Multiobjective Optimal Waste Load Allocation Models for Rivers Using Nondominated Sorting Genetic Algorithm-II

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Abstract: A multiobjective optimization framework for optimal waste load allocation in rivers is proposed, considering (1) the total treatment cost, (2) the equity among the waste dischargers, and (3) a comprehensive performance measure that reflects the dissolved oxygen (DO) violation characteristics. This framework consists of an embedded river water quality simulator that has a gradually varied flow module and a pollutant transport module, which simulates the transport process including reaction kinetics (in terms of biochemical oxygen demand-DO). The outer shell of the framework consists of the two nonseasonal, deterministic, multiobjective waste load allocation planning models, namely, cost-performance model and cost-equity-performance model. These models are solved using a powerful and recently developed multiobjective genetic algorithm technique known as the Nondominated Sorting Genetic Algorithm-II. The practical utility of the multiobjective framework in decision-making is illustrated through a realistic example of the Willamette River in the state of Oregon.

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Introduction

Water quality protection along rivers involves water quality monitoring and assessment, establishing water quality goals, and controlling pollutant discharges, so that an acceptable level of water quality is maintained. The control of water quality in any river/stream at various locations, requires the determination of the optimal pollutant removal levels at a number of point and nonpoint source locations along the river (that would yield a satisfactory water quality response) in a cost-effective, equitable, and efficient manner (Burn 1987; Burn and Yulianti 2001). This is known as "optimal waste load allocation."

Typical multiobjective optimal waste load allocation problems address minimization of the total treatment cost and minimization of the inequity among the pollutant dischargers, subject to constraints on satisfaction of a specified dissolved oxygen (DO) standard at all the check points located along the river (Brill et al. 1984; Srigiriraju 2000; Burn and Yulianti 2001). Performance measures such as number of DO violations, magnitude of maximum DO violation, and total magnitude of DO violations at the checkpoints can be expressed either as additional objectives or as constraints in the optimization model. Burn and Lence (1992) proposed four different optimization formulations of a general waste load allocation model to evaluate efficient management solutions. Two of these formulations were based on maximum deviations from a specified DO standard and the other two on total deviations from the same. Cardwell and Ellis (1993) proposed a series of optimization models with the aim to minimize the control cost and either the number of water quality standard violations or some measure of the magnitude of violations. These multiobjective models can be used to generate trade-offs between cost and either frequency or magnitude of water quality standard violations. Recently Burn and Yulianti (2001) have formulated two planning models with treatment cost as one of the objectives, and either total magnitude of DO violations or equity as the other objective.

The number of objective functions to be handled will increase to five if all three previously mentioned performance measures are to be included in the cost-equity based optimal waste load allocation model as separate objective functions, in order to ensure complete representation of the system. Obtaining the Pareto-optimal solutions would become very difficult in such a case. Also, analysis of the trade-off relationships among the various objectives would become complicated. Therefore, it is useful to derive a comprehensive performance measure that would include (1) the number of DO violations, (2) the magnitude of maximum DO violation, and (3) the total magnitude of DO violations. Also, cost-equity formulations do not offer flexibility to the decision maker in terms of allowing some prespecified violations if strict adherence to a DO standard is included as a constraint in the formulation. At times, the decision maker may wish to find out if there are reasonable cost-equity trade-off solutions for a given system, for a desired performance level that may be less than 100%. To the writers' knowledge, no studies addressing the above-mentioned issues have been reported in the literature.

The classical constraint method of multiobjective programming (Cohon 1978) is used to solve the optimal waste load allo-

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cation problem in most of the earlier studies. This method has a number of limitations (Deb 1995). The optimization of a single weighted objective function may guarantee a Pareto-optimal solution but results in a single point solution. A new optimization run has to be made each time the decision maker changes the combinations of weightages. Also, these methods may not work effectively if some of the objectives are noisy or have discontinuous variable space. The most significant drawback of these algorithms is their sensitivity toward weights.

The optimal treatment levels for a given set of pollutant sources are affected by the assimilative capacity of the receiving water body. Finding the optimal waste load allocation strategy requires a simulation model for the prediction of the steady-state water quality response in terms of DO at specified receptor locations along a river, for various possible combinations of waste loadings. One of the most commonly used water quality simulation models in waste load allocation planning studies is the Streeter-Phelps equation (Streeter and Phelps 1925; Burn and McBean 1985; Vasquez et al. 2000). However, this model does not consider the varying nature of flows and the effects due to dispersive transport. In recent times, Carmichael and Strzepek (2000), Burn and Yulianti (2001), and Maier et al. (2001) have used the Enhanced Stream Water Quality Model, QUAL2E (Brown and Barnwell 1987). This model accounts for dispersive transport, and seems to predict biochemical oxygen demand (BOD)-DO response reasonably well in case of long uninterrupted river reaches. However, this model assumes the river flows to be steady and quasi-uniform. Therefore, the backwater effects caused due to tributary flows or due to the presence of a downstream control structure are not appropriately accounted for in this model. This may lead to plausible differences in the waste load allocation decision making.

In this work, a multiobjective optimization framework for optimal waste load allocation in rivers/streams is proposed. This includes two waste load allocation optimization models: a cost-performance model and a cost-equity-performance model. An overall performance measure is proposed with regard to satisfying a prespecified DO standard along the river. These waste load allocation models use a water quality simulation model, which considers the flow to be steady, but nonuniform. The simulation model considers the advective, dispersive, and reactive transports for BOD and DO. A powerful and recently developed multiobjective genetic algorithm technique known as Nondominated Sorting Genetic Algorithm-II (NSGA-II) (Deb et al. 2000) is used for solving the multiobjective optimization models. This algorithm uses the crowding technique to ensure diversity among nondominated solutions. This method is computationally efficient and is capable of finding a good spread of Pareto-optimal solutions (Deb et al. 2000). From the trade-off surfaces generated, the decision maker can select the appropriate treatment strategy for the river system under consideration.

Overall Performance Measure

An alternative overall performance measure for the water quality of the system is proposed in this study. The proposed performance measure is expressed as a weighted sum of the individual performance measures with regard to the number of DO standard violations, magnitude of maximum DO standard violation and magnitude of total DO standard violations, with respect to a specified water quality standard, over all the checkpoints considered within the system. This overall performance measure can be computed for a given waste load allocation policy to be followed in the river system. The three individual performance measures are expressed as follows.

1. The performance measure in terms of *number of violations*, E_N is expressed as the ratio of the "difference between number of DO standard violations corresponding to no treatment, N_0 and that corresponding to actual treatment, N_a" to the "number of DO standard violations corresponding to no treatment, N_0." That is

$$E_N = \frac{N_0 - N_a}{N_0} \tag{1}$$

$$N_0 = f_1((O_j)_0, O_{\mathrm{std}}) \tag{2}$$

$$N_a = f_4((O_j)_a, O_{\mathrm{std}}) \tag{3}$$

where

$$N_a = \sum_{j=1}^{\mathrm{NC}} (y_j)_a \tag{4}$$

in which

$$(y_j)_a = \begin{cases} 1 & \text{if } O_{\mathrm{std}} > (O_j)_a \\ 0 & \text{if } O_{\mathrm{std}} \leqslant (O_j)_a \end{cases} \quad \forall\, j \tag{5}$$

and NC=number of checkpoints. The index $(y_j)_a$ keeps the count of DO standard violation at a checkpoint j (zero-one integer variable). O_j=dissolved oxygen concentration at the checkpoint j, and subscripts 0 and a indicate the level of treatment corresponding to no treatment and actual treatment, respectively. O_{std}=specified dissolved oxygen standard for the river.

2. The performance measure in terms of *magnitude of maximum violation*, E_V is expressed as the ratio of the "difference between the magnitude of maximum DO standard violation corresponding to no treatment and that corresponding to actual treatment" to the "magnitude of maximum DO standard violation corresponding to no treatment." That is

$$E_V = \frac{V_0 - V_a}{V_0} \tag{6}$$

$$V_0 = f_2((O_j)_0, O_{\mathrm{std}}) \tag{7}$$

$$V_a = f_5((O_j)_a, O_{\mathrm{std}}) \tag{8}$$

where

$$V_a = \max_j [(S_1)_a, (S_2)_a, \ldots, (S_j)_a] \tag{9}$$

in which

$$(S_j)_a = \begin{cases} (O_{\mathrm{std}} - (O_j)_a) & \text{if } O_{\mathrm{std}} > (O_j)_a \\ 0 & \text{if } O_{\mathrm{std}} \leqslant (O_j)_a \end{cases} \quad \forall\, j \tag{10}$$

In Eqs. (7)–(10), V=magnitude of maximum DO standard violation, with subscripts 0 and a, having the same meaning as defined earlier.

3. The performance measure in terms of *total magnitude of violations*, E_{TS} is expressed as the ratio of the "difference between magnitude of total DO standard violations corresponding to no treatment and that corresponding to actual treatment" to the "magnitude of total DO standard violations corresponding to no treatment." That is

$$E_{\mathrm{TS}} = \frac{\mathrm{TS}_0 - \mathrm{TS}_a}{\mathrm{TS}_0} \tag{11}$$

$$\mathrm{TS}_0 = f_3((O_j)_0, O_{\mathrm{std}}) \tag{12}$$

$$\mathrm{TS}_a = f_6((O_j)_a, O_{\mathrm{std}}) \tag{13}$$

$$\mathrm{TS}_a = \sum_{j=1}^{\mathrm{NC}} (S_j)_a \tag{14}$$

In Eqs. (11)–(14), TS = magnitude of total DO standard violations, with subscripts j, 0, and a having the same meaning as defined earlier.

The *overall performance measure* of a waste load allocation policy (E_{WLA}) is expressed as a weighted sum of the three individual performance measures, E_N, E_V, and E_{TS}, already defined. That is

$$E_{\mathrm{WLA}} = (w_N E_N + w_V E_V + w_{\mathrm{TS}} E_{\mathrm{TS}}) \tag{15}$$

where w_N, w_V, and w_{TS} = weights associated with the performance measures corresponding to the number of violations of the DO standard, the magnitude of maximum violation of the DO standard, and the magnitude of total violations of the DO standard, respectively. These weights are to be assessed by the decision maker for the particular case being studied.

Model Formulation

Two nonseasonal, deterministic, multiobjective waste load allocation planning models are formulated in this study, namely, a cost-performance model and a cost-equity-performance model.

Cost-Performance Model

The proposed cost-performance optimal waste load allocation model considers minimization of the total waste treatment cost and maximization of the overall performance [given by Eq. (15)] of the water quality system. It will be appropriate to express the performance of a water quality system for a given waste load allocation policy in terms of the resulting water quality violation characteristics against the specified water quality standards in that system. Quite often, it may not be an economically worthwhile proposition to implement a waste load allocation policy that would strictly adhere to the DO standard at all checkpoints along a river. This is relevant especially in stretches of rivers with limited assimilative capacity. In such cases, permitting a few violations may bring down the cost of treatment significantly. In such situations, it is useful for the decision maker to find out effective trade-offs between cost of treatment and overall performance.

The formulation of the proposed cost-performance optimization model is as follows:

$$\min Z_1 = \sum_{i=1}^{\mathrm{NS}} c_i(x_i) \tag{16}$$

$$\max Z_2 = E_{\mathrm{WLA}} \tag{17}$$

subject to

$$x_i \in xs_i \quad \forall\, i \tag{18}$$

$$E_{\mathrm{WLA}} = (w_N E_N + w_V E_V + w_{\mathrm{TS}} E_{\mathrm{TS}}) \tag{19}$$

$$(O_j)_a = f(\mathbf{x}_{\mathrm{a}}, \mathbf{W}, \mathbf{Q}, T, \mathbf{K}) \quad \forall\, j \tag{20}$$

in which $\mathbf{x}_{\mathrm{a}}$ = vector of waste removal levels corresponding to an arbitrary treatment; $c_i(x_i)$ = cost of the waste treatment at source i; x_i = waste removal fraction at source i; NS = number of point source locations; xs_i = set of all waste treatment options for source i. In Eq. (20), $f(\cdot)$ expresses the water quality as a function of the waste inputs and stream conditions; $\mathbf{W}$ = vector of waste inputs to the point sources; $\mathbf{Q}$ = vector of flow rates for main stream and tributaries within the river system; T = water temperature; and $\mathbf{K}$ = vector of reaction rate coefficients describing the pollutant transport process.

In this model, the values of N_0, V_0, and TS_0 have to be found in advance by simulating the DO response corresponding to no treatment level. The same should be used in the model while evaluating the individual performance measures. However, it is to be noted that the values of N_a, V_a, and TS_a are determined for a particular arbitrary treatment level during the process of optimization.

Cost-Equity-Performance Model

In waste load allocation models, it is important to consider the performance of the water quality system as an objective or a constraint in the model formulation along with the two objectives of minimization of total treatment cost and minimization of inequity among the waste dischargers. Ideally, generating the trade-off surface between total treatment cost and inequity measure, subject to a specified performance level, is of interest to decision makers in the optimal waste load allocation problem. The decision-maker may wish to find out if there are reasonable cost-equity trade-off solutions for a given system, for a desired performance level that may be less than 100%. This performance level can be prespecified as a lower limit through a constraint in the optimization model. Therefore, the model formulation is as follows:

$$\min Z_1 = \sum_{i=1}^{\mathrm{NS}} c_i(x_i) \tag{21}$$

$$\min Z_2 = \sum_{i=1}^{\mathrm{NS}} \left| \frac{x_i}{\bar{x}} - \frac{W_i}{\bar{W}} \right| \tag{22}$$

subject to

$$x_i \in xs_i \quad \forall\, i \tag{23}$$

$$E_{\mathrm{WLA}} \geqslant E_S \tag{24}$$

$$(O_j)_a = f(\mathbf{x}_{\mathrm{a}}, \mathbf{W}, \mathbf{Q}, T, \mathbf{K}) \quad \forall\, j \tag{25}$$

where E_{WLA} = overall performance measure of a waste load allocation policy given by Eq. (19); $\bar{x}$ = average waste removal level for the collection of NS number of point sources; W_i = waste input for source i; $\bar{W}$ = average waste input over NS number of point sources; and E_S = water quality system performance specified by the decision maker. $f(\cdot)$ in Eq. (25) defines the water quality as a function of the waste inputs and stream conditions.

Framework for Multiobjective Optimization

The proposed optimal waste load allocation model framework is shown in Fig. 1. It consists of the multiobjective optimization

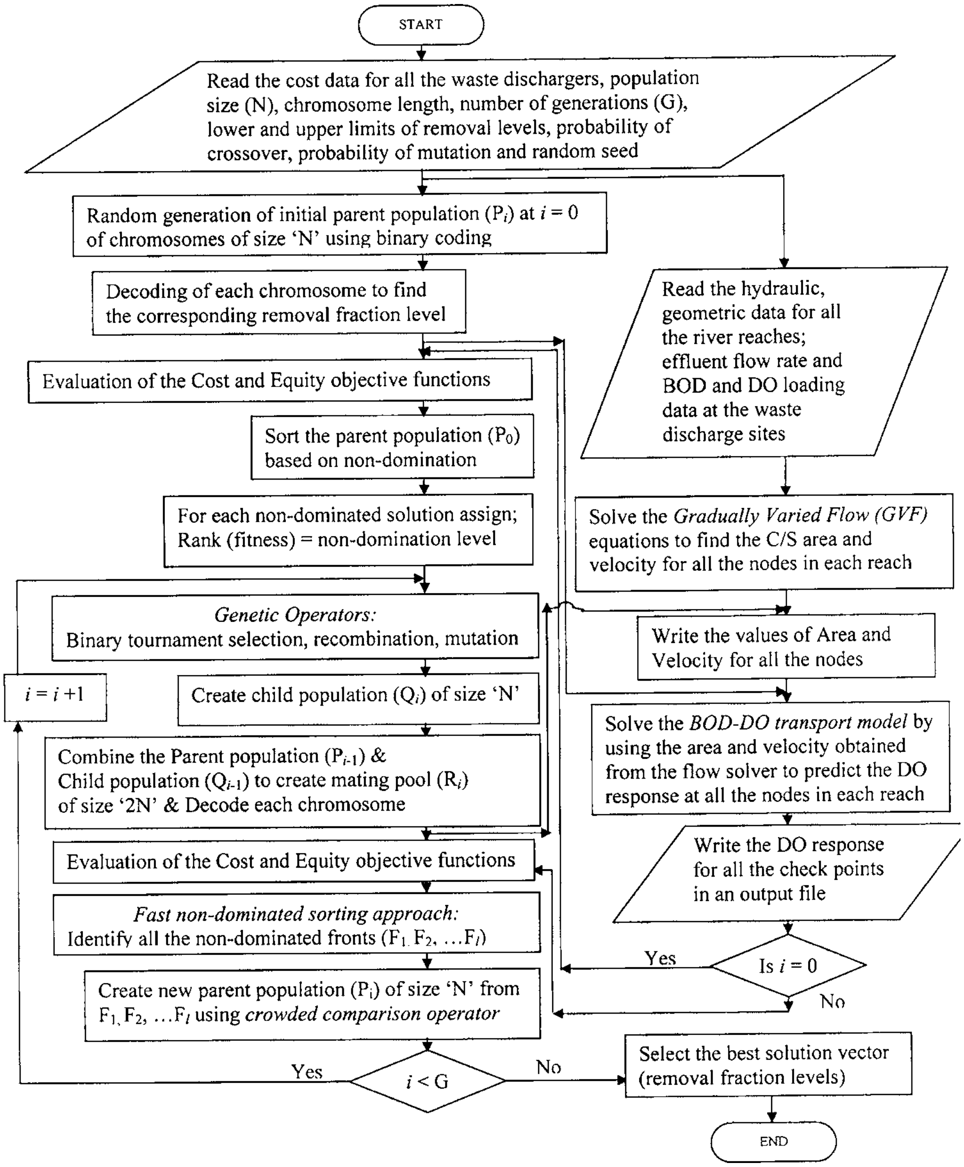

Fig. 1. Optimal waste load allocation model framework

model, with the water quality simulation model embedded into it. This framework can accommodate any one of the two optimization model formulations proposed in this study.

The multiobjective evolutionary algorithm, "Nondominated Sorting Genetic Algorithm-II (NSGA-II)" of Deb et al. (2000) is used to generate the optimal trade-offs between the objectives. Each of the alternative waste load allocation solutions generated from the NSGA-II module is sent to the water quality simulator (Fig. 1) and the predicted DO responses at all the checkpoint locations are evaluated (against the DO standard specified). Following this, the waste load allocation solution is sent to the NSGA-II module for fitness function evaluation. After this, these solutions are sorted according to the fast nondominated approach to identify different levels of nondominated fronts. Subsequently new populations are created using the Tournament selection operator and crowded comparison operator. This process is repeated until the specified stopping criterion is achieved and the final set of nondominated solutions is stored in an output file.

Nondominated Sorting Genetic Algorithm (NSGA-II)

As mentioned earlier, classical optimization techniques have certain drawbacks (Deb 1995). In contrast, multiobjective evolutionary algorithms (MOEAs) are suitable for multiobjective optimization due to their ability to handle complex problems, involving features such as discontinuities, multimodality, disjoint feasible spaces and noisy function evaluations (Fonseca and Fleming 1995). Genetic algorithms deal with a population of points and hence multiple Pareto-optimal solutions can be captured from the population in a single run. Moreover, the MOEA search procedure is also algorithmically efficient (Deb et al. 2003). Deb et al. (2000) proposed a computationally fast elitist Nondominated Sorting Genetic Algorithm-II (NSGA-II) to overcome the drawbacks (such as high computational complexity of nondominated sorting, lack of elitism and the need for specifying a sharing parameter) of the earlier nondominated sorting based MOEAs. This algorithm uses: (1) a crowding approach for diversity preservation, in which N' (as large as $2N$, where N=population size)

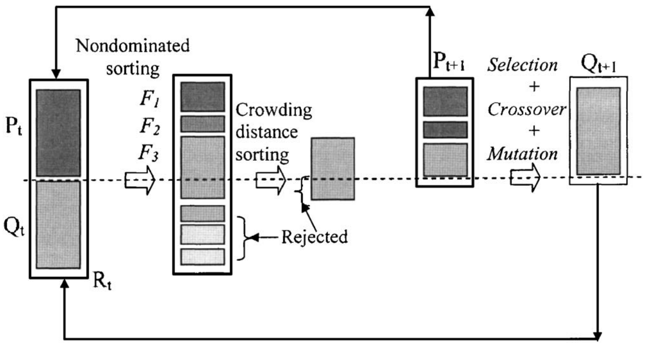

Fig. 2. Schematic of NSGA-II algorithm

solutions are processed objective-wise; (2) an elitism operator that helps in significantly speeding up the performance of the genetic algorithm, and in preserving good nondominated solutions; and (3) an objective-wise distance computation.

The steps of this algorithm are as follows (Deb et al. 2000):

1. Create a random parent population (P_0) of size N, initially.
2. Sort the random parent population based on nondomination.
3. For each nondominated solution, assign a fitness (rank) equal to its nondomination level (1 is the best level, 2 is the next-best level, and so on).
4. Create a child population (Q_0) of size N using binary tournament selection, recombination, and mutation operators.
5. From the first generation onwards, creation of each new generation constitutes the following steps:
   - Create the mating pool (R_t) of size $2N$ by combining the parent population (P_t) and the child population (Q_t).
   - Sort the combined population (R_t) according to the *fast nondominated sorting procedure* (Deb et al. 2000) to identify all nondominated fronts ($F_1, F_2, \ldots, F_l$).
   - Generate the new parent population (P_{t+1}) of size N by adding nondominated solutions starting from the first ranked nondominated front (F_1) and proceeding with the subsequently ranked nondominated fronts ($F_2, F_3, \ldots, F_l$), till the size exceeds N (Fig. 2). This means that the total count of the nondominated solutions from the fronts $F_1, F_2, \ldots, F_l$, exceeds the population size N. Now, in order to make the total count of the nondominated solutions equal to N, it is required to reject some of the lower ranked nondominated solutions from the last (F_lth) front. This is achieved through a sorting done according to the *crowded comparison operator* ($\geqslant_n$) based on the crowding distance assigned to each solution contained in the F_lth nondominated front. Thus, the new parent population (P_{t+1}) of size N is constructed.
   - Perform the selection, crossover and mutation operations on the newly generated parent population (P_{t+1}) to create the new child population (Q_{t+1}) of size N (Fig. 2).
6. Repeat Step 5 until the maximum number of generations is reached.

Water Quality Simulation Model

The water quality simulation model is used to model the physical and biochemical processes that describe the transport of BOD and DO in the river. This water quality simulation model consists of two modules, namely, a flow module and a transport module. The flow module is used to determine the gradually varied water surface profile, flow cross-sectional area and mean velocity at various nodes in the river domain, given: (1) The flow rates in the main river and the tributaries; (2) the bed profile of the river; (3) the geometric characteristics of the river cross sections; (4) the Manning's roughness coefficient; and (5) the control depth at the downstream end. The classical standard step method (Chaudhry 1993) is used for this purpose.

The transport module uses the flow area and mean velocity obtained from the flow module while solving the BOD and DO transport equations, which are given as follows (Dresnack and Dobbins 1968):

$$\frac{\partial(AB)}{\partial t} = \frac{\partial\left(AD\dfrac{\partial B}{\partial x}\right)}{\partial x} - \frac{\partial(AUB)}{\partial x} - K_B BA + \frac{S_B}{\Delta x} + L_B A \tag{26}$$

$$\frac{\partial(AO)}{\partial t} = \frac{\partial\left(AD\dfrac{\partial O}{\partial x}\right)}{\partial x} - \frac{\partial(AUO)}{\partial x} - K_D BA + K_R(O^{\mathrm{SAT}} - O)A + \frac{S_O}{\Delta x} + R_D A \tag{27}$$

where A=flow cross-sectional area (m^2); U=cross-sectional average velocity (m/s); x=distance along the river (m); t=time (s); $S_{B/O}$=point source for BOD/DO loading (mg/s); B=biochemical oxygen demand (BOD) concentration (mg/L); O=dissolved oxygen (DO) concentration (mg/L); O^{SAT}=saturation DO concentration (mg/L); D=dispersion coefficient for BOD/DO transport (m^2/s); K_B=BOD decay rate (1/day); K_D=BOD deoxygenation rate (1/day); L_B=BOD distributed source (mg/L/s); K_R=re-aeration rate (1/day); and R_D=DO distributed source (mg/L/s).

In Eqs. (26) and (27), the spatial variation of dispersion coefficient D is determined using the Seo and Cheong (1998) equation:

$$D = 5.915(hU_*)\left(\frac{W}{h}\right)^{0.620}\left(\frac{U}{U_*}\right)^{1.428} \tag{28}$$

where h=flow depth (m); W=channel width (m); and U_*=shear velocity (m/s). The re-aeration coefficient (K_R) values are determined using O'Connor and Dobbins (1958) formula, given by

$$K_R = \frac{3.90U^{0.5}}{h^{1.5}} \tag{29}$$

The transport simulation model solves Eqs. (26) and (27) to determine the BOD and DO variation with time and distance, given: (1) Initial variation (at t=0) of BOD and DO along the stream; (2) the time variation of BOD and DO at the upstream end (x=0); (3) dispersive coefficient, D; (4) reaction rates K_B, K_D, and K_R; (5) point and nonpoint source loading, S_B, S_O, L_B, and R_D; and (6) the spatial variation of flow velocity, U, and the flow area, A. A standard partial implicit finite-difference method, with spatial derivative terms approximated by backward finite differences, is used for numerically solving the transport equations. The predicted DO concentration values at all checkpoints are stored for further use in the optimization model. The simulation model has been validated using the experimental and analytical solutions available in the literature (Hann and Young 1972; Adrian and Alshawabkeh 1997; Ahmad et al. 1999). Details of validation are available in Murty (2003), and are not presented here, for the sake of brevity.

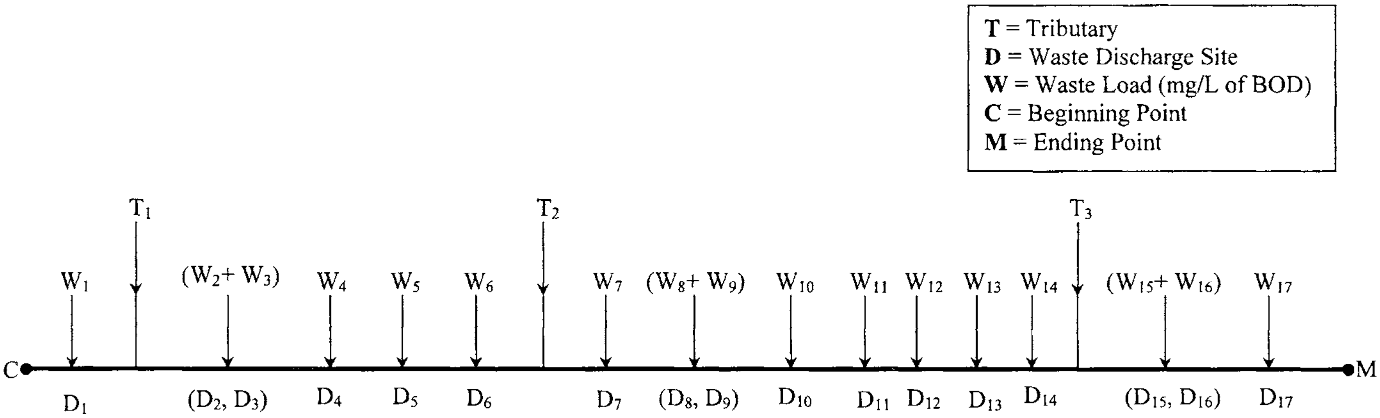

Fig. 3. Locations of waste load discharge sites and tributaries (Willamette River System, Oregon)

Data for Example River System

In this study, the usefulness of the proposed multiobjective optimal waste load allocation framework is illustrated through an example application to the realistic case of Willamette river system in the state of Oregon. Three of the state's largest cities, Portland, Salem, and Eugene, are located within this basin. The line diagram shown in Fig. 3 indicates the locations of the seventeen waste discharge sites ($D_1, D_2, \ldots, D_{17}$) and three tributaries (T_1, T_2, and T_3) along the Willamette river, considered in this study. The hydraulic and the geometric characteristics of the river system are presented in Table 1, based on the data given in the reports of Tetra Tech (1993, 1995a,b). The cross section of the river is assumed to be wide rectangular throughout. The effluent flow data at the 17 point sources and the flow from the three tributaries are given in Table 2, as extracted from Tolson (2000) and Tetra Tech (1995a). For modeling purpose, in this study, these 17 pollutant discharging sources are effectively reduced to 14 in number, by clubbing the sources 2 and 3, sources 8 and 9, and sources 15 and 16. This is because the chainage and the loading at the two sources within each of the three pairs mentioned, are found to be identical (refer to Table 1). The initial background concentrations of BOD and DO in the main river and in the tributaries are assumed to be 1.5 and 9.1 mg/L, respectively, based on the report by Tetra Tech (1993). For point sources, the DO is assumed to be 2.0 mg/L. The deoxygenation coefficient is assumed to be 0.30/day based on information from Tetra Tech reports. The estimation of the reaeration coefficient is obtained using the O'Connor and Dobbins (1958) equation, based on the finding reported in Tetra Tech (1995a). The dispersion coefficient is calculated using Seo and Cheong (1998) equation. Sixty checkpoints ($C_1, C_2, \ldots, C_{60}$) are considered along the 300 km length of the river system in a well distributed manner. The wastewater treatment plant cost data for the average influent flow rate for different point sources are given in Table 3 (constructed based on the data given in Lence et al. 1990). For the numerical simulation of the transport of pollutant, spatial step, $\Delta X = 250$ m and tempo-

Table 1. Flow and Geometric Data for the Willamette River System

Reach	Reach index	Length (km)	Width (m)	S_0	Flow (m^3/s)	Flow depth (m)	Velocity (m/s)	C/S area (m^2)
$C-D_1$	R1	15	129.5	0.00072	87.73	1.084	0.625	140.477
D_1-T_1	R2	4	137	0.00072	88.85	0.925	0.679	130.808
$T_1-D_{2/3}$	R3	45	107	0.00072	154.74	0.865	0.980	157.824
$D_{2/3}-D_4$	R4	26	102	0.00072	155.47	1.608	0.951	163.503
D_4-D_5	R5	20	126	0.00034	155.74	2.523	0.518	300.808
D_5-D_6	R6	4	152	0.00034	155.96	1.278	0.801	194.827
D_6-T_2	R7	14	183	0.00034	156.31	1.037	0.824	189.767
T_2-D_7	R8	47	243	0.00034	223.62	0.979	0.940	238.017
$D_7-D_{8/9}$	R9	45	183	0.000023	224.97	5.820	0.209	1,077.642
$D_{8/9}-D_{10}$	R10	18	190.5	0.000023	225.62	6.693	0.179	1,263.235
$D_{10}-D_{11}$	R11	9	305	0.000023	225.70	4.486	0.164	1,379.396
$D_{11}-D_{12}$	R12	9	211	0.000023	225.74	1.572	0.682	331.201
$D_{12}-D_{13}$	R13	2	122	0.000023	226.21	6.956	0.267	848.423
$D_{13}-D_{14}$	R14	2	274	0.000019	226.54	6.325	0.131	1,735.098
$D_{14}-T_3$	R15	2	274	0.000019	226.80	6.327	0.131	1,735.698
$T_3-D_{15/16}$	R16	6	305	0.000019	259.16	7.844	0.108	2,390.750
$D_{15/16}-D_{17}$	R17	3	457	0.000019	262.04	5.251	0.109	2,400.676
$D_{17}-M$	R18	29	396	0.000019	262.36	8.986	0.073	3,584.481

Note: $D_1, D_2, \ldots, D_{17}$=pollutant discharge sites; T_1, T_2, T_3=tributaries; and C, M=beginning and ending sections in the river system considered.

Table 2. Effluent Data for the Willamette River System

Waste discharge site/tributary	Point source index	Location (km)	Effluent/ tributary flow rate (m^3/s)	BOD_{ult}[a] (mg/L)	DO (mg/L)
Metropolitan Wastewater Management Commission—Eugene	D_1	285	1.124	308	2.0
Pope & Talbot Inc.	D_2	236	0.552	180	2.0
James River Paper Co. Inc.	D_3	236	0.173	33	2.0
City of Corvallis	D_4	210	0.272	528	2.0
City of Albany	D_5	190	0.224	565	2.0
Willamette Industries Inc.	D_6	186	0.346	272	2.0
City of Salem	D_7	125	1.352	740	2.0
City of Newberg	D_8	80	0.07	523	2.0
Smurfit Newsprint Corp. (Newberg)	D_9	80	0.583	521	2.0
City of Wilsonville	D_{10}	62	0.075	700	2.0
City of Canby	D_{11}	53	0.044	475	2.0
Smurfit Newsprint Corp. (Oregon City)	D_{12}	44	0.465	757	2.0
Simpson Paper Company	D_{13}	42	0.328	231	2.0
Tri-City Service District	D_{14}	40	0.263	688	2.0
City of Portland	D_{15}	32	2.765	750	2.0
Oak Lodge Sanitary District	D_{16}	32	0.114	550	2.0
Clackamas Co. Service District #1	D_{17}	29	0.316	825	2.0
McKenzie River	T_1	281	65.89	1.5	9.1
Santiam River	T_2	172	67.31	1.5	9.1
Clackamas River	T_3	38	32.37	1.5	9.1

Note: $D_1, D_2, \ldots, D_{17}$=pollutant discharge sites and T_1, T_2, T_3=tributaries.

[a]Wastewater Treatment Plant (WWTP) BOD_5 values are converted into ultimate BOD values using a conversion factor of 2.5 and Pulp and Paper Mill (PPM) BOD_5 values are converted into ultimate BOD values using a conversion factor of 4.1. [As used by the ODEQ while calibrating the QUAL2E model (Tetra Tech 1993; 1995a)].

ral step, Δt=8,640 s. The total simulation time is taken to be 7 days.

MOGA parameters

The chromosome employed in the MOGA code consists of NS genes (NS=number of point source loadings), each gene representing a coding for the required pollutant removal level at the respective point source. In this study, binary coding is used to define the discrete pollutant removal levels, ranging between 35 and 98%, at increments of 1%. Thus each decision variable in a chromosome has 64 (98–35%) possible values. The total chromosome length for any decision vector of pollutant removal levels for this case example is 102 bits, with 6 bits allotted for each binary coded decision variable.

The randomly generated initial population of size N should provide sufficient sampling of the decision space, while limiting the computational burden. Goldberg (1989) suggests the desirable range of population size to be from 30 to 100. In this study, a population size of 40 is used after making several runs within the range mentioned. The NSGA-II algorithm adopted in this study uses tournament selection for creating one or more off-springs from a pair of individuals. A single point crossover is adopted for this purpose. The crossover operation is ruled by the probability

Table 3. Treatment Cost Data

Point source	BOD (mg/L)	\$ million/year 35%	67%	90%	98%
D_1	308	1.987	2.235	2.422	5.340
$D_{2,3}$	145	0.695	0.870	1.523	3.456
D_4	528	3.406	3.832	4.152	9.155
D_5	565	3.645	4.101	4.443	9.796
D_6	272	1.303	1.632	2.856	6.483
D_7	740	4.774	5.371	5.819	12.831
$D_{8,9}$	521	3.361	3.781	4.097	9.033
D_{10}	700	4.516	5.081	5.504	12.137
D_{11}	475	3.065	3.448	3.735	8.236
D_{12}	757	3.627	4.542	7.949	18.042
D_{13}	231	1.107	1.386	2.426	5.506
D_{14}	688	4.439	4.994	5.410	11.929
$D_{15,16}$	742	4.787	5.385	5.834	12.865
D_{17}	825	5.323	5.988	6.487	14.304

of crossover (p_c), which normally ranges from 0.5 to 1.0 (Goldberg 1989). The mutation operation is applied to maintain the diversity in the population of chromosomes with a view to avoid being trapped in local optima. The mutation mechanism has a low probability (p_m), usually ranging between 0.005 and 0.020. The parameter values used in this study, namely, p_c=0.80 and p_m=0.009 have been arrived at, based on several sensitivity analysis runs. For the selected parameters mentioned above, NSGA-II has been run for a different number of generations (from 25 onwards), for the cost-equity multiobjective optimization model. It is observed from these runs that the Pareto-optimal fronts stabilize by 200 generations. Therefore, it has been decided to adopt 200 as the number of generations for all the multiobjective GA runs in this study.

Results and Discussion

Selected results for the Willamette river, Oregon. are presented herein to illustrate the application of the proposed optimal waste load allocation models for realistic systems. The focus of the discussion is on the usefulness of the multiobjective trade-off re-

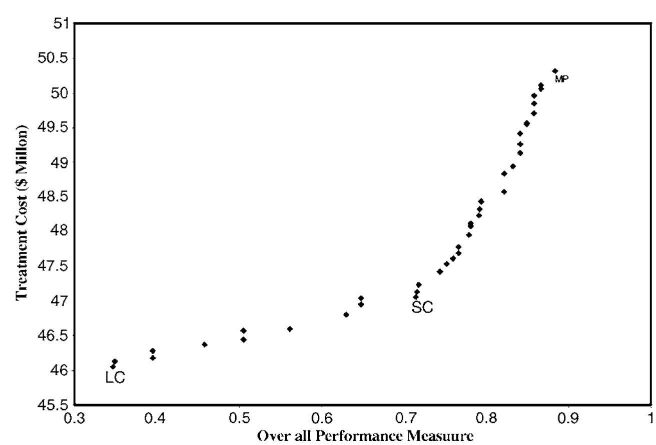

Fig. 4. Cost-performance trade-off (DO Std=7.0 mg/L)

Table 4. Cost-Performance Trade-Off-Cost-Performance Model

DO_{std} (mg/L)	Index	Solution point (TC, OSP)	Removal fraction levels $(x_1, x_2, \ldots, x_{14})$	Inequity measure
6.0	LC	46.035, 0.457	0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35	4.738
	SC	46.400, 0.677	0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.56, 0.35, 0.35	5.357
	MP	46.861, 0.976	0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.82, 0.35, 0.35	6.989
6.5	LC	46.052, 0.408	0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.36, 0.35, 0.35	4.696
	SC	46.580, 0.650	0.35, 0.35, 0.36, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.36, 0.65, 0.35, 0.35	5.904
	MP	47.197, 0.924	0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.51, 0.35, 0.35, 0.90, 0.35, 0.35	8.176
7.0	LC	46.047, 0.347	0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.35, 0.36, 0.35, 0.35, 0.35, 0.35, 0.35,	4.784
	SC	47.056, 0.715	0.35, 0.35, 0.35, 0.35, 0.36, 0.36, 0.35, 0.35, 0.36, 0.35, 0.36, 0.90, 0.35, 0.35	7.548
	MP	50.312, 0.884	0.35, 0.35, 0.35, 0.35, 0.36, 0.35, 0.37, 0.48, 0.90, 0.75, 0.66, 0.90, 0.35, 0.35	11.112

Note: TC=total cost (million $); OSP=overall system performance; LC=least cost solution; SC=selected compromise solution; and MP=maximum performance solution.

lations obtained from the framework in waste load allocation decision making.

Illustration for Cost-Performance Model

In order to illustrate the usefulness of the cost-performance model, results from three runs corresponding to prespecified DO standards of 6.0, 6.5, and 7.0 mg/L are presented in this section. A normal depth of 8.986 m is used as the control depth at the downstream boundary (Table 1) in the water quality simulation model. It is to be noted that violations of DO standard are permitted in this case. In each run, the number of violations, the vulnerability (maximum violation at a single checkpoint) and the magnitude of total violation are computed with respect to the DO standard specified. However, those solutions with more number of violations and/or a larger magnitude of violation are automatically rejected by the NSGA-II algorithm during the search process.

The Pareto-optimal front describing the cost-performance trade-off, for the prespecified DO standard of 7.0 mg/L, is presented in Fig. 4. In this trade-off curve, the two extreme points represent the least cost (LC) pareto-optimal solution and the maximum performance (MP) pareto-optimal solution. Results of these two solutions are summarized in Table 4 along with the selected compromise (SC) solution for all three runs considered. It may be observed from Table 4 that the optimal treatment levels and the corresponding treatment costs are significantly higher for MP than LC, which is to be expected. The difference in treatment costs between LC and MP solutions increases from $\$0.826 \times 10^6$ to $\$4.265 \times 10^6$ as the DO standard increases from 6.0 to 7.0 mg/L. Thus, it may be inferred that selection of the compromise solution is critical, if a higher DO standard needs to be maintained. The compromise solution (SC) for the DO standard 7.0 mg/L is shown in Fig. 4. This is selected such that there is a distinct change in the slope of the trade-off curve at that point. It can be seen from Table 4 that the total treatment cost for SC solution ($\$47.056 \times 10^6$) is not significantly higher than that of the LC solution ($\$46.047 \times 10^6$), although there is a significant

Table 5. Violation Characteristics and Performance Measures of Pareto-Optimal Solutions–Cost-Performance Model

		Violation characteristics			Performance measures			
DO_{std} (mg/L)	Index	N_a	V_a (mg/L)	TS_a (mg/L)	E_N	E_V	E_{TS}	E_{WLA}
6.0	LC	18	2.30	23.85	0.100	0.492	0.578	0.457
	SC	14	1.26	11.30	0.300	0.721	0.800	0.677
	MP	1	0.19	0.19	0.950	0.958	0.997	0.976
6.5	LC	21	2.75	32.95	0.086	0.453	0.509	0.408
	SC	19	1.32	14.00	0.174	0.738	0.787	0.650
	MP	3	0.69	1.21	0.870	0.863	0.982	0.924
7.0	LC	23	3.30	44.47	0.042	0.404	0.436	0.347
	SC	20	1.19	8.46	0.167	0.785	0.893	0.715
	MP	4	1.19	2.90	0.833	0.785	0.963	0.884

Note: LC=least cost solution; SC=selected compromise solution; MP=maximum performance solution; and E_N, E_V, E_{TS}, E_{WLA} are computed as per Eqs. (1), (6), (11), and (19), respectively.

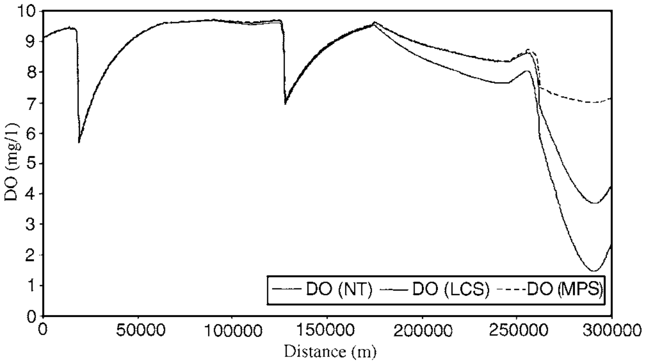

Fig. 5. DO profiles for no treatment (NT), least cost solution (LCS), maximum performance solution (MPS)-DO Std=7.0 mg/L

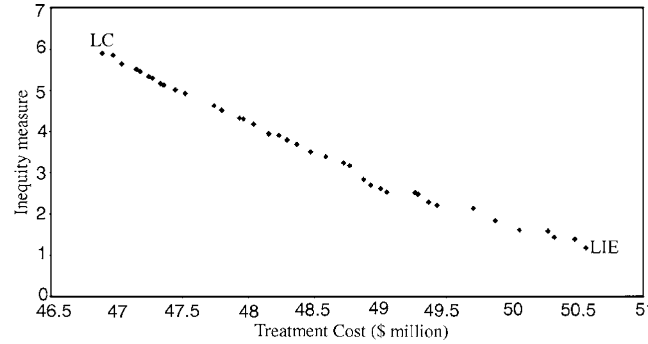

Fig. 6. Cost-inequity trade-off (DO std=7 mg/L, EWLA⩾60%)

improvement (from 0.347 to 0.715) in the performance. On the other hand, to improve the performance from 0.715 (for SC) to 0.884 (for MP), an additional amount of $\$3.256 \times 10^6$ needs to be spent. Table 4 also indicates that the maximum attainable performance level decreases with increase in DO standard (from 0.976 to 0.884 for DO standard of 6.0–7.0 mg/L).

The violation characteristics and the performance measures for the three runs are presented in Table 5. It can be observed from this table that the selected compromise solution allows less violations from the standard specified, compared to the least cost solution, and that too with significantly less vulnerability and total magnitude of violation, thus resulting in a considerably higher overall performance. The predicted DO profiles corresponding to the vector of optimal removal fraction levels for LC, SC, and MP are presented in Fig. 5, along with the DO profile for "no treatment" condition for the DO standard of 7.0 mg/L.

The cost-performance trade-off curves that are obtained from the multiobjective model proposed here will be helpful to the decision maker in choosing an appropriate waste load allocation solution for the given system, depending on the budget constraints and the desired overall performance level. The multiobjective analysis also gives information to the decision maker regarding the maximum attainable overall performance for a selected DO standard. This can be useful in deciding the DO standard that can be maintained in the river.

Table 6. Cost-Equity Trade-Offs-Cost-Equity-Performance Model

E_{WLA} (%)	Index	Solution point (TC, IEM)	Removal fraction levels ($x_1, x_2, \ldots, x_{14}$)
60	LC	46.895, 5.91	0.35, 0.35, 0.37, 0.41, 0.35, 0.36, 0.36, 0.35, 0.36, 0.35, 0.35, 0.71, 0.38, 0.36
	LIE	50.559, 1.17	0.41, 0.37, 0.53, 0.57, 0.37, 0.63, 0.49, 0.62, 0.50, 0.64, 0.35, 0.67, 0.63, 0.65
70	LC	47.759, 6.78	0.35, 0.35, 0.38, 0.35, 0.35, 0.57, 0.36, 0.35, 0.45, 0.46, 0.35, 0.82, 0.35, 0.35
	LIE	51.260, 1.56	0.45, 0.35, 0.58, 0.60, 0.41, 0.66, 0.58, 0.67, 0.48, 0.64, 0.36, 0.75, 0.67, 0.68
80	LC	49.281, 8.23	0.35, 0.36, 0.37, 0.35, 0.35, 0.67, 0.51, 0.43, 0.58, 0.67, 0.47, 0.90, 0.35, 0.35
	LIE	52.282, 2.49	0.49, 0.36, 0.62, 0.63, 0.41, 0.69, 0.62, 0.64, 0.58, 0.69, 0.40, 0.90, 0.66, 0.70

Note: TC=total cost (million $); IEM=inequity measure; LC=least cost solution; and LIE=least inequity solution.

Illustration for Cost-Equity-Performance Model

It may be observed from the results of cost-performance studies presented in Table 4 that the inequity in sharing the treatment effort is quite high, especially when the maximum performance solution is sought. This might be unacceptable to some of the waste load dischargers. For example, the maximum performance solution for DO standard of 6.5 mg/L (Table 4) indicates that the seventh and the tenth dischargers (D7 and D10 in Fig. 3) have to treat to minimum level (0.35), whereas the ninth discharger has to treat to a higher level (0.51), although D7 and D10 dischargers load the system to a greater extent (740 and 700 mg/L) than the discharger D9 (521 mg/L). In this context, it is desirable to consider minimization of inequity as one of the objectives. In this section, we discuss the results from the application of a cost-equity-performance model. It should be noted that the performance level is prespecified as a constraint, while minimization of inequity and minimization of total treatment cost are considered as objective functions.

Results of the cost-equity trade-off obtained for three overall performance levels (60, 70, and 80%), are presented in Table 6, for a DO standard of 7.0 mg/L. A normal depth of 8.986 m is used as the control depth at the d/s boundary. Figs. 6–8 show the pareto-optimal fronts for 60, 70, and 80% overall system performance, respectively. The violation characteristics and the performance measures for the pareto-optimal solutions corresponding to the least cost (LC), and the least inequity (LIE) are presented in Table 7. It is observed from Figs. 6–8 that the slope of the cost-inequity pareto-optimal front does not change significantly between the two extreme points, unlike in the case of the cost-performance trade-off. This means that the decision maker may use his/her judgment to choose the compromise solution within the range of implementation. A significant observation

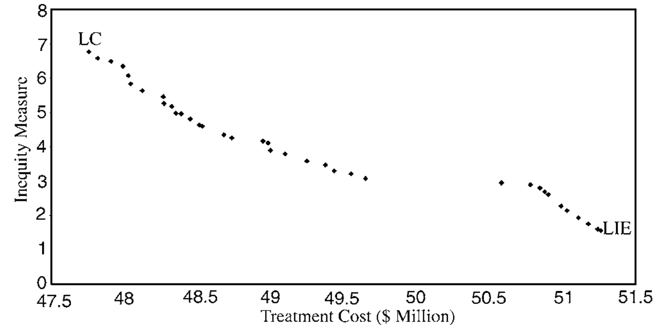

Fig. 7. Cost-inequity trade-off (DO std=7 mg/L, EWLA⩾70%)

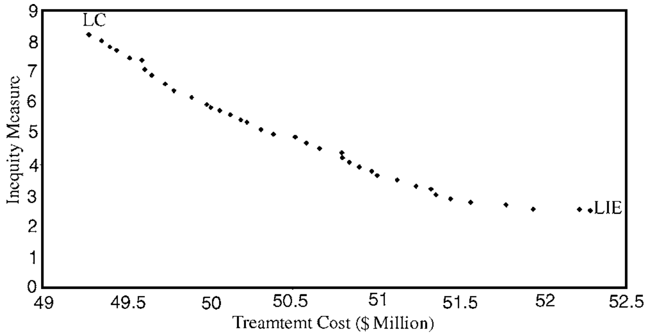

Fig. 8. Cost-inequity trade-off (DO std=7 mg/L, EWLA⩾80%)

from Table 6 is that the least inequity pareto-optimal solution demands more treatment effort and cost compared with that of the least cost pareto-optimal solution, in order to achieve equity. For example, for a specified performance level of 80%, eight out of the fourteen dischargers are required to significantly increase their treatment efforts, in order to bring down the inequity measure from 8.23 to 2.49, resulting in an increase of $3 million in the total treatment cost. This sort of detailed information obtained from the multiobjective models proposed, helps the decision maker in deciding whether the extra cost incurred in achieving the incremental degree of equity is justified, at a specified performance level. However, it may be noted that it would be easy to pick a compromise solution in cases where there is a significant change in slope in the cost-equity trade-off curve.

Summary and Conclusions

A multiobjective optimization framework for optimal waste load allocation in rivers has been developed, considering the total treatment cost, the inequity among the waste dischargers and a new comprehensive performance measure that reflects the DO violation characteristics. This framework consists of an embedded river water quality simulator that has a flow module, which simulates the gradually varied water surface profile, and a pollutant transport module, which simulates the advection, dispersion processes along with reaction kinetics for BOD and DO. The outer shell of the framework consists of the multiobjective optimization models formulated in this study. The optimization problems are solved using the multiobjective evolutionary algorithm known as "Non-Dominated Sorting Genetic Algorithm-II (NSGA-II)" of Deb et al. (2000).

The comprehensive performance measure proposed in this study is expressed as a weighted measure of: (1) the number of DO violations; (2) the magnitude of maximum DO violation; and (3) the total magnitude of DO violations, with reference to a specified DO standard, over all the checkpoints.

The proposed framework is used to obtain the cost-performance trade-off relation for a pre-specified DO standard and the cost-equity trade-off relation for a prespecified overall system performance with respect to a given DO standard. Usefulness of these relationships in decision making is illustrated through a realistic example of waste load allocation for Willamette river system in the United States.

The optimal treatment levels and the corresponding treatment costs are found to be significantly higher for the MP solution than the LC solution. The cost-performance trade-off relation developed for a prespecified DO standard is shown to be useful in obtaining reasonable compromising solutions that yield significantly higher performance at a reasonable additional cost, compared to LC.

The cost-equity trade-off obtained from the cost-equity-performance model for a specified performance level, indicates that the least inequity pareto-optimal solution demands significantly more treatment effort and cost compared with that of the least cost pareto-optimal solution. The sort of detailed information obtained from such multiobjective modeling helps the decision maker in deciding whether the extra cost to be incurred in order to achieve the incremental degree of equity is justified.

Table 7. Violation Characteristics and Performance Measures of Pareto-Optimal Solutions–Cost-Equity-Performance Model

		Violation characteristics			Performance measures			
E_{WLA} (%)	Index	N_a	V_a (mg/L)	TS_a (mg/L)	E_N	E_V	E_{TS}	E_{WLA}
60	LC	22	1.515	20.642	0.083	0.726	0.738	0.603
	LIE	21	1.438	17.706	0.125	0.740	0.775	0.635
70	LC	19	1.190	10.975	0.208	0.785	0.861	0.708
	LIE	18	1.190	12.578	0.250	0.785	0.840	0.706
80	LC	13	1.190	4.241	0.458	0.785	0.946	0.800
	LIE	12	1.190	4.049	0.500	0.785	0.949	0.810

Note: Violations characteristics for "no treatment": $N_0=24$; $V_0=5.53$ mg/L; and $TS_0=78.84$ mg/L. LC=least cost solution; LIE=least inequity solution; E_N, E_V, E_{TS}, E_{WLA} are computed as per Eqs. (1), (6), (11), and (19), respectively.

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