

Parametric optimization of high aspect ratio wing using surrogate model

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Abstract: Wing is an important component of airborne glide vehicle which provide the major portion of lift force to be generated and at the same time lift is to be produced with minimum drag. In this paper an optimum design is evolved to provide a high aerodynamically efficient wing in high subsonic regime and at the same time it is subjected to constraints like, it should provide the minimum lift (lateral acceleration as per the guidance demand) to perform flight manoeuvres. The main objective of the work is to maximize the Lift-to-Drag ratio of the wing at a design point subjected to minimum lift requirement. This aerodynamic efficiency (L / D) decides the range performance of a glide vehicle. Wing parameters like span, chord at root, taper ratio and sweep back angle are subjected to optimizer with limits to obtain the best design while the airfoil $NACA65_1 - 412$ is same for all the cases. This paper presents the study carried out using design of experiments, surrogate model, evolutionary optimization approach and gradient based optimization approach using MATLABTM solver.

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Keywords: optimization, design exploration, CFD, surrogate model, L / D ratio

NOMENCLATURE

b	-	Wing Span
CA	-	Axial force coefficient
CD	-	Drag coefficient
CL	-	Lift coefficient
CN	-	Normal force coefficient
Cr	-	Root chord
Ct	-	Tip chord
L_{ref}	-	Reference length
$\frac{L}{D}$	-	Lift to Drag ratio
M_∞	-	Free Stream Mach number
P_∞	-	Free Stream Static Pressure
S_{ref}	-	Reference Area
T_∞	-	Free Stream Static Temperature
X_{LE}	-	Wing Leading edge location from nose tip
α	-	Angle of attack
ρ_∞	-	Free stream density
λ	-	Wing taper ratio (= Ct / Cr)
Λ_{LE}	-	Leading Edge Sweepback angle

1. INTRODUCTION

The objective of this paper is to maximize aerodynamic efficiency (minimize $-\frac{L}{D}$) of the airborne vehicle configuration in turn to maximize down range of flight. The aerodynamic efficiency is mainly a function of the size, shape of the vehicle parts (body, lifting surface, control surface) and its relative locations.

1.1 Problem Description

The problem is defined as follows.

Min ($-\frac{L}{D}$) (i.e. Maximize $\frac{L}{D}$) such that

- 2500 N – Lift ≤ 0
- WingArea – $0.4m^2 \leq 0$

The design variables for varying the design space are given with lower & upper bounds as follows.

x(1) - Wing root chord $0.15 \leq Cr \leq 0.25$ m

x(2) - Wing Taper Ratio $0.2 \leq \lambda \leq 1.0$

x(3) - Wingspan $0.85 \leq b \leq 1.25$ m

x(4) - Wing leading edge sweep $0^\circ \leq \Lambda_{LE} \leq 40^\circ$

These design variables are shown in Figure 1

2. SOLUTION PROCEDURE

Based on the literature survey and prior experience on glide bombs the following design point for the current work is chosen. All the cases were simulated for this design point.

- $M_\infty = 0.7$
- $\alpha = 5^\circ$
- Altitude = 5000 m
- $P_\infty = 55436$ Pa
- $T_\infty = 270.65$ K

2.1 Design Of Experiments (DOE)

Design of Experiments has been carried out to find more sensitive parameter which affects the aerodynamic efficiency. Optimal Latin Hypercube (OLH) technique is used to generate the matrix of input parameters in combination required to characterize the wing behaviour with minimum number of runs. The combination of design variables are generated using MATLABTM. The following table shows the 51 cases for which aerodynamic characteristics are generated. Computational Fluid Dynamics (CFD) simulations have been carried out to estimate the aerodynamic characteristics of the wing at design point given below for 51 cases. The following configuration is used for baseline model which is shown in Figure 2.

- $C_r = 0.25$ m
- $\lambda = 0.8$
- $b = 1$ m
- $\Lambda_{LE} = 12.75^\circ$
- Airfoil = $NACA65_1 - 412$ (Ira H. Abbott (1959))

Meshing

An unstructured hexahedral meshing (Numeca HexpressTM) is used for creating volume mesh over the wing geometry with no-slip wall boundary condition and a pressure farfield for the free stream boundary. Initially a grid independency study on ONERA M6 Wing with Coarse, Medium, Fine, Extra fine grids is carried out for transonic regime validation and grid adequacy is finalised. (Kandasamy.S (2012)) Approximately 2 million cells are created to capture the flow conditions accurately. The surface mesh on wing geometry is shown in Figure 3.

Computational Fluid Dynamics (CFD)

The code used is ANSYSTMFLUENT which is a 3D, implicit, compressible RANS (Reynolds Averaged Navier Stokes) code with $k-\omega$ SST turbulence model. The governing equations of mass, momentum & energy conservation are solved up to a residual value of 10^{-5} . The converged steady state results are obtained approximately after 6000 iterations. The y^+ value of the converged solution is maintained below 1 ($y^+ < 1$) (Kandasamy.S (2012)). The results of the CFD simulations are the aerodynamic force coefficients in axial (C_A) and normal (C_N) directions.

From the coefficients L / D ratio is derived as,

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_N * \cos(\alpha) - C_A * \sin(\alpha)}{C_A * \cos(\alpha) + C_N * \sin(\alpha)}$$

The baseline configuration provides a L / D value of 17.84 at the design point.

Figure 4 shows the cross plots for OLH points on four dimensional (4-D) design space. Table 2.1 shows the variation of design variables using OLH technique.

The design of experiments study provided the L/D ratio (objective function) as a function of design variables (C_r , λ , b , Λ_{LE}) in the complete design space. Also the sensitivity of each parameter on the objective function is also observed.

Table 1. Design variables variation using OLH technique

Case #	C_r	λ	b	Λ_{LE}
1	0.25	0.80	1	12.75
2	0.16	0.62	0.90	31.02
3	0.18	0.33	0.96	0
4	0.19	0.53	1.14	18.78
5	0.16	0.89	1	20.41
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49	0.22	0.62	1.06	36.73
50	0.20	0.98	0.96	14.69
51	0.22	0.92	1.13	19.59

2.2 Polynomial Response Surface (PRS) - Quadratic fit

The objective function is fitted using a polynomial regression model with quadratic variation (2nd order) using 15 coefficients (Vladimir O. Balabanov (2014)) and plotted in Figure 5.

$$f(x) = k_0 + k_1 * x_1 + k_2 * x_1^2 + k_3 * x_2 + k_4 * x_2^2 + k_5 * x_3 + k_6 * x_3^2 + k_7 * x_4 + k_8 * x_4^2 + k_9 * x_1 * x_2 + k_{10} * x_1 * x_3 + k_{11} * x_1 * x_4 + k_{12} * x_2 * x_3 + k_{13} * x_2 * x_4 + k_{14} * x_3 * x_4$$

where x_1 , x_2 , x_3 , x_4 are design variables C_r , λ , b , Λ_{LE} respectively.

Error analysis (Christian Alba (2017)) carried out and error metrics obtained are within limits.

2.3 Optimization

Optimization techniques are used to estimate a set of design variables in combination to provide the best objective functional value. In this paper the optimization process has been carried out in two approaches for comparing the results from different methodologies.

- Gradient based algorithm
- Evolutionary algorithm

Gradient based algorithm (fmincon solver)

For gradient based optimization algorithm, fmincon solver (constrained nonlinear minimization (Matlab (2013))) using the interior-point algorithm in MATLABTM was used with an initial condition corresponding to baseline configuration (case1). It uses the derivative (gradient) information for obtaining the direction of optimum point. The optimum design point is obtained subjected to the constraints given in the problem description. The optimizer is tested with multiple starting point options. It is also observed that due to the quadratic nature of the PRS fit, the same optimum solution is obtained irrespective of the starting point on the design space.

The results for one such starting point is shown in Figure 6. The first figure shows the final optimized design variables of the result. Second shows the function evaluations for each iteration. Third gives the trend of functional value with iterations.

Evolutionary algorithm (Genetic Algorithm)

Genetic Algorithm (GA) is a global optimization algorithm. It uses the methods of population (generations), cross over and mutation where all of the steps are performed in a random selection. So the final value of optimum solution is improved based on the individual fitness function value over generations. Unlike gradient based algorithms GA takes more time if the population size is higher.

The following GA parameters were used to perform optimization on surrogate model.

- Population Size = 40
- Cross over rate = 0.4
- Mutation rate = 0.005

3. RESULTS & DISCUSSIONS

It is observed from Figure 7 that both GA solver & *fmincon* solver converge to the same final optimum solution since the fit is of quadratic nature. The first figure shows the function fitness value over generations and second shows the optimized design variable values for final generation result. It is also observed from both solutions that the design variables b and Λ_{LE} are fixed to a value of 1.25 and 40 respectively. The design variables C_r & λ are observed to be sensitive.

The final optimum solution details are given as follows.

Table 2. Comparison of *fmincon* & GA results

Design Variable	Baseline	<i>fmincon</i>	GA
C_r	0.25	0.1887	0.1920
λ	0.8	0.696	0.664
b	1	1.25	1.25
Λ_{LE}	12.75	40	40
$\frac{L}{D}$	17.84	22.46	22.46

The same result is graphically visualized in Figure 8. The black lines show the objective function value, the red and green lines show the lift and area constraints respectively and the point shows the optimum combination of design variables. It can be observed that the objective function is restricted by the area constraint.

4. CONCLUSION

The final result gives a wing with maximized L / D ratio of 26% more than the baseline design. An optimized wing design has been achieved to get the best aerodynamic efficiency subject to the constraints and an effective procedure is established.

Comparison is made between gradient based and evolutionary optimization algorithms performed on surrogate model and both correlate very closely. If the polynomial fit is carried out with higher order terms, then we may expect more local minima points of the cost function. i.e. In gradient based algorithm, different optimum points for various initial guess but however the GA solver will find the global optimum point.

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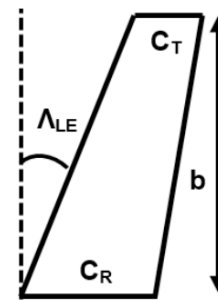


Fig. 1. Wing geometry with design variables

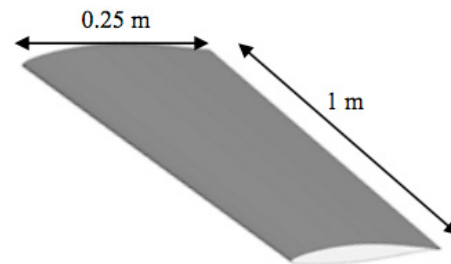


Fig. 2. Wing CAD geometry for baseline model

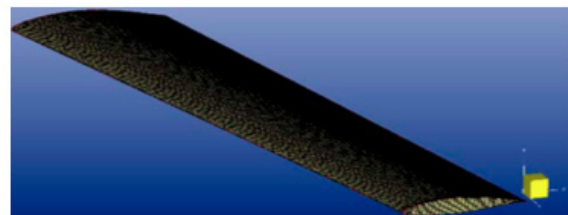


Fig. 3. Surface mesh on wing baseline model

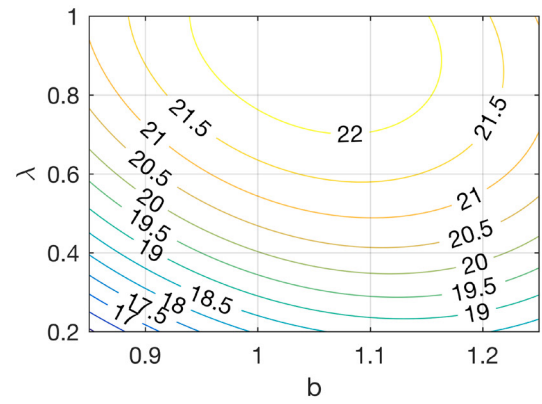
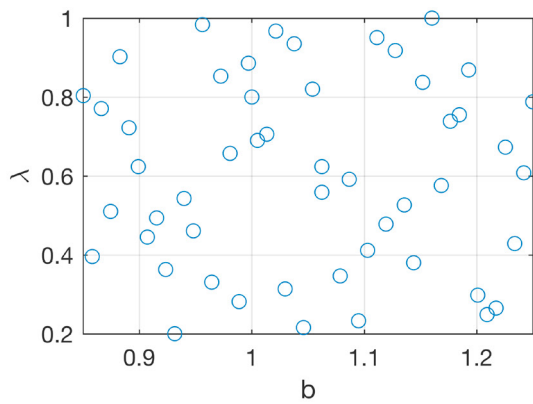
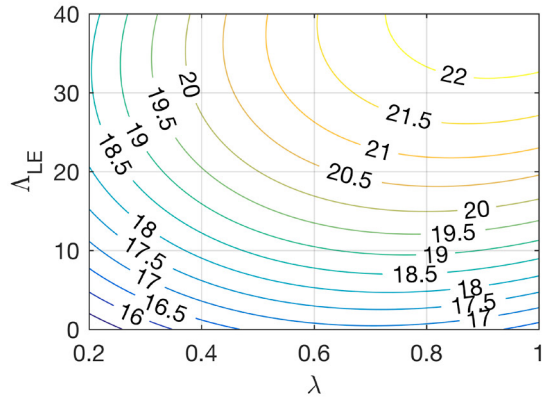
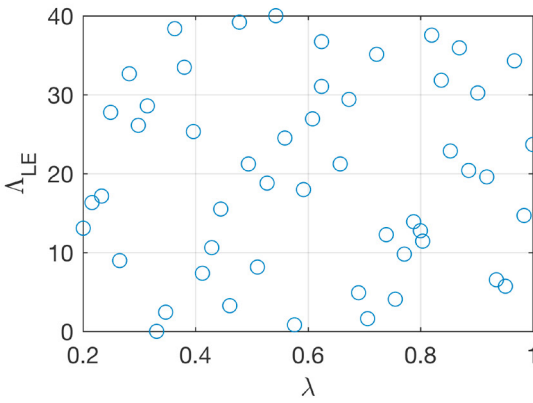
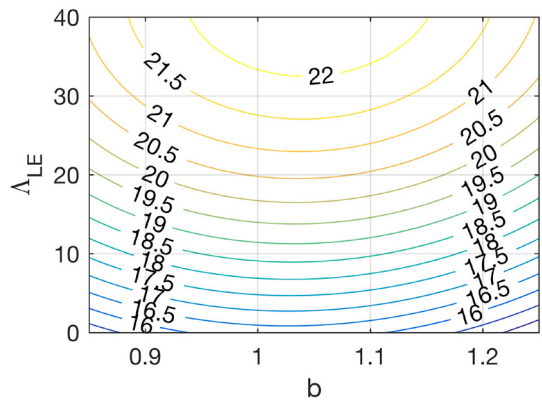
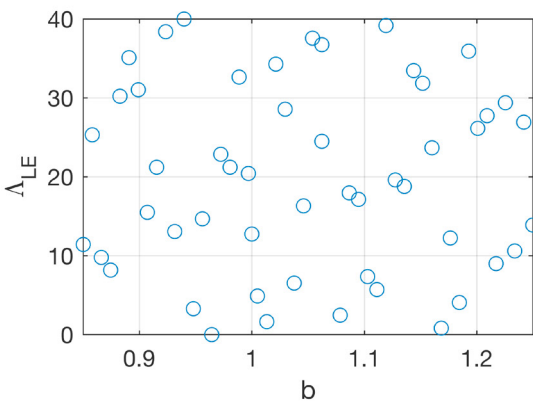
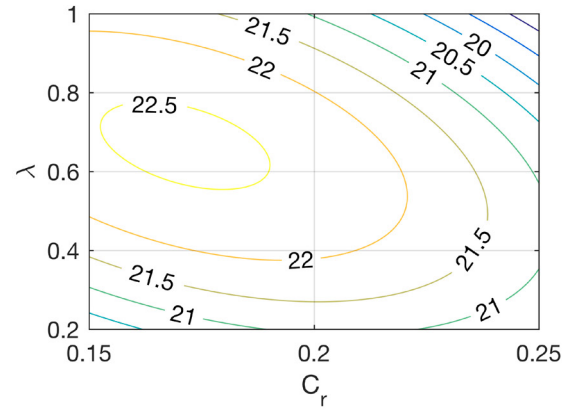
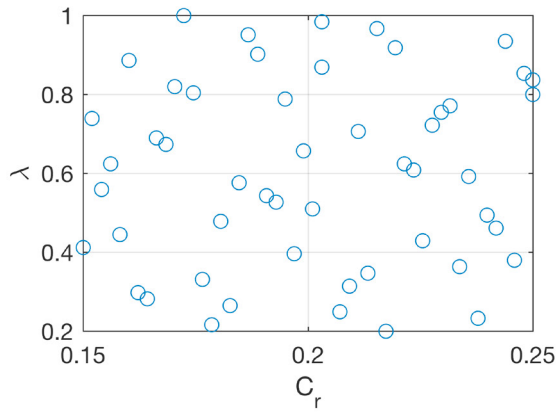


Fig. 4. Cross plots of design variables (OLH)

Fig. 5. Polynomial regression model (Quadratic fit) of $\frac{L}{D}$

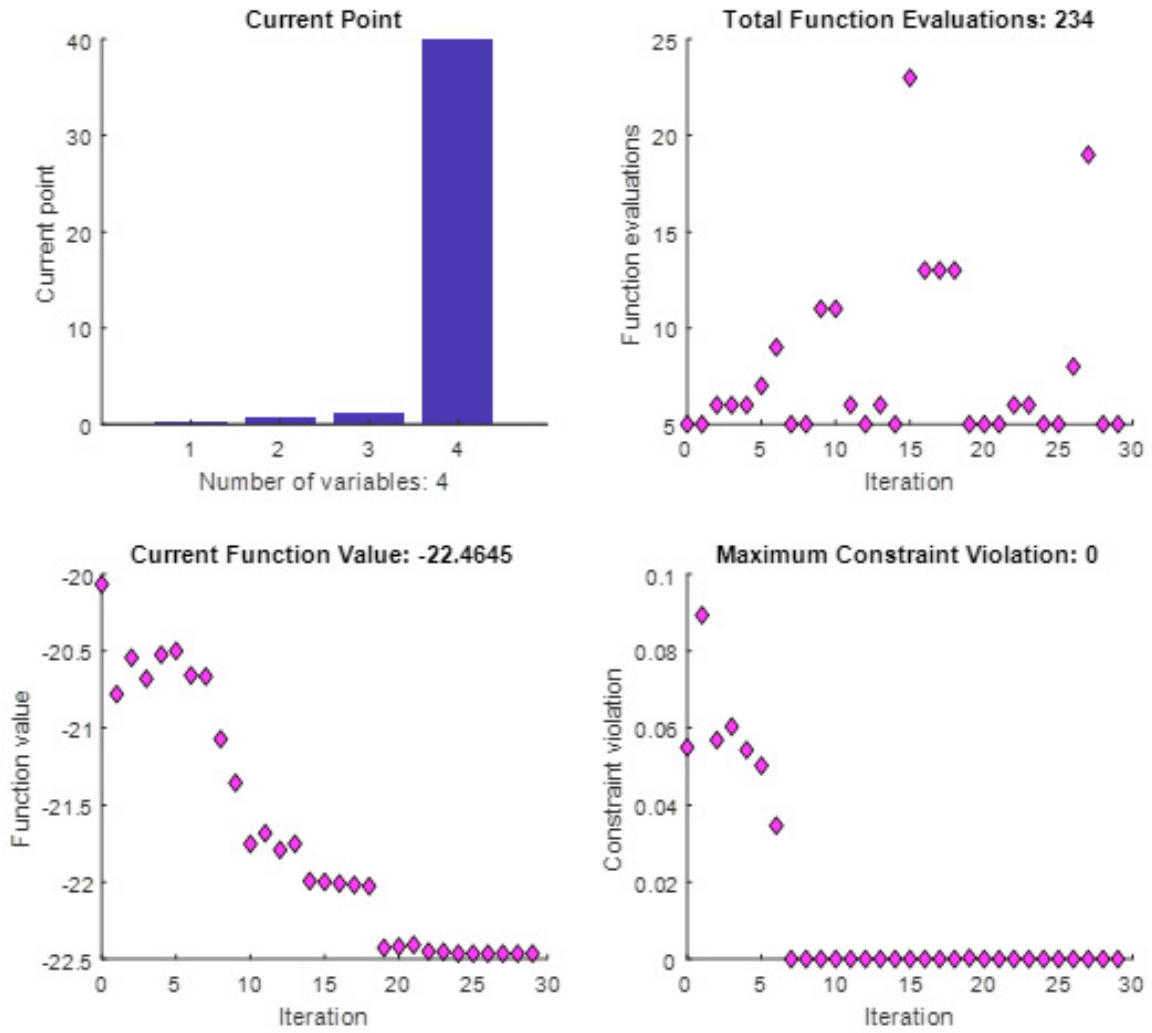


Fig. 6. Results of *fmincon* algorithm

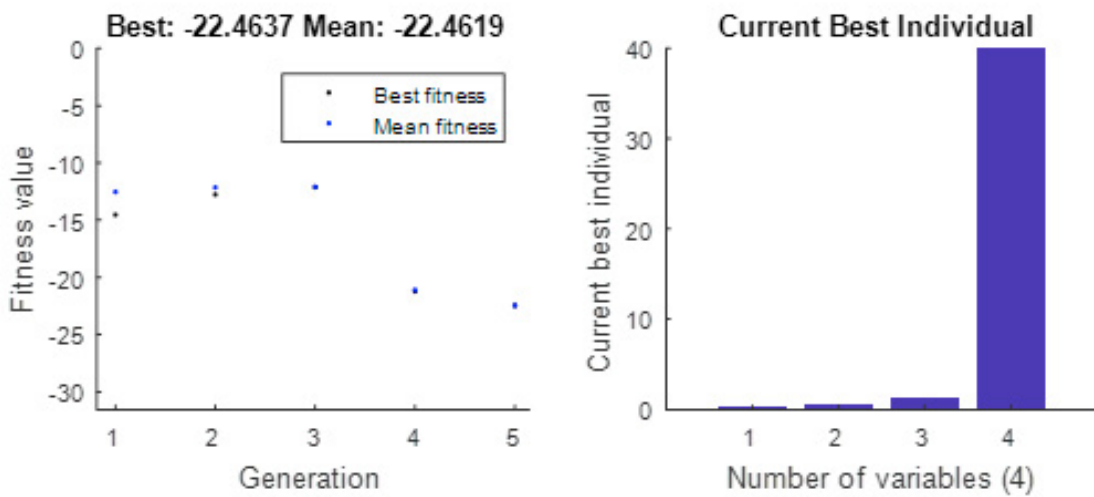


Fig. 7. Results of *GA* program

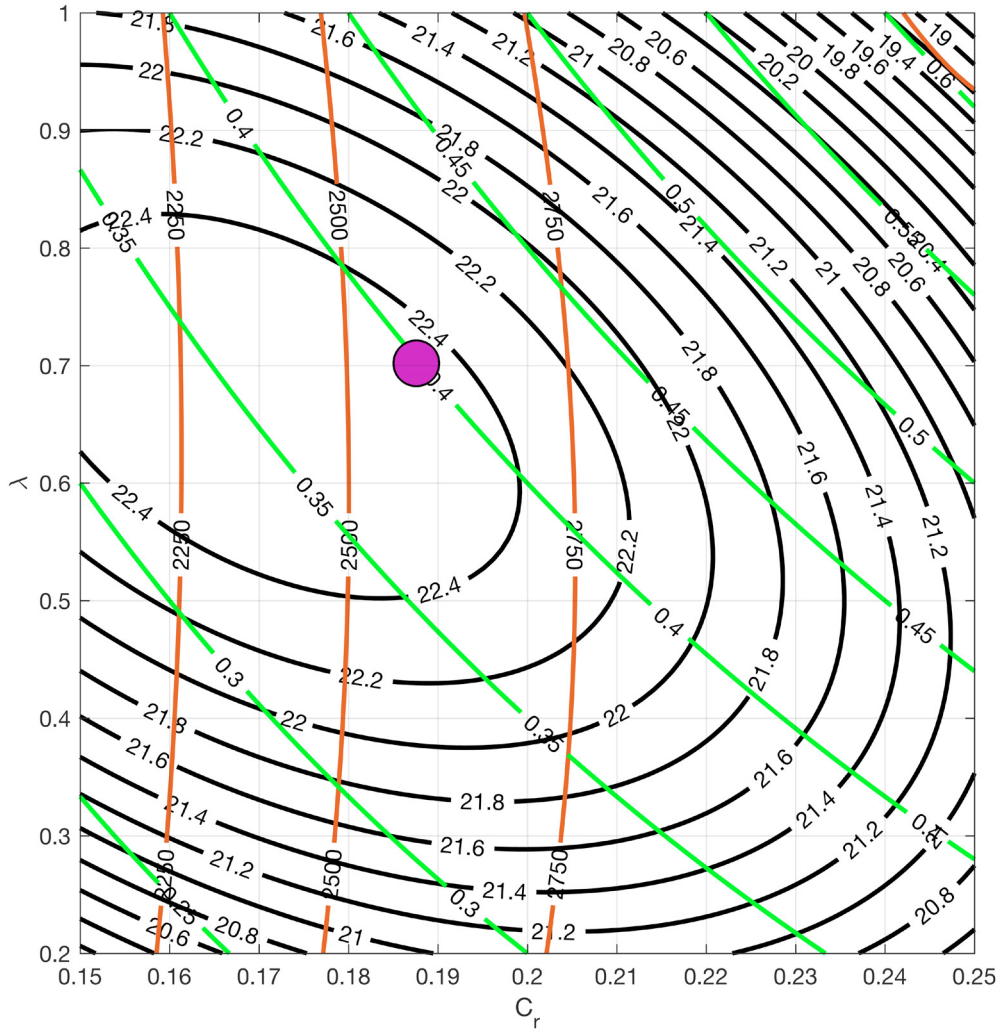


Fig. 8. Graphical illustration of the optimization problem