Plant friendly input design for system identification in closed loop

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Abstract: Optimal experiment or input design is the scientific exercise of designing informative excitation signals for the identification of a real-life dynamic system. In the least costly input design framework, the input is designed such that the identification cost is minimized while meeting desired specifications on the quality of the identified model. Identification of real-life processes require that the identification be "plant-friendly". These are typically imposed as constraints on experiment time, input and output amplitudes or input move sizes. This work focusses on an LMI based plant friendly input design in the least costly framework.

Keywords: input design; optimal experiment design, identification for control, convex optimization

1. INTRODUCTION

Optimal experiment or input design is the scientific exercise of designing informative excitation signals for the identification of a real-life dynamic system. The typical approach to this problem has been to design the power spectrum of the excitation signal in order to maximize the accuracy of the identified model (possibly with a given, say, control-oriented objective in mind) for a given experiment time and under prespecified constraints on input power (Ljung, 1999)[Chapters 12 and 13] and e.g. (Zarrop, 1979; Lindqvist and Hjalmarsson, 2000; Hildebrand and Gevers, 2003; Jansson and Hjalmarsson, 2005) and output power (Narasimhan et al., 2011).

In Bombois et al. (2004a) a new paradigm for optimal experiment design called *least costly identification experiment* for control was introduced and was further developed in Bombois et al. (2004b, 2006); Barenthin et al. (2008); Hjalmarsson (2009). In this new paradigm which is the dual of the classical approach, the goal is to design the power spectrum of the excitation signal so that the corresponding identification experiment is the *least intrusive* while guaranteeing that the identified model is sufficiently accurate for the intended application (e.g. control).

The new paradigm is quite appropriate in relation with robust control. In the least costly paradigm, the constraint on model accuracy is equivalent to imposing bounds on the size of the uncertainty region around the identified model $\hat{G}(z)$. It is clear that a large uncertainty around the model reduces the level of performance that a controller $\hat{C}(\hat{G})$ designed with this uncertain model can achieve on the unknown true system $G_0(z)$. In order words, for the controller $\hat{C}(\hat{G})$ to achieve a certain level of performance on $G_0(z)$, the modeling error must remain below a certain threshold $r_{adm}(\omega)$:

$$|\hat{G}(e^{j\omega}) - G_0(e^{j\omega})| \leq r_{adm}(\omega) \quad \forall\omega \tag{1}$$

The threshold $r_{adm}(\omega)$ in (1) can be computed using robust analysis techniques such as ν-analysis (Ferreres and Fromion, 1997a). See also Bombois et al. (2004a,b) for specific examples in the least costly setup. Note that the bound on the uncertainty of the model can be expressed in another domain than in the frequency domain. The bound can also be expressed as a bound on the covariance matrix of the identified parameter vector (see Bombois et al. (2006); Hjalmarsson (2009)). However, we here choose the frequency domain representation for its simplicity.

The definition of the cost of the identification experiment is also crucial in the least costly paradigm. In closed-loop identification, a quite realistic definition of the cost was introduced in Bombois et al. (2006). We will use the same definition in the present paper, i.e. the cost $\mathcal{J}_r$ is defined as a function of the power of the perturbations $y_r(t)$ and $u_r(t)$ induced by the excitation signal $r(t)$ on the normal operation of the closed-loop system. This cost mirrors the real cost incurred during the identification experiment.

If we suppose that the duration N of the identification experiment is fixed, the least costly input design problem can then be formulated as:

Least costly input design (fixed N). *Determine the power spectrum $\Phi_r(\omega)$ of the excitation signal $r(t)$ corresponding to the smallest cost $\mathcal{J}_r$ while guaranteeing that the model $\hat{G}(z)$ identified with this excitation signal satisfies the accuracy constraint (1) for a given threshold $r_{adm}(\omega)$.*

As shown in our previous works, this problem can be recast as a LMI optimization problem.

By its choice for the cost $\mathcal{J}_r$ of the identification experiment, the least costly framework aims at designing plant-friendly identification experiments. Indeed, Rivera et al.

(2009) define such experiments as experiments that are carried out under conditions causing minimal disruption to normal operations (see also Narasimhan and Rengaswamy (2011)). However, while limiting output excursions and input usage are directly taken into account in the least costly framework, another very important plant-friendly characteristic, i.e. constraints on the the move size of the input signal $|u(t)-u(t-1)|$ are not accounted. It is indeed a constraint expressed on the time-domain realization of the input signal while the least costly input design problem is expressed in the frequency domain (i.e. as a function of the power spectrum of the involved signals).

The absence of constraints on the move size of the input is an important drawback since a large move size increases wear and tear of actuators and valves. One solution to introduce this move size constraint is to first shape the optimal spectrum $\Phi_r(\omega)$ in either the traditional or least costly framework. This is followed by a time-domain realization which will lead to an input signal with small move size (see Godfrey (1993)). However, since $\Phi_r(\omega)$ was optimized without any consideration for the input move size, this approach is not always guaranteed to be successful.

In recent works (Narasimhan and Rengaswamy, 2011; Narasimhan et al., 2011), for the open-loop identification case, the move size constraint is relaxed by constraining the variance $\bar{E}(u(t)-u(t-1))^2$ of the move size, instead of the move size itself. This results in a convex constraint in the frequency domain and is equivalent to a weighted constraint on the input spectrum. The weighting function is theoretically equivalent to a low pass filter and hence penalizes high frequency content; leading to smoother signals.

This relaxed constraint directly accounts for the move size in the optimization problem, the solution of which results in the optimal spectrum. As mentioned above, until now, this relaxed move size constraint had only been applied for open-loop identification and had not been connected to the least costly framework. In this paper, we extend the results of Narasimhan and Rengaswamy (2011) to show that the move size constraint can be added as an extra constraint in the closed-loop least costly input design problem defined above. The resulting input design problem remains an LMI optimization problem. Moreover, using a numerical example, we illustrate the trade-offs between the conflicting requirements (minimum cost, accuracy and small move sizes).

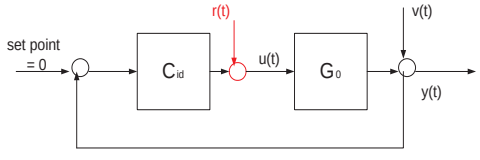

Fig. 1. Closed loop $[C_{id}\ \ G_0]$ during an identification experiment with $r(t)$ as excitation signal.

2. PREDICTION ERROR IDENTIFICATION

We consider the identification of a stable, linear, time-invariant, single input single output system with a model structure $\mathcal{M} = \{G(z,\theta),\ \ H(z,\theta)\},\ \theta \in \mathbf{R}^k$, that is able to represent the true system. Thus, the true finite-dimensional system is given by:

$$\mathcal{S}: y(t) = \overbrace{G(z,\theta_0)}^{G_0(z)} u(t) + \overbrace{H(z,\theta_0)e(t)}^{v(t)} \tag{2}$$

for some unknown parameter vector $\theta_0 \in \mathbf{R}^k$. In (2), $e(t)$ is a white noise with variance σ_e^2 and $G(z,\theta_0)$, $H(z,\theta_0)$ are stable discrete-time transfer functions. Furthermore, $H(z,\theta_0)$ is assumed to be monic and minimum-phase.

This true system is operated in closed loop with an initial controller $C_{id} : u(t) = r(t) - C_{id}(z)y(t)$. The signal $r(t)$ is zero in normal operation but is used to excite the system for a closed-loop identification experiment (see Figure 1). With $S_{id} = 1/(1+C_{id}G_0)$, the closed-loop system can be written as:

$$y(t) = S_{id}v(t) + \overbrace{G_0 S_{id} r(t)}^{y_r(t)} \tag{3}$$

$$u(t) = -C_{id}S_{id}v(t) + \underbrace{S_{id}r(t)}_{u_r(t)} \tag{4}$$

A suitable signal $r(t)$ whose spectrum $\Phi_r(\omega)$ is the design variable of our optimal experiment design problem is used to excite the system. The input-output data $Z^N = \{y(t)\ \ u(t) | t = 1...N\}$ are collected and a consistent estimate $\hat{\theta}_N$ of the true parameter vector θ_0 is obtained in the prediction error identification framework as follows:

$$\hat{\theta}_N \triangleq \arg\ \min_{\theta} \frac{1}{N}\sum_{t=1}^{N} \epsilon^2(t,\theta) \tag{5}$$

with $\epsilon(t,\theta) \triangleq H(z,\theta)^{-1}\left(y(t) - G(z,\theta)u(t)\right)$.

The identified parameter vector $\hat{\theta}_N$ is asymptotically normally distributed, $\hat{\theta}_N \sim \mathcal{N}(\theta_0, P_\theta)$ and, given the full-order model structure assumption, the covariance matrix P_θ has the following expression (Ljung, 1999): $P_\theta = \frac{\sigma_e^2}{N}\left(\bar{E}\left(\psi(t,\theta_0)\psi(t,\theta_0)^T\right)\right)^{-1}$ with $\psi(t,\theta) = -\frac{\partial \epsilon(t,\theta)}{\partial \theta}$. The asymptotic expression has been shown to be accurate for sufficiently large N, e.g., $N \geq 300$ (Ljung, 1999). The dependence of the covariance matrix P_θ on the power spectrum of the selected excitation signal $r(t)$ is evidenced by the following expression of the inverse of P_θ (Ljung, 1999):

$$P_\theta^{-1} = \left(\frac{N}{\sigma_e^2}\frac{1}{2\pi}\int_{-\pi}^{\pi} F_r(e^{j\omega},\theta_0)F_r(e^{j\omega},\theta_0)^*\Phi_r(\omega)d\omega\right) + \left(N\frac{1}{2\pi}\int_{-\pi}^{\pi} F_v(e^{j\omega},\theta_0)F_v(e^{j\omega},\theta_0)^* d\omega\right) \tag{6}$$

with $F_r(z,\theta_0) = \frac{S_{id}}{H_0}\Lambda_G(z,\theta_0)$, $F_v(z,\theta_0) = \frac{\Lambda_H(z,\theta_0)}{H_0} - C_{id}S_{id}\Lambda_G(z,\theta_0)$, $\Lambda_G(z,\theta) = \frac{\partial G(z,\theta)}{\partial \theta}$ and $\Lambda_H(z,\theta) = \frac{\partial H(z,\theta)}{\partial \theta}$. Using the asymptotic Gaussian distribution of the estimated parameter vector $\hat{\theta}_N$, it is possible to define an (additive) uncertainty region $\mathcal{D}_{r_u}(\hat{\theta}_N)$ around the identified model and containing the unknown true system $G_0(z)$ at any chosen probability level:

$$\mathcal{D}_{r_u}(\hat{\theta}_N) = \left\{G(z) \in H_\infty \mid \left|G(e^{j\omega}) - G(e^{j\omega},\hat{\theta}_N)\right| \leq r_u(\omega)\ \ \forall\omega\right\}$$

Using the following first order approximation of $G(z,\theta_0)$: $G(z,\theta_0) \approx G(z,\hat{\theta}_N) + \Lambda_G^T(z,\theta_0)(\theta_0 - \hat{\theta}_N)$ with $\Lambda_G(z,\theta)$ as defined above, $r_u(\omega)$ can be approximated as follows:

$$r_u(\omega) = \alpha \sqrt{\lambda_1 \left(T(e^{j\omega},\theta_0) P_\theta T(e^{j\omega},\theta_0)^T\right)} \quad (7)$$

where $T(e^{j\omega},\theta_0) \triangleq \begin{pmatrix} Re(\Lambda_G^T(e^{j\omega},\theta_0)) \\ Im(\Lambda_G^T(e^{j\omega},\theta_0)) \end{pmatrix} \in \mathbf{R}^{2\times k}$, $\lambda_1(A)$ denotes the largest eigenvalue of A and α is a real constant dependent on the chosen probability level: if we want $Pr(G_0 \in \mathcal{D}_{r_u}(\hat{\theta}_N)) = 0.95$, then α is chosen such that $Pr(\chi^2(k_G) < \alpha^2) = 0.95$ (k_G is the number of parameters in $G(z,\theta)$) (see Bombois et al. (2005)).

Since the true system lies in $\mathcal{D}_{r_u}$ (upto a probability level), we have that

$$|G(e^{j\omega},\hat{\theta}_N) - G_0(e^{j\omega})| \leq r_u(\omega) \quad \forall\omega \quad (8)$$

In the sequel, we will refer to $r_u(\omega)$ as the bound on the modeling error or, for the sake of simplicity, as the modeling error.

An important observation at this stage is that the modeling error $r_u(\omega)$ is a function of the covariance matrix P_θ and thus, by (6), a function of the excitation signal $r(t)$ used during the identification experiment. Obviously, given signals $r(t)$ having the same frequency content, the modelling error can be reduced by increasing the signal power. Moreover, signals $r(t)$ having the same power, but differing in frequency content will lead to different modeling errors $r_u(\omega)$. Hence, it is possible to tailor the frequency content and power of the excitation signal to achieve the desired performance.

3. OPTIMIZATION FORMULATION

In this section, we recall in a nutshell how the least costly input design problem can be formulated as a convex optimization problem and we show that adding the move size constraint also leads to a convex optimization problem.

3.1 Definition of cost

The cost of a closed-loop identification experiment can be quantified by the power $\mathcal{P}_r$ of the external signal $r(t)$. However, we define the cost such that it mirrors the actual cost incurred in the excitation of an industrial system or process. Such a process unit can be represented as in Figure 1. In normal operation the signals $u(t)$ (control signal) and $y(t)$ (the product) are given by: $y(t) = S_{id}v(t)$ and $u(t) = -C_{id}S_{id}v(t)$ ($v(t) = H_0(z)e(t)$). By applying an external signal $r(t)$ to the loop during the identification, the normal operation is affected by *disturbances* $y_r(t)$ and $u_r(t)$ as seen from (3) and (4). This results in an economic cost, e.g., loss in quality, production, higher energy consumption etc. This cost can be quantified in terms of $y_r(t)$ and $u_r(t)$:

$$\mathcal{J}_r = \beta_y \mathcal{P}_{y_r} + \beta_u \mathcal{P}_{u_r} = \frac{1}{2\pi}\int_{\pi}^{-\pi} \left(\beta_y \Phi_{y_r} + \beta_u \Phi_{u_r}\right) d\omega \quad (9)$$

where the weights β_u and β_y reflect the relative importance of the input and output perturbations. In this work, they are both chosen to be equal to one, i.e., $\beta_u = \beta_y = 1$. The cost function is clearly a linear function of the decision variable $\Phi_r(\omega)$.

3.2 Constraints

We will now formulate the accuracy constraint (1) as a function of the decision variable $\Phi_r(\omega)$. This can be done by using the relation (8) which gives a bound $r_u(\omega)$ on the modeling error between the identified model $G(z,\hat{\theta}_N)$ and the true system $G(z,\theta_0)$:

$$\alpha \sqrt{\lambda_1 \left(T(e^{j\omega},\theta_0) P_\theta T(e^{j\omega},\theta_0)^T\right)} \leq r_{adm}(\omega) \quad \forall\omega \quad (10)$$

where the covariance matrix P_θ as shown in (6), is a function of the decision variable $\Phi_r(\omega)$. The nonlinear, but convex constraint (10) can easily be recast in an affine form using Schur complements (Boyd et al., 1994). Expression (10) is indeed equivalent to: $\alpha^2 T(e^{j\omega}) P_\theta T(e^{j\omega})^T \leq r_{adm}^2(e^{j\omega}) I_2$ which in turn can be written using Schur complements as:

$$\begin{pmatrix} \frac{r_{adm}^2(e^{j\omega})}{\alpha^2} I_2 & T(e^{j\omega}) \\ T(e^{j\omega})^T & P_\theta^{-1} \end{pmatrix} > 0 \quad \forall\omega$$

The latter is also equivalent to:

$$\begin{pmatrix} P_\theta^{-1} & T^T(e^{j\omega}) \\ T(e^{j\omega}) & \frac{r_{adm}^2(e^{j\omega})}{\alpha^2} I_2 \end{pmatrix} > 0 \quad \forall\omega$$

Another application of the Schur complement now results in the following LMI (11):

$$P_\theta^{-1} \geq R_{adm}(\omega) \quad \forall\omega \quad (11)$$

with $R_{adm}(\omega) = \frac{\alpha^2}{r_{adm}^2(\omega)} T^T(e^{j\omega},\theta_0) T(e^{j\omega},\theta_0)$. The constraint (11) is now affine in the decision variable $\Phi_r(\omega)$ since P_θ^{-1} has this property (see (6)). The least costly input design defined in the introduction is thus the solution of the following convex optimization problem.

$$\min_{\Phi_r(\omega)} \mathcal{J}_r \text{ s.t. } \begin{cases} \Phi_r(\omega) \geq 0 \quad \forall\omega \\ P_\theta^{-1} \geq R_{adm}(\omega) \ \forall\omega \end{cases} \quad (12)$$

In order to address plant friendly concerns, we constrain the variance of the move size (Narasimhan and Rengaswamy, 2011):

$$\bar{E}(u(t) - u(t-1))^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |W(e^{j\omega})|^2 \ \Phi_u(\omega) d\omega \leq \xi \quad (13)$$

with $W(z) = 1 - z^{-1}$ and ξ a given constant. With (4), the constraint can be rewritten as the following affine constraint in $\Phi_r(\omega)$:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \left(|W \ S_{id}|^2 \ \Phi_r(\omega) + \sigma_e^2 |W \ C_{id} \ S_{id} \ H_0|^2\right) d\omega \leq \xi \quad (14)$$

The plant friendly optimal design problem is thus the following convex optimization problem:

$$\min_{\Phi_r(\omega)} \mathcal{J}_r$$
$$\text{s.t. } \begin{cases} P_\theta^{-1} \geq R_{adm}(\omega) \ \forall\omega \\ \frac{1}{2\pi}\int_{-\pi}^{\pi} |W \ S_{id}|^2 \left(\Phi_r(\omega) + \sigma_e^2 |\ C_{id} H_0|^2\right) d\omega \leq \xi \\ \Phi_r(\omega) \geq 0 \quad \forall\omega \end{cases} \quad (15)$$

The first issue with the optimization problems in (12) and (15) is that the number of constraints is infinite, since (11)

must hold at each frequency. Even though more elaborate solutions exist (see e.g. Bombois et al. (2004b)), a simple approach to circumvent this issue is to grid the frequency range in order to obtain a finite number of constraints. A second issue is the fact that the cost function and constraints in (12) depend on the true parameter vector θ_0 and the true noise variance σ_e^2 (via P_θ). Hence, the optimal excitation spectrum for the identification depends on the true system whose parameters have to be estimated. This is well recognized in general experiment design problems (Ljung, 1999; Antoulas and Anderson, 1999). To circumvent this issue, a short sub-optimal experiment, e.g., with white noise is performed to obtain estimates θ_{init} and $\sigma^2_{e,init}$ of the true parameter vector and noise variance respectively. The true values of θ_0 and σ_e^2, are replaced by estimates θ_{init} and $\sigma^2_{e,init}$ of respectively. For more elaborate techniques to circumvent the chicken-and-egg problem and discussion on the impact of using an estimated model in the optimization problem, we refer the reader to Bombois et al. (2006) and Gerencsér et al. (2009).

A third issue is that the decision variable, $\Phi_r(\omega)$, in the optimization problems described above (12) and (15) is infinite dimensional. Hence, in order to solve them using LMI tools such as semidefinite programming (Boyd et al., 1994), a finite linear parametrization of $\Phi_r(\omega)$ is needed. A common parametrization is the following one (Lindqvist and Hjalmarsson, 2000; Lindqvist, 2001):

$$\Phi_r(\omega) = \sum_{i=-m}^{m} c_i \; e^{j\omega i} \geq 0 \tag{16}$$

with $c_i = c_{-i}$ and m is a user defined choice. This parameterization restricts [1] the set of signals $r(t)$ to those that can be generated by a white noise of unit variance passing through an arbitrary FIR filter of length $m+1$. The parameters c_i $(i = 0, \ldots, m)$ are indeed, by construction, the finite auto-correlation sequence of the signal corresponding to $\Phi_r(\omega)$. With this choice of parametrization, the decision variables are the parameters c_i $(i = 0, \ldots, m)$ (Lindqvist and Hjalmarsson, 2000; Bombois et al., 2006). We observe that a larger m allows more flexibility in the parametrization of the spectrum and that choosing $m = 0$ is equivalent to a flat spectrum (white noise): $\Phi_r(\omega) = c_0 \; \forall \omega$.

An additional advantage of the parametrization (16) is that it is very easy to generate a time-domain input sequence $r(t)$ having the spectrum (16) (see (Lindqvist and Hjalmarsson, 2000; Lindqvist, 2001) for details).

4. NUMERICAL ILLUSTRATIONS

Consider the following ARX system (Landau et al., 1995):

[1] The positivity of the spectrum $\Phi_r(\omega)$ parametrized by (16) can be guaranteed by the existence of a symmetric matrix Q satisfying the following LMI constraint that can be added to (15) (Lindqvist and Hjalmarsson, 2000; Bombois et al., 2006):

$$\begin{pmatrix} Q - A^T Q A & C^T - A^T Q B \\ C - B^T Q A & D + D^T - B^T Q B \end{pmatrix} \geq 0$$

with the following definitions of A, B, C, D:

$$A = \begin{pmatrix} 0 & 0 \\ I_{m-1} & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & \ldots & 0 \end{pmatrix}$$
$$C = \begin{pmatrix} c_1 & c_2 & \ldots & c_m \end{pmatrix} \quad D = \frac{c_0}{2}$$

$$y(t) = G_0(z)u(t) + H_0(z)e(t) = \frac{z^{-3}B_0(z)}{A_0(z)}u(t) + \frac{1}{A_0(z)}e(t) \tag{17}$$

with $B_0(z) = 0.10276 + 0.18123z^{-1}$, $A_0(z) = 1 - 1.99185z^{-1} + 2.20265z^{-2} - 1.84083z^{-3} + 0.89413z^{-4}$, and $e(t)$ a realization of a white noise process of variance $\sigma_e^2 = 0.5$. The true system operates in closed loop with a controller C_{id} which has been designed using the 4-block H_∞ control design method of Ferreres and Fromion (1997b) and an initial estimate of the true system. This initial estimate of the true system originates from an open-loop experiment and is equal to:

$$y(t) = \frac{z^{-3}B_{init}(z)}{A_{init}(z)}u(t) + \frac{1}{A_{init}(z)}e(t)$$

with $B_{init}(z) = 0.0817 + 0.172z^{-1}$ and $A_{init}(z) = 1 - 1.9755z^{-1} + 2.1965z^{-2} - 1.8495z^{-3} + 0.8881z^{-4}$. The estimate $\sigma^2_{e,init}$ of σ_e^2 is 0.5265.

We wish to improve the performance of the controller by re-identifying a model using a direct *closed-loop* experiment of duration $N = 500$. The desired accuracy $r_{adm}(\omega)$ for the to-be-identified model is based on robust control specifications on the sensitivity function and is represented by the red curve in Figure 2. We choose $\alpha = 3.55$ to compute (7) and hence, we ensure that this bound is valid at a probability of 95 %.

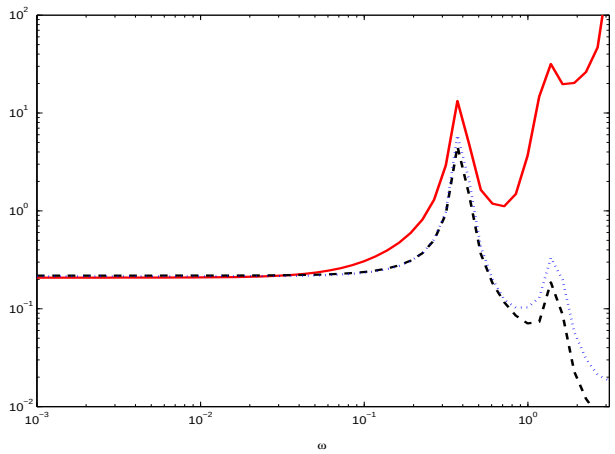

Fig. 2. Comparison of $r_{adm}(\omega)$ (red solid) with the modeling error $r_u(\omega)$ obtained with $\Phi_{r,opt,white}(\omega)$ (black dashed) and with $\Phi_{r,opt}(\omega)$ (blue dotted)

In order to optimally shape the spectrum $\Phi_r(\omega)$ of the excitation signal and obtain the spectrum inducing the lowest cost $\mathcal{J}_r = \mathcal{P}_{u_r} + \mathcal{P}_{y_r}$, we consider the least costly design problem (12). The parametrization for the power spectrum $\Phi_r(\omega)$ is chosen as in (16) with $m = 10$. The optimal spectrum $\Phi_{r,opt}(\omega)$ with $m = 10$ is depicted in Figure 3. The cost $\mathcal{J}_r$ incurred by the application of an external signal having the spectrum $\Phi_{r,opt}(\omega)$ is equal to 10. If we would have limited ourselves to white noise, i.e., flat spectrum with $m = 0$ and solved (12), we would obtain the optimal spectrum $\Phi_{r,opt,white}(\omega)$ which corresponds to an induced cost $\mathcal{J}_r = 22$ (approximately two times larger than with the flexible spectrum).

Realizations $r(t)$ of length $N = 500$ of the two spectra $\Phi_{r,opt}(\omega)$ and $\Phi_{r,opt,white}(\omega)$ are generated. These signals

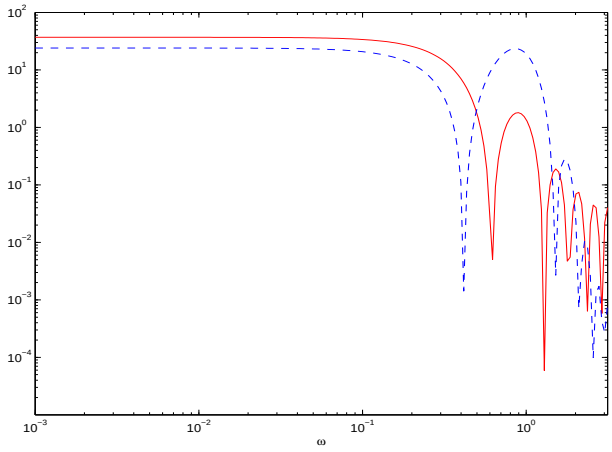

Fig. 3. Flexible spectrum $\Phi_{r,opt}(\omega)$ without move size constraint (blue dashed) and flexible spectrum $\Phi_{r,opt,move}(\omega)$ with move size constraints (red solid)

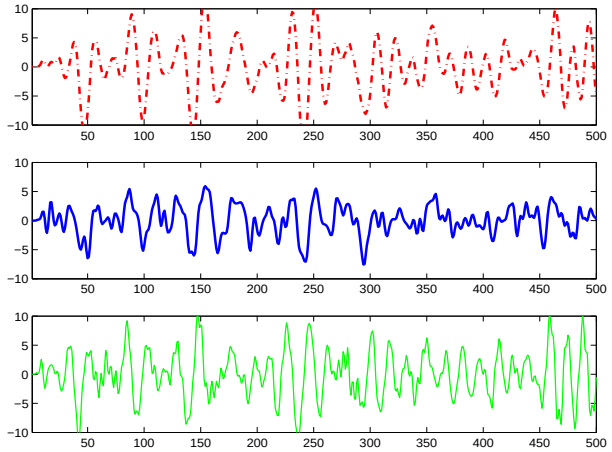

Fig. 4. Induced y_r with with $\Phi_{r,opt,move}(\omega)$ (red, top), with $\Phi_{r,opt}(\omega)$ (blue, middle), and with $\Phi_{r,opt,white}(\omega)$ (green, bottom)

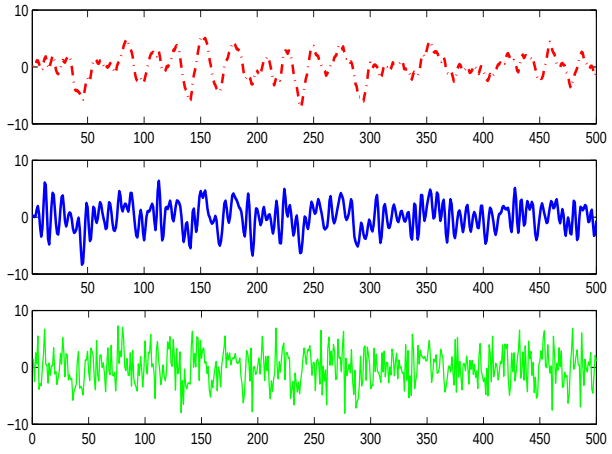

Fig. 5. Induced u_r with $\Phi_{r,opt,move}(\omega)$ (red, top), with $\Phi_{r,opt}(\omega)$ (blue, middle), and with $\Phi_{r,opt,white}(\omega)$ (green, bottom)

$r(t)$ have been applied to the closed-loop $[C_{id}\ G_0]$ and the induced disturbances $y_r(t)$ and $u_r(t)$ corresponding to these two signals $r(t)$ are depicted in Figures 4 and 5, respectively. We observe that the induced perturbation $y_r(t)$ is much smaller when $\Phi_{r,opt}(\omega)$ is used, i.e. when the spectrum is shaped appropriately (the perturbation u_r are similar in both cases). Thus, this corroborates the substantially higher cost incurred in using white noise as the excitation signal as compared to the optimal input. The extra flexibility that is available is used to reduce the cost while meeting the desired constraints.

We have also used the two realizations of $r(t)$ to identify a model using direct closed-loop identification and we observe that, as expected, the bound $r_u(\omega)$ on the modeling error of these two models satisfy the accuracy threshold $r_{adm}(\omega)$ (see Figure 2). However, recall that the signal corresponding to $\Phi_{r,opt}(\omega)$ induced a performance degradation $\mathcal{J}_r$ which is two times smaller than the performance degradation induced by one corresponding to $\Phi_{r,opt,white}(\omega)$.

Now, consider the plant friendly design problem. When $\Phi_{r,opt}(\omega)$ is used, the move size variance $\bar{E}(u(t)-u(t-1))^2$ is 2.7. The smallest value for ξ such that (15) is feasible is 0.45. Hence, to uncover the trade-off between cost and the move size, it is sufficient to consider $\xi \in [0.45, 2.76]$. The optimal spectrum obtained by solving (15) with $\xi = 0.45$ and $m = 10$ is shown in Figure 3. The move size constraint (essentially a filter) penalizes high frequency contents of the signal as seen in Figure 3. The cost function $\mathcal{J}_r$ is for $\Phi_{r,opt,move}(\omega)$ equal to 20.

Hence, with the constraint on the move size, in order to obtain an identified model of reasonable accuracy, the system has to be excited in a way which results in a higher cost, and specifically in an higher value of $\mathcal{P}_{y_r}$. The increase in output perturbation y_r is evident in Figure 4 where the output perturbation y_r with $\Phi_{r,opt,move}(\omega)$ is in red. However, the extra constraint results in a signal u_r which is much smoother (as expected) as seen in Figure 5 where the resulting u_r with $\Phi_{r,opt,move}(\omega)$ is in red. Thus, while the input is friendlier (i.e., the input is smoother) as compared to the one corresponding to the optimal spectrum $\Phi_{r,opt}(\omega)$, the cost is higher. As compared to white noise, it is friendlier and also results in a lower cost.

In order to find the optimal trade-off between $\mathcal{J}_r$ and the move size of $u(t)$, let ξ vary in $[0.45, 2.76]$. An acceptable trade-off between the cost and input friendliness can be found with $\xi = 1$ which leads to an optimal spectrum $\Phi_{r,opt,move,\xi=1}(\omega)$ corresponding to $\mathcal{J}_r = 14$. The induced perturbations y_r and u_r for this $\Phi_{r,opt,move,\xi=1}(\omega)$ are depicted in red in Figures 6 and 7, respectively and compared with the ones induced by $\Phi_{r,opt}(\omega)$ and $\Phi_{r,opt,white}(\omega)$.

5. CONCLUSIONS

We presented an optimal input design problem for identification in closed loop in the least costly framework with constraints on the input move size and desired model accuracy. The trade-off between the identification cost and input move size variance was quantified. The move size constraint restricts the spectrum of the excitation signal

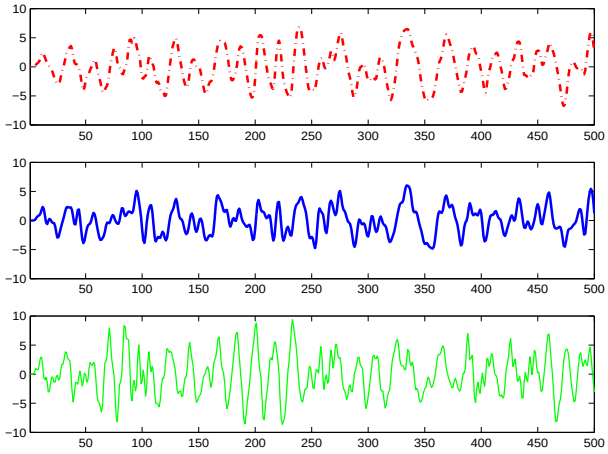

Fig. 6. Induced y_r with $\Phi_{r,opt,move,\xi=1}(\omega)$ (red, top), with $\Phi_{r,opt}(\omega)$ (blue, middle) and with $\Phi_{r,opt,white}(\omega)$ (green, bottom)

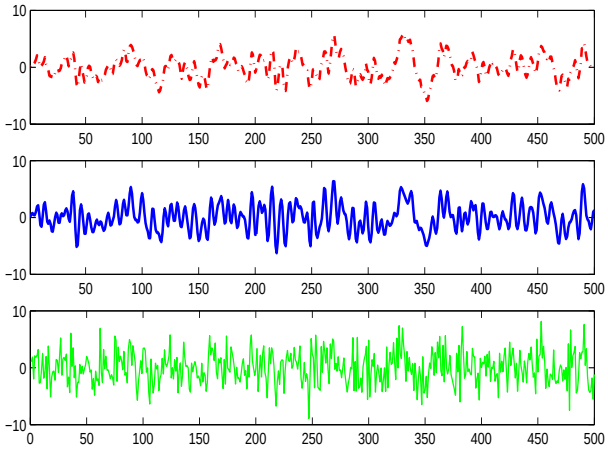

Fig. 7. Induced u_r with $\Phi_{r,opt,move,\xi=1}(\omega)$ (red, top), with $\Phi_{r,opt}(\omega)$ (blue, middle) and with $\Phi_{r,opt,white}(\omega)$ (green, bottom)

and hence the system is excited to a larger extent in order to obtain a model of sufficient accuracy.

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