On the response of detectors in classical electromagnetic backgrounds

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Abstract

I study the response of a detector that is coupled non-linearly to a quantized complex scalar field in different types of classical electromagnetic backgrounds. Assuming that the quantum field is in the vacuum state, I show that, when in *inertial* motion, the detector responds *only* when the electromagnetic background produces particles. However, I find that the response of the detector is *not* proportional to the number of particles produced by the background.

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1 The concept of a detector

The idea of the detector originated in literature with the aim of providing an operational definition for the concept of a particle and also for being utilized as a probe to study the phenomenon of particle production in classical gravitational backgrounds. A detector is an idealized point like object whose motion is described by a classical worldline, but which nevertheless possesses internal energy levels. Such detectors are essentially described by the interaction Lagrangian for the coupling between the degrees of freedom of the detector and the quantum field. The response of detectors that are coupled to the quantum field through a linear [1, 2] or a derivative coupling [3, 4] and also detectors that are coupled to the energy-momentum tensor of the quantum field [5] have been studied in a variety of situations in different classical gravitational backgrounds (also see Ref. [6], Secs. 3.3, 5.4 and 8.3 in this context).

Phenomena such as vacuum polarization and particle production that occur in gravitational backgrounds take place in electromagnetic backgrounds too. It will be interesting to examine as to how detectors respond to such phenomena in classical electromagnetic backgrounds. But, the response of detectors on non-inertial trajectories turns out to be non-zero even in the Minkowski vacuum in flat spacetime [7, 8]. Therefore, in order to avoid effects due to non-inertial motion and also to isolate the effects that arise due to the electromagnetic background, it is essential that we restrict our attention to inertial trajectories. With this motivation, in this Letter, I shall study the response of an *inertial* detector in different types of classical electromagnetic backgrounds. (I shall set $\hbar = c = 1$ and I shall denote complex and Hermitian conjugation by an asterisk and a dagger, respectively.)

2 The non-linearly coupled detector

The quantum field I shall consider is a *complex* scalar field Φ described by the action

$$S[\Phi] = \int d^4x \, \left[(D_\mu\Phi)(D^\mu\Phi)^* - m^2\Phi\Phi^*\right], \tag{1}$$

where $D_\mu \equiv (\partial_\mu + iqA_\mu)$, A^μ is the vector potential describing the classical electromagnetic background and q and m are the charge and the mass of a single quanta of the scalar field. Varying this action leads to the following equation of motion for the complex scalar field Φ:

$$\left(D_\mu D^\mu + m^2\right)\Phi = 0. \tag{2}$$

The simplest of the different possible detectors is the detector due to Unruh and DeWitt [1, 2]. Consider a Unruh-DeWitt detector that is moving along a trajectory $\tilde{x}(\tau)$, where $\tilde{x}$ denotes the set of four coordinates x^μ and τ is the proper time in the frame of the detector. The interaction of the Unruh-DeWitt detector with a *real* scalar field Φ is described by the Lagrangian

$$\mathcal{L}_{\rm int} = c\,\mu(\tau)\,\Phi\left[\tilde{x}(\tau)\right], \tag{3}$$

where c is a small coupling constant and $\mu(\tau)$ is the detector's monopole moment. For the case of the complex scalar field I shall be considering here, this interaction Lagrangian can be generalized to

$$\mathcal{L}_{\rm int} = c\left(\mu(\tau)\,\Phi[\tilde{x}(\tau)] + \mu^{*}(\tau)\,\Phi^{*}[\tilde{x}(\tau)]\right). \tag{4}$$

But, under a gauge transformation of the form: $A^{\mu} \to (A^{\mu} + \partial^{\mu}\chi)$, the complex scalar field transforms as: $\Phi \to (\Phi\, e^{-iq\chi})$. Clearly, the interaction Lagrangian (4) will *not* be invariant under such a gauge transformation, unless I assume that the monopole moment transforms as follows: $\mu \to (\mu\, e^{iq\chi})$. However, I would like to treat the detector part of the coupling, viz. the monopole moment $\mu(\tau)$, as a quantity that transforms as a scalar under gauge transformations. In such a case, the simplest of the Lagrangians that is *explicitly* gauge invariant is the *non-linear* interaction

$$\mathcal{L}_{\rm int} = c\,\mu(\tau)\left(\Phi[\tilde{x}(\tau)]\,\Phi^{*}[\tilde{x}(\tau)]\right). \tag{5}$$

In what follows, I shall study the response of a detector that is coupled to the field through such an interaction Lagrangian in different types of classical electromagnetic backgrounds. It is important to note here that demanding gauge invariance naturally leads to non-linear interactions. A physical manifestation of gauge invariance is charge conservation. As we shall see later, the non-linear and gauge invariant interaction Lagrangian (5) leads to the excitation of a particle-anti-particle pair thereby conserving charge.

In an electromagnetic background, the quantized complex scalar field $\hat{\Phi}$ satisfying the Klein-Gordon equation (2) can, in general, be decomposed as follows (see Ref. [9] and references therein):

$$\hat{\Phi}(\tilde{x}) = \sum_{i}\left[\hat{a}_i\, u_i(\tilde{x}) + \hat{b}_i^{\dagger}\, v_i(\tilde{x})\right], \tag{6}$$

where $u_i(\tilde{x})$ and $v_i(\tilde{x})$ are positive and negative *norm* modes, respectively[1]. These modes are normalized with respect to the following gauge invariant scalar product (see, for e.g., Ref. [9])

$$(u_i, u_j) = -i\int\limits_{t=0} d^3x\,\left(u_i\left[\partial_t - iqA_t\right]u_j^{*} - u_j^{*}\left[\partial_t + iqA_t\right]u_i\right), \tag{7}$$

[1]The only non-trivial commutation relations satisfied by the two sets of operators $\left\{\hat{a}_i, \hat{a}_i^{\dagger}\right\}$ and $\left\{\hat{b}_i, \hat{b}_i^{\dagger}\right\}$ are: $\left[\hat{a}_i, \hat{a}_j^{\dagger}\right] = \left[\hat{b}_i, \hat{b}_j^{\dagger}\right] = \delta_{ij}$. All other commutators vanish.

where $\partial_t \equiv (\partial/\partial t)$ and A_t is the zeroth component of the vector potential A^μ. The vacuum state $|0\rangle$ of the quantum field $\hat{\Phi}$ is defined as the state that is annihilated by *both* the operators $\hat{a}_i$ and $\hat{b}_i$ for *all* i.

Now, assume that the quantized complex scalar field $\hat{\Phi}$ is initially in the vacuum state $|0\rangle$. Then, up to the first order in perturbation theory, the amplitude of transition of the detector that is coupled to the field through the interaction Lagrangian (5) is given by

$$\mathcal{A}(\mathcal{E}) = \left(\frac{\mathcal{M}}{2}\right) \int_{-\infty}^{\infty} d\tau\, e^{i\mathcal{E}\tau} \,\langle\Psi| \Big(\hat{\Phi}[\tilde{x}(\tau)]\,\hat{\Phi}^\dagger[\tilde{x}(\tau)] + \hat{\Phi}^\dagger[\tilde{x}(\tau)]\,\hat{\Phi}[\tilde{x}(\tau)]\Big)|0\rangle, \qquad (8)$$

where $\mathcal{M} \equiv ic\,\langle E|\hat{\mu}(0)|E_0\rangle$, $\mathcal{E} = (E - E_0)$, E_0 and E are the energy eigen values corresponding to the ground state $|E_0\rangle$ and the excited state $|E\rangle$ of the detector and $|\Psi\rangle$ is the state of the quantum field after its interaction with the detector. (Since the term $\mathcal{M}$ depends only on the internal structure of the detector and not on its motion, I shall drop this term hereafter.) The transition amplitude $\mathcal{A}(\mathcal{E})$ above involves products of the field $\hat{\Phi}$ at the *same* spacetime point and hence we will encounter divergences when evaluating this transition amplitude. In order to avoid the divergences, I shall normal order the creation and the annihilation operators in the matrix element in the transition amplitude (8). On substituting the decomposition (6) for the field $\hat{\Phi}$ in the transition amplitude (8) and normal ordering the creation and the annihilation operators, I obtain that

$$\mathcal{A}^*(\mathcal{E}) = \sum_i \sum_j \int_{-\infty}^{\infty} d\tau\, e^{-i\mathcal{E}\tau}\, u_i[\tilde{x}(\tau)]\, v_j^*[\tilde{x}(\tau)]\, \langle 0|\hat{a}_i \hat{b}_j|\Psi\rangle. \qquad (9)$$

This transition amplitude will be non-zero *only* when $|\Psi\rangle = \hat{a}_i^\dagger \hat{b}_j^\dagger|0\rangle = |1_i, 1_j\rangle$. This implies that the interaction of the field with the detector leads to the excitation of a particle-anti-particle pair. Since the quantum field I am considering here is a *charged* scalar field, the excitation of a particle-anti-particle pair is essential for charge conservation. As I had pointed out before, it is the non-linear and the gauge invariant nature of the interaction Lagrangian (5) that ensures that such a pair is indeed excited.

Before I go on to study the response of inertial detectors in electromagnetic backgrounds, let me briefly discuss the response of an inertial detector in the Minkowski vacuum. (The arguments I shall present here will prove to be useful for our discussion later on.) Consider an inertial detector stationed at a point, say, $\mathbf{a}$. In the absence of an electromagnetic background, the positive and negative norm modes are related as follows: $v_i(\tilde{x}) = u_i^*(\tilde{x})$. Moreover, in the Minkowski coordinates, the definition of positive norm modes *match* the definition of positive frequency modes. Then, it is clear from Eq. (9) that it is *only* the positive frequency modes $u_i(\tilde{x})$ that contribute to the transition amplitude $\mathcal{A}^*(\mathcal{E})$ in such a situation. Therefore, the transition amplitude of the detector corresponding to a pair of modes, say, $\mathbf{k}$ and $\mathbf{l}$, of the quantum field is given by

$$\mathcal{A}^*(\mathcal{E}) = \left(\frac{e^{i(\mathbf{k}+\mathbf{l}).\mathbf{a}}}{\sqrt{(2\pi)^4\, 4\omega_k\omega_l}}\right)\, \delta^{(1)}(\mathcal{E} + \omega_k + \omega_l), \tag{10}$$

where, for a given mode $\mathbf{k}$, $\omega_k = (|\mathbf{k}|^2 + m^2)^{1/2}$. The quantities ω_k and ω_l are always $\geq m$ and, since $\mathcal{E} > 0$ as well, the argument of the delta function above is a positive definite quantity and, hence, the transition amplitude $\mathcal{A}^*(\mathcal{E})$ reduces to zero for *all* $\mathbf{k}$ and $\mathbf{l}$. In other words, the non-linearly coupled detector will *not* respond in the Minkowski vacuum state when in inertial motion.

The transition probability of the non-linearly coupled detector to all possible final states $|\Psi\rangle$ of the field can now be evaluated from the transition amplitude (9). I find that

$$\mathcal{P}(\mathcal{E}) = \sum_{|\Psi\rangle} |\mathcal{A}(\mathcal{E})|^2 = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau'\, e^{-i\mathcal{E}(\tau-\tau')}\, \tilde{G}\left[\tilde{x}(\tau), \tilde{x}(\tau')\right], \tag{11}$$

where $\tilde{G}\left[\tilde{x}(\tau), \tilde{x}(\tau')\right]$ is a four point function given by

$$\tilde{G}[\tilde{x}, \tilde{x}'] = \sum_i \left[u_i(\tilde{x}) u_i^*(\tilde{x}')\right]\, \sum_j \left[v_j^*(\tilde{x})\, v_j(\tilde{x}')\right]. \tag{12}$$

In cases wherein the four point function $\tilde{G}[\tilde{x}, \tilde{x}']$ is invariant under translations in the proper time in the frame of the detector, a transition probability rate for the detector can be defined as follows:

$$\mathcal{R}(\mathcal{E}) = \int_{-\infty}^{\infty} d(\tau - \tau')\, e^{-i\mathcal{E}(\tau-\tau')}\, \tilde{G}(\tau - \tau'). \tag{13}$$

I had pointed out above that, in the absence of an electromagnetic background, the positive and negative norm modes are related by the following expression: $v_i(\tilde{x}) = u_i^*(\tilde{x})$. It is then useful to note that, in such a case, the four point function $\tilde{G}[\tilde{x}, \tilde{x}']$ is given by square of the Wightman function in the Minkowski vacuum. Therefore, when in inertial motion, the transition probability rate $\mathcal{R}(\mathcal{E})$ of the non-linearly coupled detector in the Minkowski vacuum is identically zero (for exactly the same reasons) as it is in the case of the Unruh-DeWitt detector (see Ref. [6], pp. 50–53 in this context).

In the following three sections, I shall study the response of the non-linearly coupled detector (when it is in *inertial* motion) in: (i) a time-dependent electric field, (ii) a time-independent electric field and (iii) a time-independent magnetic field, backgrounds. I shall then conclude this Letter with a few summarizing remarks.

3 In time-dependent electric field backgrounds

Consider a time-dependent electric field background described by vector potential

$$A^{\mu} = (0, A(t), 0, 0), \tag{14}$$

where $A(t)$ is an arbitrary function of t. This vector potential gives rise to the electric field $\mathbf{E} = -(dA/dt)\,\hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$ is the unit vector along the positive x-direction. The modes of a quantum field evolving in such a time-dependent electric field background are of the form

$$u_{\mathbf{k}}(t, \mathbf{x}) = g_{\mathbf{k}}(t)\; e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{15}$$

In general, the modes at early and late times will be related by a non-zero Bogolubov coefficient β (see Ref. [10] and references therein). In fact, the expectation value of the number operator (corresponding to a given mode of the quantum field) at late times in the in-vacuum will be proportional to $|\beta|^2$.

Now, consider a detector that is stationed at a particular point. Along the world line of such a detector, the four point function (12) corresponding to the modes (15) is given by

$$\tilde{G}(t, t') = \sum_{\mathbf{k}} \sum_{\mathbf{l}} \left[g_{\mathbf{k}}(t)\, g_{\mathbf{l}}(t)\, g_{\mathbf{k}}^{*}(t')\, g_{\mathbf{l}}^{*}(t')\right] \tag{16}$$

and the transition probability of the detector reduces to

$$\mathcal{P}(\mathcal{E}) = \sum_{\mathbf{k}} \sum_{\mathbf{l}} |g_{\mathbf{kl}}(\mathcal{E})|^2, \quad \text{where} \quad g_{\mathbf{kl}}(\mathcal{E}) = \int_{-\infty}^{\infty} dt\, e^{-i\mathcal{E}t}\, \left[g_{\mathbf{k}}(t)\, g_{\mathbf{l}}(t)\right]. \tag{17}$$

Clearly, the response of the inertial detector will, in general, be non-zero.

Let me now assume that the function $A(t)$ behaves such that the electric field vanishes in the past and future infinity. Also, let the detector be switched on for a finite time interval in the future asymptotic domain. Let me further assume that the effects that arise due to switching [11, 12, 13] can be neglected. Then, by relating the modes at future and past infinity, I can express the transition probability rate of the detector (in the in-vacuum) in terms of the Bogolubov coefficients α and β as follows:

$$\begin{aligned}\mathcal{R}(\mathcal{E}) \;&=\; (2\pi) \sum_{\mathbf{k}} \sum_{\mathbf{l}} \Big(2\, |\alpha_{\mathbf{k}}|^2\; |\beta_{\mathbf{l}}|^2\; \delta^{(1)}(\mathcal{E} + \omega_k - \omega_l) \\ &\qquad\qquad + |\beta_{\mathbf{k}}|^2\; |\beta_{\mathbf{l}}|^2\; \delta^{(1)}(\mathcal{E} - \omega_k - \omega_l)\Big),\end{aligned} \tag{18}$$

where ω_k and ω_l are positive definite (in fact $\geq m$) frequencies corresponding to the modes $\mathbf{k}$ and $\mathbf{l}$ in the out-region. Clearly, the detector responds *only* when the Bogolubov coefffient β turns out to be non-zero (i.e. *only* when particle production takes place). However, it is evident that the transition probability rate of detector I have obtained above is *not* proportional to the number of particles produced by the time-dependent electric field background.

4 In time-independent electric field backgrounds

Consider the vector potential

$$A^{\mu} = (A(x), 0, 0, 0), \tag{19}$$

where $A(x)$ is an arbitrary function of x. Such a vector potential gives rise to a time-independent electric field along the x-direction given by $\mathbf{E} = -(dA/dx)\,\hat{\mathbf{x}}$. In such a case, the modes of the quantum field $\hat{\Phi}$ can be decomposed as follows:

$$u_{\omega\mathbf{k}_{\perp}}(t, \mathbf{x}) = e^{-i\omega t}\, f_{\omega\mathbf{k}_{\perp}}(x)\, e^{i\mathbf{k}_{\perp}\cdot\mathbf{x}_{\perp}}, \tag{20}$$

where $\mathbf{k}_{\perp}$ is the wave vector along the perpendicular direction. Due to lack of time dependence, the Bogolubov coefficient β relating these modes at two different times is trivially zero. Though the Bogolubov coefficient β is zero, particle production takes place in such backgrounds due to a totally different phenomenon. It is well-known that if the depth of the potential $[qA(x)]$ is greater than $(2m)$, then the corresponding electric field will produce particles due to Klein paradox (see Ref. [9] and references therein). It is then interesting to examine whether an inertial detector in a time-independent electric field background will respond under the same condition.

Consider a detector that is stationed at a particular point. It is easy to see from the form of the modes (20) that the transition amplitude $\mathcal{A}^{*}(\mathcal{E})$ of such a detector will be proportional to a delta function as in the case of an inertial detector in the Minkowski vacuum (cf. Eq. (10)). But, unlike the Minkowski case wherein the definition of positive frequency modes match the definition of positive norm modes, in a time-independent electric field background there exist negative frequency modes which have a positive norm whenever the depth of the potential $[qA(x)]$ is greater than $(2m)$. In other words, when Klein paradox occurs in an electric field background, ω_k and ω_l appearing in the argument of the delta function in Eq. (10) can be negative and, hence, there exists a range of values of these two quantities for which this argument can be zero. These modes excite the detector as a result of which the response of an inertial detector proves to be non-zero in such a background.

I shall now show (for the special case of the step potential) as to how there exist negative frequency modes which have a positive norm when the depth of the potential is greater than $(2m)$. In order to show that, let me evaluate the norm of the mode $u_{\omega\mathbf{k}_{\perp}}(t, \mathbf{x})$. On substituting the mode (20) and the vector potential (19) in the scalar product (7), I obtain that

$$(u_{\omega\mathbf{k}_{\perp}}, u_{\omega\mathbf{k}_{\perp}}) = 2\,(2\pi)^2\,\delta^{(2)}(0) \int\limits_{-\infty}^{\infty} dx\ [\omega - qA(x)]\ |f_{\omega\mathbf{k}_{\perp}}(x)|^2. \tag{21}$$

Let me now assume that $A(x) = -[\Theta(x)\, V]$, where $\Theta(x)$ is the step-function and V is a constant. For such a case, the function $f_{\omega\mathbf{k}_\perp}$ is given by

$$f_{\omega\mathbf{k}_\perp}(x) = \Theta(-x)\left(e^{ik_L x} + R_{\omega\mathbf{k}_\perp}\, e^{-ik_L x}\right) + \Theta(x)\, T_{\omega\mathbf{k}_\perp}\, e^{ik_R x}, \tag{22}$$

where

$$k_R = \left[(\omega + qV)^2 - |\mathbf{k}_\perp|^2 - m^2\right]^{1/2} \quad \text{and} \quad k_L = \left[\omega^2 - |\mathbf{k}_\perp|^2 - m^2\right]^{1/2}. \tag{23}$$

The quantities $R_{\omega\mathbf{k}_\perp}$ and $T_{\omega\mathbf{k}_\perp}$ are the usual reflection and tunnelling amplitudes. They are given by the expressions

$$R_{\omega\mathbf{k}_\perp} = \left(\frac{k_L - k_R}{k_L + k_R}\right) \quad \text{and} \quad T_{\omega\mathbf{k}_\perp} = \left(\frac{2k_L}{k_L + k_R}\right). \tag{24}$$

If I now assume that k_R and k_L are real quantities, then, for the case of the step potential I am considering here, the scalar product (21) is given by

$$(u_{\omega\mathbf{k}_\perp}, u_{\omega\mathbf{k}_\perp}) = (2\pi)^3\, \delta^{(3)}(0)\, \left[\omega\left(1 + R^2_{\omega\mathbf{k}_\perp}\right) + (\omega + qV)\, T^2_{\omega\mathbf{k}_\perp}\right]. \tag{25}$$

Let me now set $\mathbf{k}_\perp = 0$. Also, let me assume that $\omega = -(m + \varepsilon)$ and $(qV) = (2m+\varepsilon)$, where ε is a positive definite quantity. For such a case, $R_{\omega 0} = 1$, $T_{\omega 0} = 2$ and the scalar product (25) reduces to

$$(u_{\omega 0}, u_{\omega 0}) = 2\,(m - \varepsilon)\,(2\pi)^3\, \delta^{(3)}(0) \tag{26}$$

which is a positive definite quantity if I choose ε to be smaller than m. I have thus shown that there exist negative frequency modes (i.e. modes with $\omega \leq -m$) which have a positive norm. Moreover, this occurs *only* when (qV) is greater than $(2m)$ (note that $(qV) = (2m + \varepsilon)$) which is exactly the condition under which Klein paradox is expected to arise. As I have discussed in the last paragraph, it is this feature of the Klein paradox that is responsible for exciting the detector.

5 In time-independent magnetic field backgrounds

A time-independent magnetic field background can be described by the vector potential

$$A^\mu = (0, 0, A(x), 0), \tag{27}$$

where $A(x)$ is an arbitrary function of x. This vector potential gives rise to the magnetic field $\mathbf{B} = (dA/dx)\, \hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector along the positive z-axis. It can be shown that the effective Lagrangian corresponding to such a time-independent magnetic field background does *not* have an imaginary part which then implies that such backgrounds do *not* produce particles [14].

The modes of the quantum field $\hat{\Phi}$ in a time-independent magnetic field background can be decomposed exactly as I did in Eq. (20) in the case of the time-independent electric field background. Hence, the transition amplitude $\mathcal{A}^*(\mathcal{E})$ of an inertial detector in a time-independent magnetic field background will also be proportional to a delta function as in Eq. (10). However, on substituting the mode (20) and the vector potential (27) in the scalar product (7), I find that

$$(u_{\omega\mathbf{k}_\perp}, u_{\omega\mathbf{k}_\perp}) = (2\omega)\ (2\pi)^2\,\delta^{(2)}(0) \int_{-\infty}^{\infty} dx\,|f_{\omega\mathbf{k}_\perp}(x)|^2 \tag{28}$$

which is clearly a positive definite quantity whenever $\omega \geq m$. In other words, unlike the case of the time-independent electric field background, in a time independent magnetic field background, the definition of positive frequency modes *always* match the definition of positive norm modes. Therefore, as in the case of an inertial detector in the Minkowski vacuum, an inertial detector will *not* respond in the vacuum state in a time-independent magnetic field background.

6 Concluding remarks

It is clear that, when the quantum field is in the vacuum state, the non-linearly coupled detector, while in *inertial* motion, responds *only* when the classical electromagnetic background produces particles. However, as we have seen in the case of the time-dependent electric field background, the detector response does *not* reflect the amount of the particles produced by the background. This feature should not come as a surprise and, in fact, it can be attributed to the non-linearity of the interaction Lagrangian (5) for the following two reasons. Firstly, it is known that in a time-dependent gravitational background with asymptotically static domains, the response of the Unruh-DeWitt detector (which is coupled to the quantum field through a linear interaction) in the out-region *is* proportional to the number of particles produced by the background (see Ref. [6], pp. 57-59). Secondly, it has been shown that the response of a detector that is coupled to the energy-momentum tensor of the quantum field (which is evidently a non-linear interaction) does *not* reflect the particle content of the field [5]. As I have discussed earlier, demanding gauge invariance naturally leads to non-linear interaction Lagrangians. Therefore, quite generically, we can expect that the response of detectors in classical electromagnetic backgrounds will *not* be proportional to the amount of particles produced by the background.

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