if we can determine the nearly optimal derivate mode weighting. Nasution et.al (2011) were able to

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determine that but it is highly intricate. It can be derived in a simple way by using the equating coefficient method. In this paper, a simple method is proposed to calculate set point weighting parameters for unstable first order plus time delay systems.

2. CONTROLLER DESIGN

The structure of IMC control, where $G_p(s)$ is the transfer function of the unstable process, $G_m(s)$ is the corresponding transfer function model and Q_C is the transfer function of the IMC controller is demonstrated in Fig.1. Analytical tuning method has been developed for PID controller for unstable first order plus time delay (UFOPTD) processes based on optimal H_2 framework by Nasution et al. (2011) recently. The The present method uses this methodology and hence is briefly discussed here.

$$G_p = \frac{k_p e^{-\theta s}}{\tau_p s - 1} \quad (1)$$

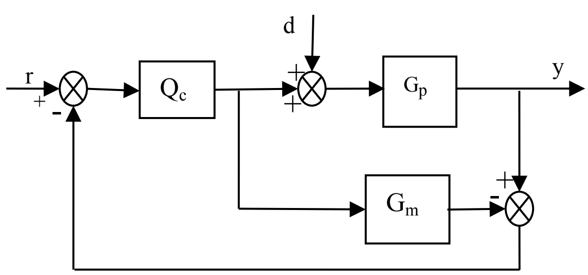

Fig.1.IMC control

Based on IMC principle, the IMC controller Q_C is equivalent to

$$Q_C = \tilde{Q}_C F \quad (2)$$

Where, F is a filter which is used for altering the robustness of the controller.

$$G_m = G_{m-} G_{m+} \quad and \quad v = v_- v_+ \quad (3)$$

Where the subscript "–"refers to minimum phase part and "+" refers to non-minimum phase part. The Blaschke product of RHP poles of G_m and v are defined as

$$b_m = \prod_{i=1}^{k} \frac{-s + p_i}{s + \bar{p}_i} \quad and \quad b_v = \prod_{i=1}^{\tilde{k}} \frac{-s + p_i}{s + \bar{p}_i} \quad (4)$$

Where p_i and $\bar{p}_i$ are the ith RHP pole and its conjugate respectively. Based on this, the H_2 optimal controller is derived by using the following formula, Morari and Zafiriou (1989).

$$\tilde{Q}_C = b_m (G_{m-} b_v v_-)^{-1} \{(b_m G_{m+})^{-1} b_v v_-\}|_* \quad (5)$$

Where {...}|* is defined as the operator that operates by omitting all terms involving the poles of $(G_{m+})^{-1}$ after taking a partial fraction expansion.

Here, F is the filter, which is selected as

$$F(s) = (\alpha s + 1)/(\lambda s + 1)^3$$

Substitute F(s) in Eq. (2), the IMC controller is obtained as

$$Q_C = \frac{(\tau_P s - 1)}{k_P} \left\{ \left(e^{\theta/\tau_P} - 1 \right) \tau_P s + 1 \right\} \frac{(\alpha s + 1)}{(\lambda s + 1)^3} \quad (6)$$

Here, λ is the closed loop tuning parameter. The value of α is obtained from the conditions of internal stability for IMC structure. The conditions to be followed for internal stability are

Condition 1: Q_C must be stable and should cancel the right half plane poles of G_m

Condition 2: $Q_C G_m$ should be stable

Condition 3: $(1-G_m Q_C)$ at the RHP poles of the process should be zero

The first two conditions are satisfied from the above design procedure and third condition can be applied as

$$(1 - Q_C G_m)|_{s=1/\tau_P} = 0$$

Substituting QC, the value of α is obtained as

$$\alpha = \left\{ \left(\lambda/\tau_P \right)^2 + 3\left(\lambda/\tau_P \right) + 3 \right\} \lambda$$

Now, this IMC controller is converted in to a unity feedback control system and the corresponding unity feedback controller G_C is obtained as

$$G_C = \frac{Q_C}{1 - Q_C G_m} \quad (7)$$

Substituting all the terms, we will get

$$G_C = \frac{\{(e^{\theta/\tau_P} - 1)\tau_P s + 1\}(\alpha s + 1)(\tau_P s - 1)}{k_P \left[(\lambda s + 1)^3 - \{(e^{\theta/\tau_P} - 1)\tau_P s + 1\}(\alpha s + 1) e^{-\theta s} \right]} \quad (8)$$

Vanavil et al. (2014) simplified the above form into a PID with lead-lag controller. In this work, it is simplified as a PID controller. To simplify this expression to a PID controller form, Maclaurin series or Laurent series can be used. To do that, let us define J(s) = s $G_c(s)$. Expand J(s) using Maclaurin series expansion to obtain the controller G_c as

$$G_c = \frac{1}{s} \left(J(0) + J'(0)s + \frac{J''(0)s^2}{2!} + ... \right) \quad (9)$$

By considering this as a PID controller in the form

$$G_c = k_c \left(1 + \frac{1}{\tau_I s} + \tau_D s \right) \quad (10)$$

$$K_c = J'(0) \quad (11)$$

$$\tau_I = \frac{J'(0)}{J(0)} \quad (12)$$

$$\tau_D = \frac{J''(0)}{2J'(0)} \quad (13)$$

3. SET POINT WEIGHTING PARAMETERS

Set point weighted PID control law is given by

$$u(t) = k_c \left[e_p(t) + (1\backslash\tau_I)\int e\,dt + \tau_D \frac{de_d}{dt}\right] \quad (14)$$

Where

$$e_p = \beta y_r - y \qquad e = y_r - y \quad e_d = \gamma y_r - y \quad (15)$$

Here y is the system output, y_r the set point, β and γ are set point weighting parameters.

Let us consider the UFOPTD process as

$$G_p(s) = \frac{[k_p e^{-\theta s}]}{[(\tau_p s - 1)]} \quad (16)$$

The controller is assumed to be a PID. The transfer function relating y to y_r is then given by

$$\frac{y}{y_r} = \frac{[k_c k_p(\beta s + m + \gamma s^2)e^{-\theta s}]}{[s(\tau_p s - 1) + k_c k_p(s + m + s^2)e^{-\theta s}]} \quad (17)$$

Due to the presence of delay term in the numerator, there is a delayed response in y. Instead, the above expression can be first considered without this term in the numerator in order to simplify the mathematical treatment. Using first order Pade's Approximation for $e^{-\theta s}$ in the denominator, Eq. (17) becomes

$$\frac{y}{y_r} = \frac{[k_c k_p(\beta s + m + \gamma s^2)(1 + 0.5\theta s)e^{-\theta s}]}{[s(\tau_p s - 1)(1 + 0.5\theta s) + k_c k_p(s + m + s^2)(1 - 0.5\theta s)]} \quad (18)$$

Eq. (18) can be rewritten as

$$\frac{y}{y_r} = \frac{[k_c k_p(m + c_1 s + c_2 s^2 + c_3 s^3)(1 + 0.5\theta s)\exp(-\theta s)]}{[-s + c_4 s^2 + c_5 s^3 + k_c k_p(m + s c_6 + s^2 c_7 + s^3 c_8)]} \quad (19)$$

Where

$$c_1 = \beta + 0.5m\theta \quad (20)$$

$$c_2 = 0.5\beta\theta + \gamma\tau_D \quad (21)$$

$$c_3 = 0.5\gamma\tau_D \quad (22)$$

$$c_4 = \tau_p - 0.5\theta \quad (23)$$

$$c_5 = -0.5\theta\tau_p \quad (24)$$

$$c_6 = 1 - 0.5\theta m \quad (25)$$

$$c_7 = \tau_D - 0.5\theta \quad (26)$$

$$c_8 = -0.5\tau_D\theta \quad (27)$$

The set point weighting parameter is selected here based on the numerator and denominator as polynomial in s and s^3 coefficients.

Let L_1 be defined as the ratio of corresponding coefficient of *s* in the numerator to that in the denominator when there is no set point weighting (β=1)

$$L_1 = \frac{[k_c k_p(1 + 0.5m\theta]}{[-1 + k_c k_p(1 - 0.5m\theta)]} \quad (28)$$

If $L_1 > 1$, then we equate the corresponding coefficient of *s* in the numerator to that in the denominator i.e.

$$k_c k_p(\beta + 0.5m\theta) = (-1 + k_c k_p(1 - 0.5m\theta)) \quad (29)$$

From Eq. (29), we get

$$\beta = 1 - \frac{1}{k_c k_p} - \frac{\theta}{\tau_I} \quad (30)$$

If $L_1 \le 1$, then equate the corresponding coefficient of *s* in the numerator to L_1 times that in the denominator i.e.

$$k_c k_p(\beta + 0.5m\theta) = L_1(-1 + k_c k_p(1 - 0.5m\theta))$$

It is found by simulation on various transfer function models that $\beta = 0.7L_1$ gives the reduced overshoot.

Let L_2 be defined as the ratio of corresponding coefficient of s^3 in the numerator to that in the denominator when there is no set point weighting (γ=1)

$$L_2 = \frac{k_c k_p[0.5\tau_D\theta]}{[0.5\theta\tau_p + k_c k_p(-0.5\tau_D\theta)]} \quad (31)$$

If $L_2 > 1$, then we equate the corresponding coefficient of s^3 in the numerator to that in the denominator i.e.

$$k_c k_p[0.5\gamma\tau_D\theta] = [0.5\theta\tau_p + k_c k_p(-0.5\tau_D\theta)] \quad (32)$$

From Eq. (32), we get

$$\gamma = -1 + \frac{\tau_p}{\tau_D k_c k_p} \quad (33)$$

If $L_2 \le 1$, then equate the corresponding coefficient of s^3 in the numerator to L_2 times that in the denominator i.e.

$$k_c k_p[0.5\gamma\tau_D\theta] = L_2[0.5\theta\tau_p + k_c k_p(-0.5\tau_D\theta)] \quad (34)$$

It is found by simulation on various transfer function models that $\gamma = 0.3L_2$ gives the reduced overshoot.

4. SIMULATION RESULTS

Simulation studies have been performed on different unstable FOPTD processes and the closed loop performances are compared with modern and more accepted PID tuning method Nasution et al. (2011). Integral of absolute error (IAE) of the controlled variable as well as total variation (TV) of manipulated variable at a particular value of maximum sensitivity are measured as performance indices for fair assessment. λ needs to be chosen appropriately for all the methods in order to compare the closed performances. In this case, λ for all the methods is selected for same value of M_s so that the desired robustness index is same. It is important to note that all the methods allow for λ to be suitably selected. By using these controller settings and using eqns. 30 and 33, the set-point weighting parameters (β and γ) are obtained.

Example-1: Let us consider the UFOPTD process as

$$G_p(s) = \frac{4e^{-2s}}{4s - 1}$$

According to Dasari et al. (2016), λ is obtained as 4.5 which corresponds to M_s Value of 2.68. Table – 1 shows the corresponding controller settings. By using these controller settings and using eqns. 30 and 33, the set-point weighting parameters are obtained. The control system is simulated by giving a unit step change in set point. The corresponding closed loop and control action response is illustrated in Fig.

2. To analyse the performances of the all the methods for robustness, perturbations of, +10% in time delay and -10% in time constant, are considered and the consequent responses are shown in Fig. 3. It can be seen that the present method provides enhanced performances with smoother control action. The corresponding IAE and TV values for perfect model condition and perturbed condition are given in Table – 1.

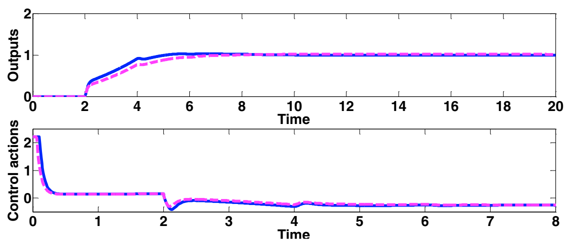

Fig. 2. Comparison of closed loop servo response for perfect condition for example 1, solid - Proposed method, dash – Nasution et.al.

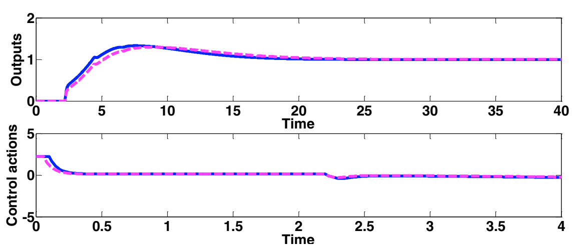

Fig. 3. Comparison of closed loop servo response for perturbations of +10% in time delay and -10% in time constant for example 1, solid - Proposed method, dash – Nasution et.al,

Example-2: Let us consider the UFOPTD process as

$$G_p(s) = \frac{e^{-1.2s}}{s-1}$$

Here, λ is selected as 3.28 corresponding to M_s value of 9. Note that smaller Ms values cannot be obtained for delay dominant unstable systems. The corresponding controller settings are given in Table – 1. By using these controller settings and using eqns. 30 and 33, the set-point weighting parameters are obtained. The control system is simulated by giving a unit step change in set point. The corresponding closed loop and control action response is shown in Fig. 4. To analyse the performances of the all the methods for robustness, perturbations of, +5% in time delay and -5%, in time constant are considered and the corresponding responses are shown in Fig. 5. It can be observed that the present method provides improved performances with smoother control action. The corresponding IAE and TV values for perfect model condition and perturbed condition are given in Table – I.

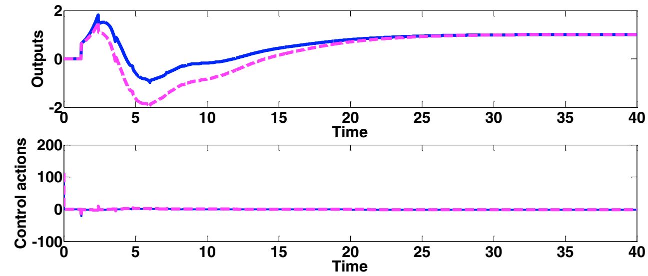

Fig. 4. Comparison of closed loop servo response for perfect condition for example 2, solid - Proposed method, dash – Nasution et.al.

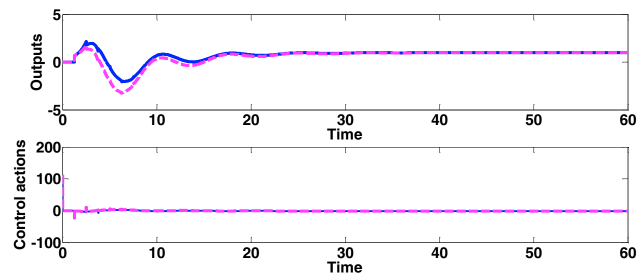

Fig. 5. Comparison of closed loop servo response for perturbations of +10% in time delay and -10% in time constant for example 2, solid - Proposed method, dash – Nasution et al.

Example-3: Let us consider the UFOPTD process as

$$G_p(s) = \frac{-5.89e^{-2.4s}}{5.86s-1}$$

Here, λ is considered as 2.56 for an M_s Value of 3.78. The corresponding controller settings are shown in Table – 1. By using these controller settings, and by making use of eqns. 30 and 33, the set-point weighting parameters are calculated. The control system is simulated by giving a unit step change in set point. The corresponding closed loop and control action response is shown in Fig. 6. To analyse the performances of the all the methods for robustness, perturbations of, -10%in process gain,+10% in time delay and -10% in time constant are considered and the corresponding responses are shown in Fig. 7. It can be observed that the present method provides robust performances with smoother control action. The corresponding IAE and TV values for perfect model condition and perturbed condition are given in Table – 1.

Example-4: Let us consider the UFOPTD process as

$$G_p(s) = \frac{e^{-0.5s}}{s-1}$$

In this case, λ is selected as 0.8 corresponding to M_s Value of 2.8. The corresponding controller settings are given in Table – I.

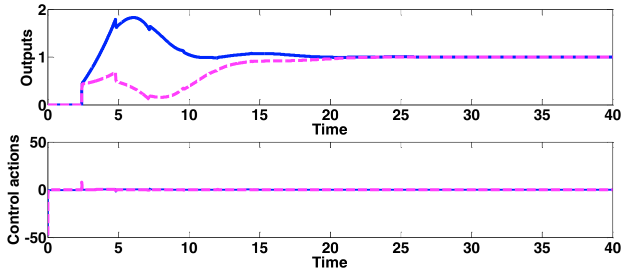

Fig. 6. Comparison of closed loop servo response for perfect condition for example 3, solid - Proposed method, dash – Nasution et.al

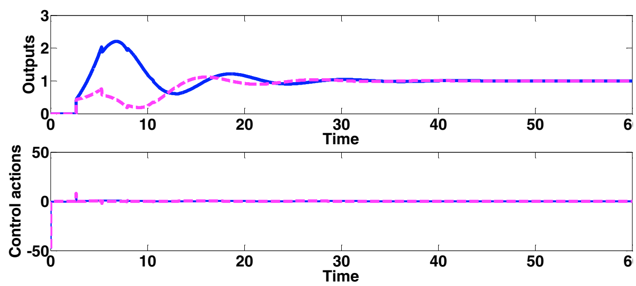

Fig. 7. Comparison of closed loop servo response for perturbations of +10% in time delay and -10% in time constant for example 3, solid - Proposed method, dash – Nasution et.al,

By using these controller settings and with the use of eqns. 30 and 33, the set-point weighting parameters are obtained. The control system is simulated by giving a unit step change in set point. The corresponding closed loop and control action response is shown in Fig. 8. To analyse the performances of the all the methods for robustness, perturbations of,-10%in process gain, +10% in time delay and -10% in time constant are considered and the corresponding responses are shown in Fig. 9. It can be observed that the present method provides more robust performances with smoother control action. The corresponding IAE and TV values for perfect model condition and perturbed condition are given in Table – 1.

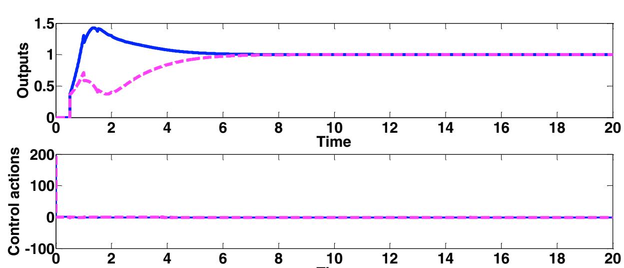

Fig. 8. Comparison of closed loop servo response for perfect condition for example 4, solid - Proposed method, dash – Nasution et.al.

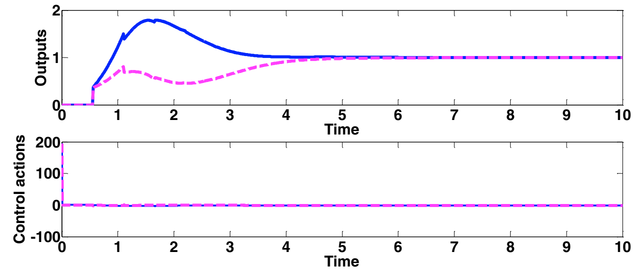

Fig. 9. Comparison of closed loop servo response for perturbations of +10% in time delay and -10% in time constant for example 4, solid - Proposed method, dash – Nasution et.al,

4. CONCLUSIONS

The proposed method is simple for the calculation of the set point weighting parameters β and γ of a PID controller for unstable first order systems with time delay. From the simulation results, it can be observed that the overshoot and undershoot are reduced considerably by the proposed method. Comparison of the proposed method with that of the Nasution et al. (2011) illustrates that, though the methods give similar performances, the proposed method is simpler to carry out as β and γ can be obtained by simple hand calculations. An additional advantage of the proposed method over existing methods is its ability to provide improved stable closed loop response even when there are large amount of perturbations in the process parameters. Quantitative comparison is carried out using IAE and TV values. Furthermore, the control action being smoother and providing low TV values which is recommended for any control system.

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Table 1. Comparison of IAE and TV for closed loop responses for different methods.

	Method	λ	k_c	τ_I	τ_D	M_s	β	γ	Perfect model		Perturbations of -10%in process gain +10% in time delay and -10% in time constant	
									IAE	**TV**	**IAE**	**TV**
Example 1 $G_p(s)=\frac{4e^{-2s}}{4s-1}$	Proposed	4.5	0.4074	35.0024	0.6099	**2.68**	0.2957	2.7257	**3**	**3.5**	**5.52**	**3.6**
	Nasution et al.	4.5	0.4074	35.0024	0.6099	**2.68**	0.3007	1.3458	**3.6**	**3.3**	**6.06**	**3.5**
											+5% in time delay and -5% in time constant	
Example 2 $G_p(s)=\frac{e^{-1.2s}}{s-1}$	Proposed	3.28	1.13	77.9	0.55	**9**	0.099	0.609	**17.83**	**213**	**19.4**	**231**
	Nasution et al.	3.28	1.13	77.9	0.55	**9**	0.09	0.371	**26.79**	**214**	**27.37**	**232**
											-10%in process gain +10% in time delay and -10% in time constant	
Example 3 $G_p(s)=\frac{-5.89e^{-2.4s}}{5.86s-1}$	Proposed	2.56	-0.48	12.57	0.9	**3.78**	0.45	1.3	**6.28**	**72**	**10**	**73**
	Nasution et al.	2.56	-0.48	12.57	0.9	**3.78**	0.378	0.451	**9.01**	**72**	**9.53**	**72.5**
											-10%in process gain +10% in time delay and -10% in time constant	
Example 4 $G_p(s)=\frac{e^{-0.5s}}{s-1}$	Proposed	0.8	1.9465	5.0235	0.1849	**2.8**	0.38	1.77	**1.42**	**235**	**1.7**	**224**
	Nasution et al.	0.8	1.9465	5.0235	0.1849	**2.8**	0.18	0.19	**2.18**	**230**	**1.9**	**220**

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