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## Observation of Z_\{b\}(10610) and Z_\{b\}(10650) Decaying to B Mesons

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\text { Study of } e^{+} e^{-} \rightarrow B^{(*)} \bar{B}^{(*)} \pi^{ \pm} \text {at } \sqrt{s}=10.866 \mathrm{GeV}
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#### Abstract

We report the analysis of the three-body $e^{+} e^{-} \rightarrow B \bar{B} \pi^{ \pm}, B \bar{B}^{*} \pi^{ \pm}$, and $B^{*} \bar{B}^{*} \pi^{ \pm}$processes, including the first observations of the $Z_{b}^{ \pm}(10610) \rightarrow\left[B \bar{B}^{*}+\text { c.c. }\right]^{ \pm}$and $Z_{b}^{ \pm}(10650) \rightarrow\left[B^{*} \bar{B}^{*}\right]^{ \pm}$transitions that are found to dominate the corresponding final states. We measure Born cross sections for the three-body production of $\sigma\left(e^{+} e^{-} \rightarrow\left[B \bar{B}^{*}+\text { c.c. }\right]^{ \pm} \pi^{\mp}\right)=(17.4 \pm 1.6$ (stat. $) \pm 1.9$ (syst. $)$ ) pb and $\sigma\left(e^{+} e^{-} \rightarrow\left[B^{*} \bar{B}^{*}\right]^{ \pm} \pi^{\mp}\right)=(8.75 \pm 1.15($ stat. $) \pm 1.04$ (syst.) $) \mathrm{pb}$ and set a $90 \%$ C.L. upper limit of $\sigma\left(e^{+} e^{-} \rightarrow[B \bar{B}]^{ \pm} \pi^{\mp}\right)<2.9 \mathrm{pb}$. The results are based on a $121.4 \mathrm{fb}^{-1}$ data sample collected with the Belle detector at a center-of-mass energy near the $\Upsilon(10860)$ peak.


Analysis of the quark composition of the initial and final110 states reveals that these hadronic objects have an exotic ${ }^{11}$ nature: $Z_{b}$ should be comprised of (at least) four quarks 12 including a $b \bar{b}$ pair. Several models [3] have been pro-113 posed to describe the internal structure of these states. In ${ }^{114}$ Ref. [4], it was suggested that $Z_{b}(10610)$ and $Z_{b}(10650)^{115}$ states might be loosely bound $B \bar{B}^{*}$ and $B^{*} \bar{B}^{*}$ systems, ${ }^{116}$ respectively. If so, it is natural to expect the $Z_{b}$ states ${ }^{117}$
to decay to final states with $B^{(*)}$ mesons at substantial rates.

Evidence for the three-body $\Upsilon(10860) \rightarrow B \bar{B}^{*} \pi$ decay has been reported previously by Belle, based on a data sample of $23.6 \mathrm{fb}^{-1}$ [5]. In this analysis, we use a data sample with an integrated luminosity of $121.4 \mathrm{fb}^{-1}$ collected near the peak of the $\Upsilon(10860)$ resonance $(\sqrt{s}=10.866 \mathrm{GeV})$ with the Belle detector [6] at the KEKB asymmetric-energy $e^{+} e^{-}$collider [7]. Note that we reconstruct only three-body $B^{(*)} \bar{B}^{(*)} \pi$ combinations with a charged primary pion. For brevity, we adopt the following notations: the set of $B^{+} \bar{B}^{0} \pi^{-}$and


FIG. 1: The (a) invariant mass and (b) $M_{\text {miss }}^{*}(B \pi)$ distribu- ${ }^{167}$ tion for $B$ candidates in the $B$ signal region. Points with error ${ }^{168}$ bars represent the data. The open histogram in (a) shows the169 result of the fit to data. The solid line in (b) shows the result ${ }_{170}$ of the fit to the RS $B \pi$ data; the dashed line represents the ${ }_{171}$ background level.
$B^{-} B^{0} \pi^{+}$final states is referred to as $B B \pi$; the set of ${ }_{175}^{174}$ $B^{+} \bar{B}^{* 0} \pi^{-}, B^{-} B^{* 0} \pi^{+}, B^{0} B^{*-} \pi^{+}$and $\bar{B}^{0} B^{*+} \pi^{-}$final ${ }_{176}^{175}$ states is referred to as $B B^{*} \pi$; and the set of $B^{*+} \bar{B}^{* 0} \pi^{-{ }^{176}}$ and $B^{*-} B^{* 0} \pi^{+}$final states is denoted as $B^{*} B^{*} \pi$. The in ${ }^{1777}$ clusion of the charge conjugate mode is implied through- ${ }_{179}$ out this report.

We use Monte Carlo (MC) events generated with Evt- ${ }_{18}^{180}$ Gen [8] and then processed through a detailed detector ${ }_{182}$ simulation implemented in GEANT3 [9]. The simulated ${ }_{183}$ samples for $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s, c$, or $b)$ are equiv- ${ }_{184}$ alent to six times the integrated luminosity of the data ${ }_{185}$ and are used to develop criteria to separate signal events ${ }_{186}$ from backgrounds, identify types of background events ${ }_{187}$ determine the reconstruction efficiency and parameterize ${ }_{188}$ the distributions needed for the extraction of the signal ${ }_{189}$ decays.
$B$ mesons are reconstructed in the following decay ${ }_{191}$ channels: $B^{+} \rightarrow J / \psi K^{(*)+}, B^{+} \rightarrow \bar{D}^{(*) 0} \pi^{+}, B^{0} \rightarrow_{192}$ $J / \psi K^{(*) 0}, B^{0} \rightarrow D^{(*)-} \pi^{+}$. We use Belle standard tech- ${ }_{193}$ niques [10] to reconstruct primary particles such as pho- ${ }_{194}$ tons, pions, kaons, and leptons. The $K^{* 0}\left(K^{*+}\right)$ is re- ${ }_{195}$ constructed in the $K^{+} \pi^{-}\left(K^{0} \pi^{+}\right)$final state; the invari ${ }^{-196}$ ant mass of the $K^{*}$ candidate is required to be within $\mathrm{n}_{197}$ $150 \mathrm{MeV} / c^{2}$ of the nominal $K^{*}$ mass [11]. The invari- ${ }_{198}$ ant mass of a $J / \psi \rightarrow \ell^{+} \ell^{-}$candidate is required $\mathrm{to}_{199}$ be within $30(50) \mathrm{MeV} / c^{2}$ for $\ell=e(\mu)$, of the nom- ${ }^{200}$ inal $J / \psi$ mass. Neutral (charged) $D$ mesons are re- ${ }_{201}$ constructed in the $K^{-} \pi^{+}, K^{-} \pi^{+} \pi^{0}$, and $K^{-} \pi^{-} \pi^{+} \pi^{+}{ }_{202}$ $\left(K^{-} \pi^{+} \pi^{+}\right)$modes. To identify $D^{*}$ candidates, we require ${ }_{203}$ $\left|M(D \pi)-M(D)-\Delta m_{D^{*}}\right|<3 \mathrm{MeV} / c^{2}$, where $M(D \pi)_{204}$ and $M(D)$ are the reconstructed masses of the $D^{*}$ and $D_{205}$ candidates, respectively, and $\Delta m_{D^{*}}=m_{D^{*}}-m_{D}$ is the ${ }_{206}$ difference between the nominal $D^{*}$ and $D$ masses. The ${ }_{207}$ mass windows for narrow states quoted above correspond ${ }_{208}$ to a $\pm 2.5 \sigma$ requirement.

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The dominant background comes from $e^{+} e^{-} \rightarrow c \bar{c}$ con-210 tinuum events, where true $D$ mesons produced in $e^{+} e^{-}{ }_{211}$ annihilation are combined with random particles to form ${ }_{212}$ a $B$ candidate. This type of background is suppressed us-213
ing variables that characterize the event topology. Since the momenta of the two $B$ mesons produced from a threebody $e^{+} e^{-} \rightarrow B^{(*)} B^{(*)} \pi$ decay are low in the center-ofmass (c.m.) frame (below $0.9 \mathrm{GeV} / c$ ), the decay products of different $B$ mesons are essentially uncorrelated so that the event tends to be spherical. In contrast, hadrons from continuum events tend to exhibit a back-to-back jet structure. We use $\theta_{\mathrm{thr}}$, the angle between the thrust axis of the $B$ candidate and that of the rest of the event, to discriminate between the two cases. The distribution of $\left|\cos \theta_{\mathrm{thr}}\right|$ is strongly peaked near $\left|\cos \theta_{\mathrm{thr}}\right|=1.0$ for $c \bar{c}$ events and is nearly flat for $B^{(*)} B^{(*)} \pi$ events. We require $\left|\cos \theta_{\mathrm{thr}}\right|<0.80$ for the $B \rightarrow D^{(*)} \pi$ final states; this eliminates about $81 \%$ of the continuum background and retains $73 \%$ of the signal events.

We identify $B$ candidates by their reconstructed invariant mass $M(B)$ and momentum $P(B)$ in the c.m. frame. We require $P(B)<1.35 \mathrm{GeV} / c$ to retain $B$ mesons produced in both two-body and multibody processes. The $M(B)$ distribution for $B$ candidates is shown in Fig. 1(a). We perform a binned maximum likelihood fit of the $M(B)$ distribution to the sum of a signal component parameterized by a Gaussian function and two background components: one related to other decay modes of $B$ mesons and one due to continuum $e^{+} e^{-} \rightarrow q \bar{q}$ processes, where $q=u, d, s, c$. The shape of the $B$-related background is determined from a large sample of generic MC; the shape of the $q \bar{q}$ background is parameterized with a linear function. The parameters of the signal Gaussian, the normalization of the $B$-related background and the parameters of the $q \bar{q}$ background float in the fit. We find $12263 \pm 168$ fully reconstructed $B$ mesons. The $B$ signal region is defined by requiring $M(B)$ to be within 30 to $40 \mathrm{MeV} / c^{2}$ (depending on the $B$ decay mode) of the nominal $B$ mass.

Reconstructed $B^{+}$or $\bar{B}^{0}$ candidates are combined with $\pi^{-}$'s - the right-sign (RS) combination - and the missing mass, $M_{\text {miss }}(B \pi)$, is calculated as $M_{\text {miss }}(B \pi)=$ $\sqrt{\left(\sqrt{s}-E_{B \pi}\right)^{2} / c^{4}-P_{B \pi}^{2} / c^{2}}$, where $E_{B \pi}$ and $P_{B \pi}$ are the measured energy and momentum of the reconstructed $B \pi$ combination. Signal $e^{+} e^{-} \rightarrow B B^{*} \pi$ events produce a narrow peak in the $M_{\text {miss }}(B \pi)$ spectrum around the nominal $B^{*}$ mass while $e^{+} e^{-} \rightarrow B^{*} B^{*} \pi$ events produce a peak at $m_{B^{*}}+\Delta m_{B^{*}}$, where $\Delta m_{B^{*}}=m_{B^{*}}-m_{B}$, due to the missed photon from the $B^{*} \rightarrow B \gamma$ decay. It is important to note here that, according to signal MC, $B B^{*} \pi$ events, where the reconstructed $B$ is the one from the $B^{*}$, produce a peak in the $M_{\text {miss }}(B \pi)$ distribution at virtually the same position as $B B^{*} \pi$ events, where the reconstructed $B$ is the primary one. To remove the correlation between $M_{\text {miss }}(B \pi)$ and $M(B)$ and to improve the resolution, we use $M_{\text {miss }}^{*}=M_{\text {miss }}(B \pi)+M(B)-m_{B}$ instead of $M_{\text {miss }}(B \pi)$. The $M_{\text {miss }}^{*}$ distribution for the RS combinations is shown in Fig. 1(b), where peaks corresponding to the $B B^{*} \pi$ and $B^{*} B^{*} \pi$ signals are evident. Combinations with $\pi^{+}$- the wrong sign (WS) combi-
nations - are used to evaluate the shape of the combinatorial background. (The $B \rightarrow J / \psi K^{0}$ mode is not included in the WS sample but both combinations with $\pi^{+}$and $\pi^{-}$are added to the RS sample.) We apply factor of $1.19 \pm 0.01$ [12] to the WS distribution to normalize it to the expected number of the background events in the RS sample. There is also a hint for a peaking structure in the WS $M_{\text {miss }}^{*}$ distribution, shown as a hatched histogram in Fig. 1(b). Due to $B^{0}-\bar{B}^{0}$ oscillations, we expect a fraction of the produced $B^{0}$ mesons to decay as $\bar{B}^{0}$ given by $0.5 x_{d}^{2} /\left(1+x_{d}^{2}\right)=0.1861 \pm 0.0024$, where $x_{d}$ is the $B^{0}$ mixing parameter [11].

Note that the momentum spectrum of $B$ mesons produced in events with initial-state radiation (ISR), $e^{+} e^{-} \rightarrow \gamma B \bar{B}$, overlaps significantly with that for $B$ mesons from the three-body $e^{+} e^{-} \rightarrow B^{(*)} B^{(*)} \pi$ processes. However, ISR events do not produce peaking structures in the $M_{\text {miss }}^{*}$ distribution.

A binned maximum likelihood fit is performed to fit the $M_{\text {miss }}^{*}$ distribution to the sum of three Gaussian functions to represent three possible signals and two threshold components $A_{k}\left(x_{k}-M_{\text {miss }}^{*}\right)^{\alpha_{k}} \exp \left\{\left(M_{\text {miss }}^{*}-x_{k}\right) / \delta_{k}\right\}$ $(k=1,2)$ to parameterize the $q \bar{q}$ and two-body $B^{(*)} \bar{B}^{(*)}$ backgrounds. The means and widths of the signal Gaussian functions are fixed from the signal MC simulation. The parameters $A_{k}, \alpha_{k}, \delta_{k}$ of the background functions are free parameters of the fit; the threshold $\mathrm{pa}_{-270}$ rameters $x_{k}$ are fixed from the generic MC. ISR events produce an $M_{\text {miss }}^{*}$ distribution similar to that for $q \bar{q}$ events; these two components are modeled by a single threshold function. The resolution of the signal peaks ${ }^{271}$ in Fig. 1(b) is dominated by the c.m. energy spread and ${ }^{272}$ is fixed at $6.5 \mathrm{MeV} / c^{2}$ and $6.2 \mathrm{MeV} / c^{2}$ for the $B B^{*} \pi^{273}$ and $B^{*} B^{*} \pi$, respectively as determined from the signal ${ }^{274}$ MC. The fit to the RS spectrum yields $N_{B B \pi}=13 \pm 25,{ }^{275}$ $N_{B B^{*} \pi}=357 \pm 30$ and $N_{B^{*} B^{*} \pi}=161 \pm 21$ signal events. ${ }^{276}$ The statistical significance of the observed $B B^{*} \pi$ and ${ }^{277}$ $B^{*} B^{*} \pi$ signal is $9.3 \sigma$ and $8.1 \sigma$, respectively. The statis- ${ }^{278}$ tical significance is calculated as $\sqrt{-2 \ln \left(\mathcal{L}_{0} / \mathcal{L}_{\text {sig }}\right)}$, where ${ }^{279}$ $\mathcal{L}_{\text {sig }}$ and $\mathcal{L}_{0}$ denote the likelihood values obtained with ${ }^{280}$ the nominal fit and with the signal yield fixed at zero, ${ }^{281}$ respectively.
For the subsequent analysis, we require $\mid M_{\text {miss }}^{*}-^{283}$ $m_{B^{*}} \mid<15 \mathrm{MeV} / c^{2}$ to select $B B^{*} \pi$ signal events and ${ }^{284}$ $\left|M_{\text {miss }}^{*}-\left(m_{B^{*}}+\Delta m_{B}\right)\right|<12 \mathrm{MeV} / c^{2}$, where $\Delta m_{B}=$ $m_{B^{*}}-m_{B}$, to select $B^{*} B^{*} \pi$ events. For the selected $B^{(*)} B^{*} \pi$ candidates, we calculate $M_{\text {miss }}(\pi)==_{285}$ $\sqrt{\left(\sqrt{s}-E_{\pi}\right)^{2} / c^{4}-P_{\pi}^{2} / c^{2}}$, where $E_{\pi}$ and $P_{\pi}$ are the re-286 constructed energy and momentum, respectively, of the 287 charged pion in the c.m. frame. The $M_{\text {miss }}(\pi)$ distribu-288 tions are shown in Fig. 2 [13]. We perform a simultaneous289 binned maximum likelihood fit to the RS and WS sam-200 ples, assuming the same number (after normalization) $2_{291}$ and distribution of background events in both samples $2_{22}$ and known fraction of signal events in the RS sample293 that leaks to the WS sample due to mixing. To fit the $2_{24}$


FIG. 2: The $M_{\text {miss }}(\pi)$ distribution for the (a) $B B^{*} \pi$ and (b) $B^{*} B^{*} \pi$ candidate events. Normalization factor is applied for the WS distributions.
$M_{\text {miss }}(\pi)$ spectrum, we use the function

$$
\begin{equation*}
F(m)=\left[f_{\mathrm{sig}} S(m)+B(m)\right] \epsilon(m) F_{\mathrm{PHSP}}(m), \tag{1}
\end{equation*}
$$

where $m \equiv M_{\text {miss }}(\pi) ; f_{\text {sig }}=1.0(0.1366 \pm 0.0032,[14])$ for the RS (WS) sample; $S(m)$ and $B(m)$ are the signal and background PDFs, respectively; and $F_{\text {PHSP }}(m)$ is the phase space function. To account for the instrumental resolution, we smear the function $F(m)$ with a Gaussian function with $\sigma=6.0 \mathrm{MeV} / c^{2}$ that is dominated by the c.m. energy spread. The reconstruction efficiency is parametrized as $\epsilon(m) \sim \exp \left(\left(m-m_{0}\right) / \Delta\right)\left(1-m / m_{0}\right)^{3 / 4}$, where $m_{0}=10.718 \pm 0.001 \mathrm{GeV} / c^{2}$ is an efficiency threshold and $\Delta=0.094 \pm 0.002 \mathrm{GeV} / c^{2}$.

The distribution of background events is parameterized as $B_{B^{(*)} B^{*} \pi}(m)=b_{0} e^{-\beta \delta_{m}}$, where $b_{0}$ and $\beta$ are fit parameters and $\delta_{m}=m-\left(m_{\left.B^{*}\right)}+m_{B^{*}}\right)$. A general form of the signal PDF is written as

$$
\begin{equation*}
S(m)=\left|\mathcal{A}_{Z_{b}(10610)}+\mathcal{A}_{Z_{b}(10650)}+\mathcal{A}_{\mathrm{nr}}\right|^{2}, \tag{2}
\end{equation*}
$$

where $\mathcal{A}_{\mathrm{nr}}=a_{\mathrm{nr}} e^{i \phi_{\mathrm{nr}}}$ is the non-resonant amplitude parameterized as a complex constant and the two $Z_{b}$ amplitudes, $\mathcal{A}_{Z_{b}}$, are parameterized with BreitWigner functions $\mathcal{A}_{Z_{b}}=a_{Z} e^{i \phi_{Z}} /\left(m^{2}-m_{Z}^{2}-i \Gamma_{Z} m_{Z}\right)$. The masses and widths of the $Z_{b}$ states are fixed at the values obtained from the analyses of $e^{+} e^{-} \rightarrow$ $\Upsilon(n \mathrm{~S}) \pi^{+} \pi^{-}$and $e^{+} e^{-} \rightarrow h_{b}(m \mathrm{P}) \pi^{+} \pi^{-}: M_{Z_{b}(10610)}=$ $10607.2 \pm 2.0 \mathrm{MeV} / c^{2}, \Gamma_{Z_{b}(10610)}=18.4 \pm 2.4 \mathrm{MeV}$ and $M_{Z_{b}(10650)}=10652.2 \pm 1.5 \mathrm{MeV} / c^{2}, \Gamma_{Z_{b}(10650)}=$ $11.5 \pm 2.2 \mathrm{MeV}[1]$.

TABLE I: Summary of fit results to the $M_{\text {miss }}(\pi)$ distributions for the three-body $B B^{*} \pi$ and $B^{*} B^{*} \pi$ final states.

| Mode | Parameter | Model-0 | Model-1 |  | Model-2 |  | Model-3 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Solution 1 | Solution 2 | Solution 1 | Solution 2 |  |
| $B B^{*} \pi$ | $f_{Z_{b}(10610)}$ | 1.0 | $1.45 \pm 0.24$ | $0.64 \pm 0.15$ | $1.01 \pm 0.13$ | $1.18 \pm 0.15$ | - |
|  | $f_{Z_{b}(10650)}$ | - | - | - | $0.05 \pm 0.04$ | $0.24 \pm 0.11$ | - |
|  | $\phi_{Z_{b}(10650)}$, rad. | - | - | - | $-0.26 \pm 0.68$ | $-1.63 \pm 0.14$ | - |
|  | $f_{\mathrm{nr}}$ | - | $0.48 \pm 0.23$ | $0.41 \pm 0.17$ | - | - | 1.0 |
|  | $\phi_{\mathrm{nr}}, \mathrm{rad}$. | - | $-1.21 \pm 0.19$ | $0.95 \pm 0.32$ | - | - | - |
|  | $-2 \log \mathcal{L}$ | -304.7 | -300.6 | -300.5 | -301.4 | -301.4 | -344.5 |
| $B^{*} B^{*} \pi$ | $f_{Z_{b}(10650)}$ | 1.0 | $1.04 \pm 0.15$ | $0.77 \pm 0.22$ |  |  | - |
|  | $f_{\mathrm{nr}}$ | - | $0.02 \pm 0.04$ | $0.24 \pm 0.18$ |  | 1.0 |  |
|  | $\phi_{\mathrm{nr}}, \mathrm{rad}$. | - | $0.29 \pm 1.01$ | $1.10 \pm 0.44$ |  | - |  |
|  | $-2 \log \mathcal{L}$ | -182.4 | -182.4 | -182.4 |  | -209.7 |  |

We first analyze the $B B^{*} \pi\left[B^{*} B^{*} \pi\right]$ data with the ${ }_{33}$ simplest hypothesis, Model-0, that includes only the ${ }_{334}$ $Z_{b}(10610)\left[Z_{b}(10650)\right]$ amplitude. Results of the fit are $e_{35}$ shown in Fig. 2; the numerical results are summarized ${ }_{336}$ in Table I. The fraction $f_{X}$ of the total three-body sig-337 nal attributed to a particular quasi-two-body intermedi-338 ate state is calculated as $f_{X}=\int\left|\mathcal{A}_{X}\right|^{2} d m / \int S(m) d m, 339$ where $\mathcal{A}_{X}$ is the amplitude for a particular component ${ }_{340}$ $X$ of the three-body amplitude. Next, we extend the hy-341 pothesis to include a possible non-resonant component, ${ }_{342}$ Model-1, and repeat the fit to the data. Then the $B B^{*} \pi_{343}$ data is fit to a combination of two $Z_{b}$ amplitudes, Model ${ }_{-344}$ 2. In both cases, the addition of an extra component $\mathrm{to}_{345}$ the amplitude does not give a statistically significant im-346 provement in the data description: the likelihood value is ${ }_{347}$ only marginally improved (see Table I). The addition of $\mathrm{f}_{348}$ extra components to the amplitude also produces multi-349 ple maxima in the likelihood function. As a result, we use ${ }_{350}$ Model-0 as our nominal hypothesis. Finally, we fit both ${ }_{351}$ samples to a pure non-resonant amplitude (Model-3). $\mathrm{In}_{352}$ this case, the fit is significantly worse.

If the parameters of the $Z_{b}$ resonances are allowed ${ }_{354}$ to float, the fit to the $B B^{*} \pi$ data with Model-0 gives ${ }_{355}$ $10605 \pm 6 \mathrm{MeV} / c^{2}$ and $25 \pm 7 \mathrm{MeV}$ for the $Z_{b}(10610)_{356}$ mass and width, respectively, and the fit to the $B^{*} B^{*} \pi_{357}$ data gives $10648 \pm 13 \mathrm{MeV} / c^{2}$ and $23 \pm 8 \mathrm{MeV}$ for the ${ }_{358}$ $Z_{b}(10650)$ mass and width, respectively. The large errors ${ }_{359}$ here reflect the strong correlation between the resonance ${ }_{360}$ parameters.

The three-body Born cross sections are calculated as

$$
\begin{equation*}
\sigma\left(e^{+} e^{-} \rightarrow f\right)=\frac{N_{f}}{L \mathcal{B}_{f} \alpha \eta\left(1+\delta_{\mathrm{ISR}}\right)|1-\Pi|^{2}} \tag{3}
\end{equation*}
$$

where $N_{f}$ is the three-body signal yield and $L=$ $121.4 \mathrm{fb}^{-1}$ is the total integrated luminosity. The efficiency-weighted sum of $B$-meson branching fractions $\mathcal{B}_{f}$ is determined using both signal MC and two-body $e^{+} e^{-} \rightarrow B^{(*)} \bar{B}^{(*)}$ events in data. To avoid the large systematic uncertainties associated with the determination of reconstruction efficiencies for $B$ and $D$ decays to multibody final states, we select a subset of two-body
modes: $B^{+} \rightarrow \bar{D}^{0}\left[K^{+} \pi^{-}\right] \pi^{+}$and $B \rightarrow J / \psi\left[\ell^{+} \ell^{-}\right] K$, and calculate $\mathcal{B}_{f}=\mathcal{B}_{f}^{\text {sel }} \times N_{B^{(*)} \bar{B}^{(*)}}^{\text {all }} / N_{B^{(*)} \bar{B}^{(*)}}^{\text {sel }}$, where the superscripts "sel" and "all" refer to quantities determined for the selected subset of $B$ decay modes and for the full set of modes, respectively. Two-body $e^{+} e^{-} \rightarrow B^{(*)} \bar{B}^{(*)}$ events are selected with the requirement $0.90 \mathrm{GeV} / c$ $<P(B)<1.35 \mathrm{GeV} / c$; the $B$ yield is determined from the fit to the $M(B)$ distribution. We find $N_{B^{(*)} \bar{B}^{(*)}}^{\text {all }}=$ $10131 \pm 152$ and $N_{B^{(*)} \bar{B}^{(*)}}^{\mathrm{sel}}=2406 \pm 62$. (MC studies show no significant dependence of the reconstruction efficiency on the $B$ momentum.) To account for the non-uniform distribution of signal events over the phase space, we introduce an efficiency correction factor $\eta$ determined from the MC simulation with signal events generated according to the nominal model. Since we do not observe a signal in the $B B \pi$ final state, no correction is made for this channel. A factor $\alpha=0.897 \pm 0.002$ is introduced to correct for the effect of neutral $B$-meson oscillations that is determined using the known $B^{0}$ mixing parameter $x_{d}$ and the yield ratio in data of two-body events with a reconstructed neutral vs. charged $B$ meson. The ISR correction, $1+\delta_{\text {ISR }}$, for the $B^{(*)} B^{*} \pi$ final states is calculated using recent results on $\sigma\left(e^{+} e^{-} \rightarrow h_{b}(m P) \pi^{+} \pi^{-}\right)[15]$ and an observation that the $\Upsilon(5 S) \rightarrow h_{b}(m P) \pi^{+} \pi^{-}$transitions are saturated by the intermediated $Z_{b}$ production [1]; for the $B B \pi$ final state we assume constant cross section. For the vacuum polarization correcrection we use $1 /|1-\Pi|^{2}=0.928[16]$. The results are summarized in Table II.

TABLE II: Summary of results on three-body cross sections. The first (or sole) uncertainty is statistical; the second is systematic.

| Parameter | $B B \pi$ | $B B^{*} \pi$ | $B^{*} B^{*} \pi$ |
| :--- | :---: | :---: | :---: |
| $N_{f}$, Events | $13 \pm 25$ | $357 \pm 30$ | $161 \pm 21$ |
| $\mathcal{B}_{f}, 10^{-6}$ | $293 \pm 22$ | $276 \pm 21$ | $223 \pm 17$ |
| $\eta$ | 1.0 | 1.066 | 1.182 |
| $1+\delta_{\text {ISR }}$ | $0.720 \pm 0.017$ | $0.598 \pm 0.016$ | $0.594 \pm 0.016$ |
| $\sigma, \mathrm{pb}$ | $<2.9$ | $17.4 \pm 1.6 \pm 1.9$ | $8.75 \pm 1.15 \pm 1.04$ |

TABLE III: $B$ branching fractions for the $Z_{b}^{+}(10610)$ and $_{403}^{40}$ $Z_{b}^{+}(10650)$ decays. The first quoted uncertainty is statisti- ${ }^{403}$ cal; the second is systematic.

| Channel | Fraction, $\%$ |  |
| :--- | :---: | :---: |
|  | $Z_{b}(10610)$ | $Z_{b}(10650)$ |
| $\Upsilon(1 \mathrm{~S}) \pi^{+}$ | $0.54_{-0.13-0.08}^{+0.16+0.11}$ | $0.17_{-0.06-0.02}^{+0.07+0.03}$ |
| $\Upsilon(2 \mathrm{~S}) \pi^{+}$ | $3.62_{-0.59-0.79}^{+0.763}$ | $1.39_{-0.38-0.34}^{+0.48+0.33}$ |
| $\Upsilon(3 \mathrm{~S}) \pi^{+}$ | $2.15_{-0.42-0.43}^{+0.55+.60}$ | $1.63_{-0.42+0.28}^{+0.53+0.39}$ |
| $h_{b}(1 \mathrm{P}) \pi^{+}$ | $3.45_{-0.71-0.63}^{+0.87+0.86}$ | $8.41_{-2.12-1.49}^{+2.43+1.49}$ |
| $h_{b}(2 \mathrm{P}) \pi^{+}$ | $4.67_{-1.00-0.18}^{+1.24+18}$ | $14.7_{-2.8-2.3}^{+3.2+2.8}$ |
| $B^{+} \bar{B}^{* 0}+\bar{B}^{0} B^{*+}$ | $85.6_{-2.0-2.1}^{+1.5+59}$ | --.4 |
| $B^{*+} \bar{B}^{* 0}$ | - | $73.7_{-4.4-3.5}^{+3.4+2.7}$ |

The dominant sources of systematic uncertainties for ${ }^{413}$ the three-body production cross sections are the uncer- ${ }^{414}$ tainties in the signal yield extraction $\left(6.9 \%\right.$ for $B B^{*} \pi_{416}^{415}$ and $8.7 \%$ for $B^{*} B^{*} \pi$ ), in the reconstruction efficiency $y_{417}$ (7.6\%) (including secondary branching fractions [11]), $\mathrm{in}_{418}$ the correction factor $\alpha(1 \%)$, in the integrated luminos-419 ity $(1.4 \%)$ and in the ISR correction ( $2.7 \%$ ). The overall420 systematic uncertainties for the three-body cross sections ${ }^{421}$ are estimated to be $7.9 \%, 10.8 \%$, and $12.0 \%$ for the $B B \pi{ }^{422}$ $B B^{*} \pi$, and $B^{*} B^{*} \pi$ final states, respectively.

Using the results of the fit to the $M_{\text {miss }}(\pi) \operatorname{spectra}_{425}^{424}$ with the nominal model (Model-0 in Table I) and the ${ }_{426}$ results of the analyses of $e^{+} e^{-} \rightarrow \Upsilon(n \mathrm{~S}) \pi^{+} \pi^{-}[1]_{427}$ and $e^{+} e^{-} \rightarrow h_{b}(m \mathrm{P}) \pi^{+} \pi^{-} \quad[15, \quad 17]$, we cal-428 culate the ratio of the branching fractions ${ }^{429}$ $\mathcal{B}\left(Z_{b}^{+}(10610) \rightarrow \bar{B}^{0} B^{*+}+B^{+} \bar{B}^{* 0}\right) / \mathcal{B}\left(Z_{b}^{+}(10610) \rightarrow{ }_{431}^{430}\right.$ bottomonium $)=5.93_{-0.69-0.73}^{+0.99+1.01}$ and $\mathcal{B}\left(Z_{b}^{+}(10650) \rightarrow_{432}^{431}\right.$ $\left.B^{*+} \bar{B}^{* 0}\right) / \mathcal{B}\left(Z_{b}^{+}(10650) \rightarrow\right.$ bottomonium $) \quad={ }_{433}$ $2.80_{-0.40}^{+0.69+0.54}$. We also calculate the relative frac-434 tions for $Z_{b}$ decays, assuming that they are saturated ${ }^{435}$ by the already observed $\Upsilon(n \mathrm{~S}) \pi, h_{b}(m \mathrm{P}) \pi$, and $B^{(*)} B^{* 436}$ channels. The results are presented in Table III.

To summarize, we report the first observations of the ${ }^{438}$ three-body $e^{+} e^{-} \rightarrow B B^{*} \pi$ and $e^{+} e^{-} \rightarrow B^{*} B^{*} \pi$ pro- $_{440}^{439}$ cesses with a statistical significance above $8 \sigma$. Measured d41 Born cross sections are $\sigma\left(e^{+} e^{-} \rightarrow\left[B \bar{B}^{*}+\text { c.c. }\right]^{ \pm} \pi^{\mp}\right)={ }_{442}$ $(17.4 \pm 1.6 \pm 1.9) \mathrm{pb}$ and $\sigma\left(e^{+} e^{-} \rightarrow\left[B^{*} \bar{B}^{*}\right]^{ \pm} \pi^{\mp}\right)=443$ $(8.75 \pm 1.15 \pm 1.04) \mathrm{pb}$. For the $e^{+} e^{-} \rightarrow B B \pi$ process, ${ }^{444}$ we set a $90 \%$ confidence level upper limit of $\sigma\left(e^{+} e^{-} \rightarrow^{445}\right.$ $\left.[B \bar{B}]^{ \pm} \pi^{\mp}\right)<2.9 \mathrm{pb}$. The analysis of the $B^{(*)} B^{*} \operatorname{mass}_{447}^{446}$ spectra indicates that the total three-body rates are dom- ${ }_{448}$ inated by the intermediate $e^{+} e^{-} \rightarrow Z_{b}(10610)^{\mp} \pi^{ \pm} \operatorname{and}_{449}$ $e^{+} e^{-} \rightarrow Z_{b}(10650)^{\mp} \pi^{ \pm}$transitions for the $B B^{*} \pi$ and 450 $B^{*} B^{*} \pi$ final states, respectively.

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[13] The bin-by-bin information is available in the on-line supplementary material.
[14] Determined using mixing parameter $x_{d}$ and the ratio of charged to neutral $B$ yields measured in data from twobody $e^{+} e^{-} \rightarrow B^{(*)} \widehat{B^{(*)}}$ processes. Renormalization factor of $1.19 \pm 0.01$ is also applied.
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