Rupture Life of Materials Obeying Exponential and Power Law Creep*

By V. M. RADHAKRISHNAN**

Synopsis

Based on the void growth and damage mechanics, the rupture time t_r is derived as a function of the creep exponent n and the reference stress σ_o and the corresponding time t_o, for materials obeying exponential and power law creep. The analysis is extended to obtain parameters for prediction of long time behaviour. For materials which obey a simple power law creep, the parameter is shown as

$$P = \log t_r + (n - n_r) \log (1 - D_c)$$

and for materials which obey an exponential function the parameter is given as

$$P = \log t_r + (n_r - n) D_c$$

where n_r is the reference creep exponent and D_c is the critical size of the void at fracture, which is related to the applied stress σ as

$$D_c = 1 - \frac{\sigma}{\sigma_o}$$

I. Introduction

The expected life of many high temperature components is around 10^5 hr and normally it will be very difficult to carry out laboratory scale experiments to determine the stress and creep rate in those time ranges. So it is usual to carry out short time tests and analyse the results to predict long time behaviour. As many as 15 parameters, notable among them being those due to Larson–Miller.[1] Sherby–Dorn[2] and Manson–Haferd[3] have been proposed to predict long time life, each giving the best correlation for the material investigated. However, no parameter is universal and what is applicable to one material fails in other cases.[4]

Based on the void growth and damage mechanics, an analysis has been presented[5] which resulted in a simple parameteric approach to creep rupture life. In the present paper the analysis is extended to different steels which obey exponential as well as power law creep.

II. Analysis

Void growth in power law creep can be both transgranular and intergranular.[6] A schematic representation of the element containing the void is shown in Fig. 1, defining the void size. The stable initial void size is D_i which grow with time. The time taken by the void of initial size D_i at nucleation to grow to any given size D is given by[5]

$$t = t_o \ln (D/D_i)(\sigma_o/\sigma)^n \quad \text{.................(1)}$$

where, t_o, σ_o: constants
exponent n: dependent on temperature
σ: the applied stress.

When the void reaches the critical size D_c the stress on the remaining area of cross-section will reach the ultimate stress σ_o at the temperature and rupture will take place. The rupture time is given by

$$t_r = t_o \ln (D_c/D_i)(\sigma_o/\sigma)^n \quad \text{.................(2)}$$

In the case of materials which obey exponential power function, the above relation can be given as

$$t_r = t_o \ln (D_c/D_i) \exp 2.3n\left(1 - \frac{\sigma}{\sigma_o}\right) \quad \text{...........(3)}$$

The constant σ_o is the reference stress, very nearly equal to the ultimate strength σ_u and the corresponding t_o is very small say of the order of 1 hr and the corresponding critical damage D_c is taken as $2.7\,D_i$. The exponent n is the inverse slope of log (σ/σ_o) *vs.* log t_r in the power law function and of (σ/σ_o) *vs.* log t_r in the exponential function relations of Eqs. (2) and (3), respectively. Typical relations between log σ and log t_r for 35–15 stainless steel and 25–20 stainless steel are shown in Figs. 2 and 3, raw data taken from Ref. 4). Over a wide range of stress and temperature, straight line relations are obtained between stress and rupture time on log–log plot in these alloys. In the second category of materials, log σ *vs.* log t_r plot will show drooping curves—a typical example is shown in Fig. 4. for A 286 steel—indicating a drastic reducting in stress level at longer rupture times.[7] For such materials, a semi-log plot of stress *vs.* log t_r will yield straight line relations. Typical examples are shown in Figs. 5, 6 and 7 for 347 stainless steel, Cr–Mo steel and Cr–Mo–V steel.[8] This is also true in the case of A 286 steel. In general, the materials can be classified as those whose deformation follows a simple power function (type A) and those that follow an exponen-

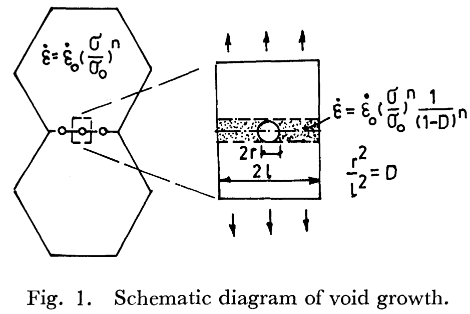

Fig. 1. Schematic diagram of void growth.

* Manuscript received on November 14, 1985; accepted in the final form on April 18, 1986.
** Metallurgy Department, Indian Institute of Technology, Madras-600036, India.

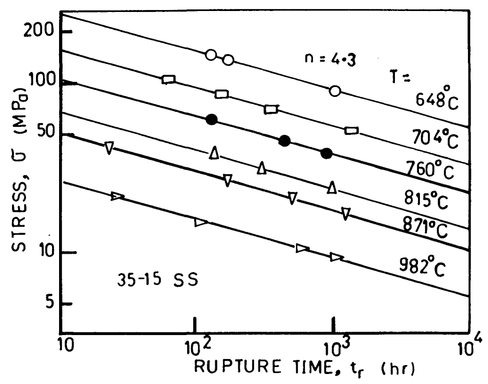

Fig. 2. Stress-rupture relation for 35–15 stainless steel.
Note the lines are parallel to each other.

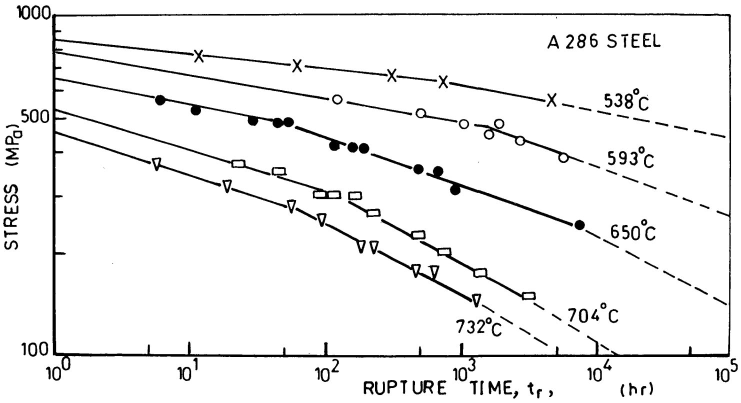

Fig. 4. Stress-rupture relation for A 286 steel.
Note the curves are drooping. The dotted lines are predicted values based on parameter.

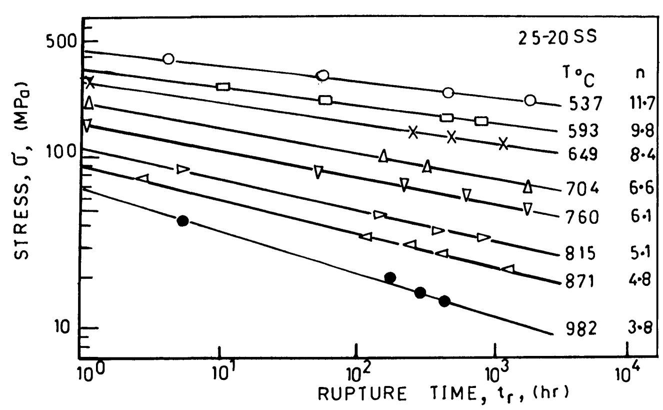

Fig. 3. Stress-rupture relation for 25–20 stainless steel.

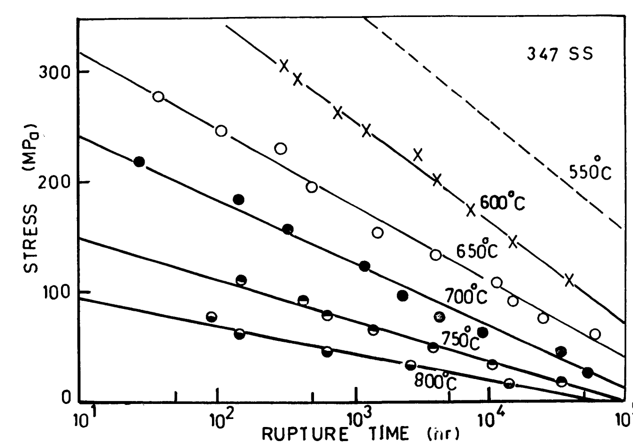

Fig. 5. Stress-rupture relation for 347 stainless steel on semi-log plot.

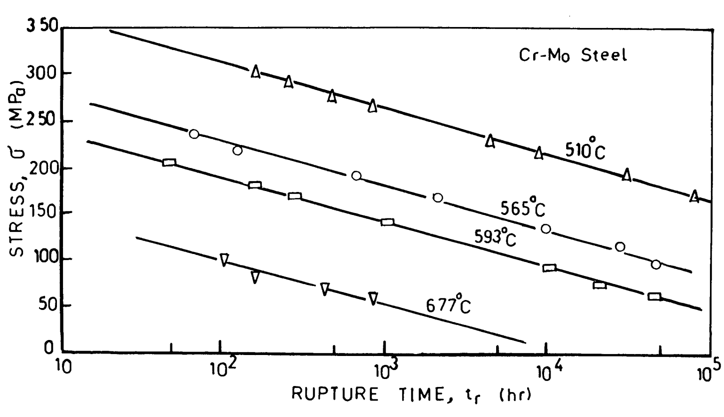

Fig. 6.
Stress-rupture relation for Cr–Mo steel on semi-log plot.

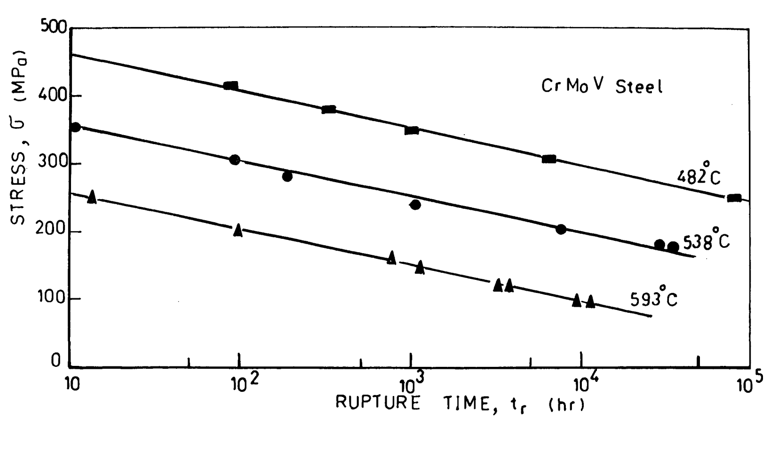

Fig. 7.
Stress-rupture relation for Cr–Mo–V steel on semi-log plot.

tial function (type B).

III. Parameter Development

Schematic representations of normalized stress (σ/σ_o) and the rupture time t_r of the two types of materials are shown in Fig. 8. Since the stress is normalised by the reference stress σ_o which is taken as the fracture stress at rupture time $t_r = t_o$, all the lines start from the same point where $D_c = 2.7\ D_i$. However, this value of D_c at t_o and σ_o will be very small compared to the critical sizes at other stress levels. It is assumed that as the void grows the net section stress gets increased and when it reaches the ultimate strength $\sigma_u(=\sigma_o)$ failure of the material takes place.[5] So we have

$$D_c = 1 - \frac{\sigma}{\sigma_0} \quad \text{..........................(4)}$$

Or at a given σ/σ_o ratio, the same critical damage results. So, for a given critical damage D_c we can write from Eqs. (2) and (3)

$$t_{r1}(\sigma_1/\sigma_{o1})^{n_1} = t_{r2}(\sigma_2/\sigma_{o2})^{n_2} \quad \text{.................(5)}$$

in the first case and

$$t_{r1} \exp 2.3n_1\left(1 - \frac{\sigma_1}{\sigma_{o1}}\right) = t_{r2} \exp 2.3n_2\left(1 - \frac{\sigma_2}{\sigma_{o2}}\right) \quad \text{..........................(6)}$$

in the second case. Using Eq. (4) in Eqs. (5) and (6) we get the parameters in the two cases as

$$P = \log t_r + (n - n_r) \log (1 - D_c) \quad \text{..............(7)}$$

for materials of type A and

$$P = \log t_r + (n_r - n) D_c \quad \text{....................(8)}$$

for materials of type B. n_r is the inverse slope of any reference line on the normalized stress *vs.* rupture time plot. On this reference line the data points of σ–t_r relation at other temperatures will be transferred. The exponent n will be intercept on the $\log t_r$ scale and will give the time if t_o is taken as 1 hr and σ_o as the rupture stress corresponding to 1 hr rupture time.

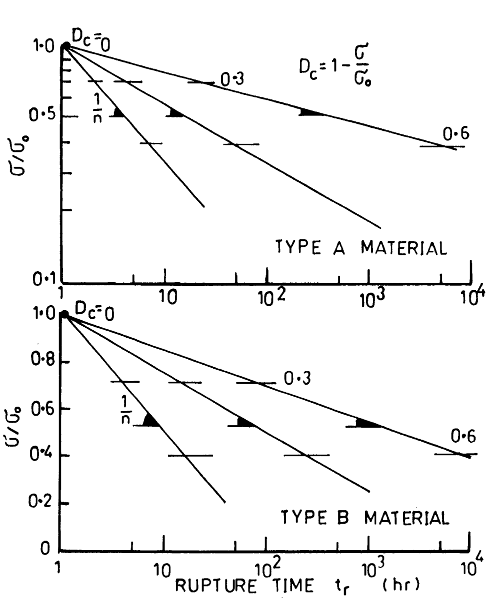

Fig. 8. Schematic representation of normalized stress and rupture time for the two types of materials.

IV. Parameter Application

Materials 35–15 stainless steel and 25–20 stainless steel obey the power law creep and hence the parameter given in Eq. (7) can be applied. Figure 9 shows the parametric correlations of the two materials. In both the cases t_o is taken as 1 h and the corresponding stress (by extrapolation) is taken as the reference stress σ_o. In the case of 35–15 stainless steel, the log σ–long t_r lines are parallel to each other with a slope $n = 4.3$. In such a case mere normalization of the stress by σ_o will give the Master Curve. In the case of 25–20 SS, the line corresponding to 871 °C temperature with $n_r = 4.8$ has been taken as the reference line and all other data points have been transferred on to the Master Curve by using Eq. (7). The correlation obtained is very good and there is no systematic error.

Figure 10 shows the semi-log plot of σ/σ_o *vs.* log t_r based on the data in Fig. 4 for A 286 steel and Fig.

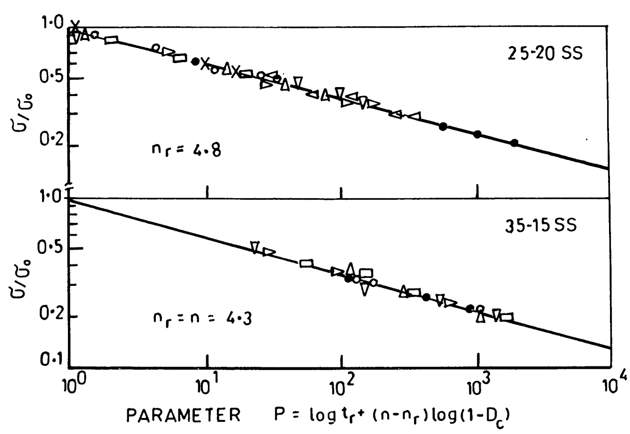

Fig. 9. Normalised stress *vs.* the parameter for 35–15 stainless steel and 25–20 stainless steel.

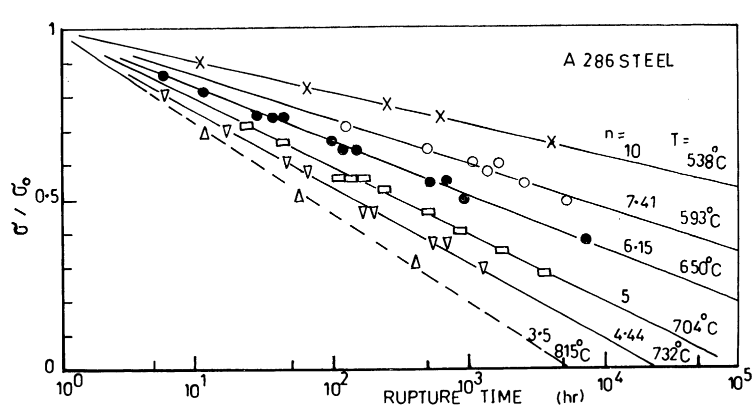

Fig. 10. Normalized stress *vs.* the rupture time for A 286 steel on semi-log plot.

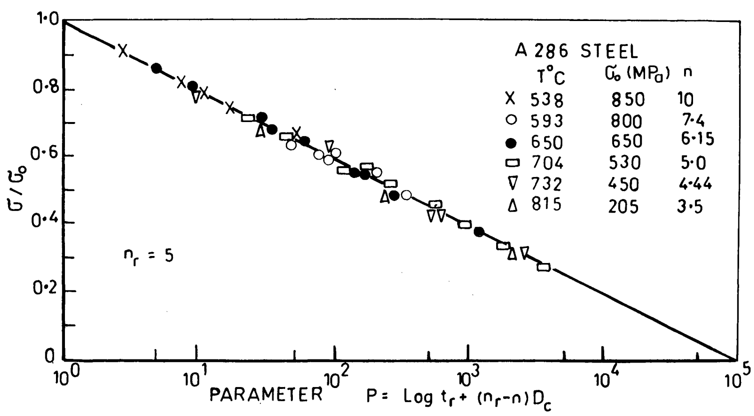

Fig. 11. Normalized stress *vs.* the parameter for A 286 steel.

11 gives the parametric correlation. So also, Figs. 12 and 13 show the parametric correlations for 347 type stainless steel and Cr–Mo–V steels. The reference n_r values are indicated in the figures. Since these four materials follow exponential function for their deformation rate, Eq. (8) will be applicable in these cases. Here again, one can see very good correlations without any scatter.

The reference n_r can be any value. It can also be chosen arbitrarily, say, as $n_r=5$ and the other data points can be transferred on to this line. The relation between the exponent n and temperature must be known for the effective application of the present parameter.

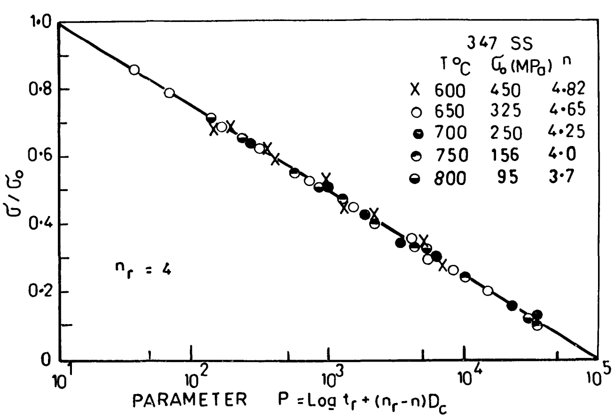

Fig. 12. Normalized stress *vs.* the parameter for 347 stainless steel.

V. Discussion

The exponent n in both the types of materials decreases with increasing temperature. This is the creep exponent which governs the deformation rate $\dot{\varepsilon}$. In the case of simple power law creep, (type A materials), the exponent can be given as

$$\frac{\delta \log \dot{\varepsilon}}{\delta \log \sigma} = n = \frac{A}{kT} \quad \cdots\cdots\cdots\cdots\cdots\cdots(9)$$

where, A: a constant dependent on temperature.

In the case of type B materials, the creep rate can be given by the exponential function in the form

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp -\frac{(Q - A^* b\sigma)}{kT} \quad \cdots\cdots\cdots\cdots\cdots(10)$$

where, $\dot{\varepsilon}_0$: a constant
Q: the activation energy
A^*: the activation area
b: the Burgers Vector
kT: its usual meaning.

Activation area A^* is the area swept by the dislocation segment of length λ between two pinning points like precipitates. It is dependent on temperature—increases with increasing temperature. Now taking the activation energy Q in the form $Q=A^*b\sigma_0$ we can rewrite the relation (10) in the form

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp -\frac{A^* b\sigma_0}{kT}\left(1-\frac{\sigma}{\sigma_0}\right) \quad \cdots\cdots\cdots\cdots(11)$$

$$= \dot{\varepsilon}_0 \exp -n\left(1-\frac{\sigma}{\sigma_0}\right)$$

with the value of $n=A^*b\sigma_0/kT$. The exponent n which is the slope of log $\dot{\varepsilon}$ *vs.* (σ/σ_0) depends on the temperature through the quantities A^*, σ_0 and T in the above equation. The reference stress $\sigma_0(\sim\sigma_u)$ decreases with increasing temperature whereas, the activation area A^* will increase with temperature. The exact nature of dependence of n on temperature will depend on the alloy. A semi-log plot of log n *vs.* $1/T$ for the materials investigated is shown in Fig. 14. A straight line relation is obtained according to

$$\log n = \frac{A_1}{T} + \log n_0$$

or

$$n = n_0 \exp \frac{Q_1}{RT} \quad \cdots\cdots\cdots\cdots\cdots\cdots(12)$$

where, n_0: the intercept
Q_1/R: the slope of the lines.

The values of these constants are given in Table 1 for the materials analysed. Once the relation between n and T has been established, the value of the exponent can be fixed up for the operating temperature of the component. Using this value of n in the parametric relation the allowable stress (σ/σ_0) can be computed. Another way of approaching the problem is like this. n is the intercept on the log t_r axis of the stress–reupture life plot, if log t_0 is taken as zero or $t_0=1$ hr. So the value of n on the X-axis against

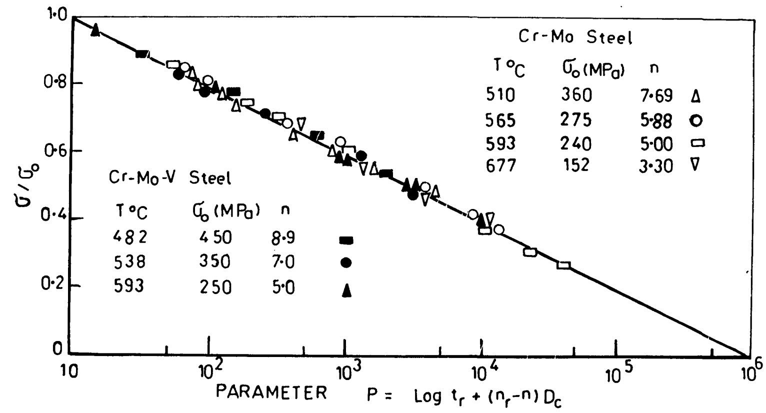

Fig. 13.
Normalized stress *vs.* the parameter for Cr–Mo and Cr–Mo–V steels.

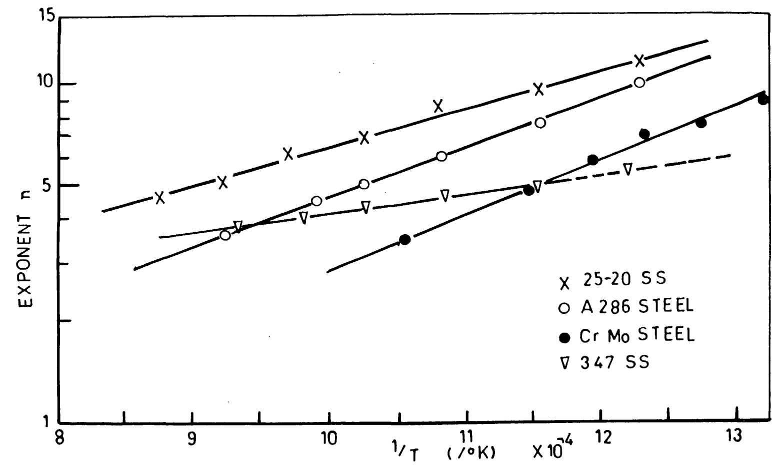

Fig. 14.
Relation between the creep exponent n and the inverse temperature $(1/T)$.

Table 1. Values of n_0 and Q_1 in Eq. (12).

Material	n_0	Q_1 (kcal/mol)
25–20 stainless steel	0.49	4 884
A 286 steel	0.193	6 087
347 type stainless steel	1.15	2 390
Cr–Mo–V steel	0.067	7 053

$\sigma/\sigma_o=0.1$ in type A material and $\sigma/\sigma_o=0$ in type B material will fix up the σ/σ_o *vs.* log t_r line (by joining this point with the point $\sigma/\sigma_o=1$ at log $t_o=0$). From this normalized stress–rupture time relation the life of the component can be computed.

VI. Conclusions

Based on void growth and damage mechanics approach, the rupture life of materials in power law creep is derived as a function of the creep exponent n and the reference stress σ_o. The analysis leads to a simple parametric approach for prediction of long time behaviour. The parameter gives a very good correlation for both type of materials which obey a simple power function and those which obey an exponential power function, without any systematic scatter.

REFERENCES

1) F. R. Larson and J. M. Miller: *Trans. ASME*, **74** (1952), 765.
2) O. D. Sherby, R. L. Orr and J. E. Dorn: *Trans. ASM*, **46** (1954), 113.
3) S. S. Manson and A. M. Haferd: NACA TN 2890 (1953).
4) F. Clauss: Engineers' Guide to High Temperature Materials, Addison Wesley Pub. Co., Mass., (1969), 280.
5) V. M. Radhakrishnan: *Scripta Met.*, **19** (1985), 259.
6) A.C.F. Cocks and M. F. Ashby: *Progress in Mat. Sci.*, **27** (1982), 189.
7) J. B. Conway: Stress-Rupture Parameters-Origin, Calculation and Use, Gordon and Breach Sci. Pub., New York, (1969), 297.
8) R. M. Goldhoff: *J. Testing and Evaluation*, **2** (1974), 387.