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Investigating Bowing of Hot Wire during cutting of EPS

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Abstract

During hot wire cutting of expanded polystyrene foam blocks, it is observed that the wire adopts a characteristic bow shape as the cutting proceeds. Due to this the cutting process loses its precision. The present study investigates the factors responsible for the bowing phenomenon via a series of experiments involving variations in current and wire feed rate. Based on the contribution of these two parameters, a regime of ‘No Bowing’ has been identified. This regime will be useful to a process planner to select working conditions which would allay bowing and build more precise prototypes. In this regard, the role of kerf width is also examined. An analytical model is suggested to quantify the bowing effect at different current and feed rates under steady state cutting conditions. Further a novel finding to predict the influence of gases in bowing has been analyzed and quantified.

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Keywords: EPS; Hot Wire Cutting; Bowing; Kerfwidth

1. Introduction

The rapid prototyping and manufacturing industry in recent years has seen a rapid growth of hot wire cutting techniques. Developments in CNC controlled thermo-mechanical cutting along with progress in CAM technology have greatly increased the complexities of geometries that can be sculpted. The technique involves use of a heated wire to melt and cut soft materials such as Expanded Polystyrene (EPS), Extruded Polystyrene (XPS), rubber and wax. The most popular material used in this technique is EPS, whose appealing features include its lightness, affordability and easy availability.

The early history of hot wire cutting encompasses its diverse uses, such as cutting of optical fibres by Khoe et al. [1] to the making of polystyrene moulds by Vishwanathan et al. [2]. The cutting process has

evolved from making relatively simple shapes to the use of robotic arms to control complex motions as in the work of Hamade et al. [3]. These processes were developed by working with small scale workpieces. The fabrication of large sized EPS with Variable Laminated Manufacturing (VLM) was first attempted by Ahn et al. [4], [5], [6]. Parallely, researchers such as Mehta et al. [7] have studied the behavior of EPS during thermal degradation.

While early research was mainly experimental, later researchers have numerically modeled and elaborated on the thermo-electro-mechanical effect of the hot wire cutting process such as in Petkov et al. [8]. Ahn et al. in [4] has discussed the need of a minimum gap between the wire and the foam to restrict the wire foam interaction. Further more, Bain [9] studied the different stages of wire foam interaction at high feed rates and low current. More recently, finite element simulation was also attempted for a 2D steady state analysis of the hot wire cutting of foams to investigate the kerf width offline [10]. Along with the simulation, Brooks [11] also characterized the force that arises due to wire foam

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interaction. The wire-foam interactions will give rise to a force. This force will tend to bend the wire in a bow shape.

Bowing of the wire is one of the practical challenges involved in hot-wire cutting process. The bow shape of the wire is shown by the schematic of the top view of the set up in Figure 1. During hot wire cutting of EPS, as the cutting proceeds the straight wire traverses along positive X-direction and assumes a characteristic bow shape. When the cutting velocity is kept low and the wire temperature is sufficient, the wire cuts EPS only by thermal means. There is no ‘physical’ contact between the wire and the foam. In these conditions the wire is expected to get enough time to ablate the foam ahead of it as it moves. Hence, it maintains its straight form throughout the length of stroke. However, as the speed increases, the feed rate becomes greater than the ablation rate of the foam, which results in foam and wire interaction. Such cutting condition also prevails at lower values of wire current. When the current is lower than its optimum value at a certain feed, the wire requires more time to ablate the EPS. This foam-wire interaction leads to bowing of the wire.

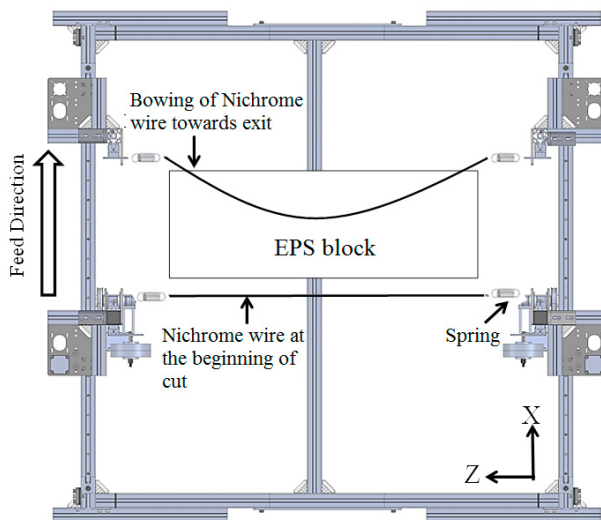


Fig. 1. Schematic of bowing from top view

As the straight form of the wire advances further inside the foam, the wire starts taking a curve form. The curvature of the wire increases and attains a steady state bow shape. And the bow shape is consistent hereafter.

This is called the bowing phenomenon of the wire.

Sometimes the motor vibrations induce vibration in the wire. Due to these vibrations the trajectory of the bow shape was imprinted on the cut EPS piece as seen in Figure 2. It also provides some credence to the assertion that the wire attains a steady state after which the magnitude of bowing is invariant of the travel distance.

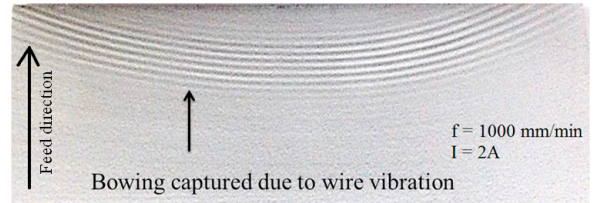


Fig. 2. Combined bowing and vibrational effects on EPS

Nomenclature

| | |
|----------|--|
| T | tension in the wire (N) |
| V | vertical (x) component of tension (N) |
| H | horizontal (z) component of tension (N) |
| F | drag force per unit length (N/mm) |
| F_s | drag force per unit arc length (N/mm) |
| s | arc length (mm) |
| x | bowing (deflection) (mm) |
| z | distance along the length of the wire (mm) |
| L | length of EPS block (mm) |
| θ | angle of bowing |
| T_m | melting temperature of EPS (K) |
| T_g | glass transition temperature of EPS (K) |
| d | depth of cut for vent holes (mm) |

The present work aims at quantifying the bowing phenomenon through experimental investigation and analysis. Such an investigation has not been reported in the literature and hence this study is expected to add knowledge to this area and help resolve problems associated with bowing. This paper reports investigations of the following:

- Experimentally quantifying the bow under steady state condition.
- Quantifying the influence of the contributing factors on the bow shape formed by the wire.

- Developing a regime of ‘No Bowing’ for making industrial decisions.
- Providing an analytical formulation of the deflection in the hot wire based on mechanics.
- Quantifying the effect of gases on the bowing of the wire.

2. Experimental Setup

2.1. Hot-wire CNC machine

A CNC controlled hot wire machine with Nichrome (NiCr) wire as the heating element was used to cut EPS foam in the desired shape. The machine consists of a rigid frame of Aluminium extrusions. A NiCr wire was attached on the vertical axis mount plates which were fixed to the timer belt on vertical columns with the help of springs that act as roller supports. Figure 3 shows the actual experimental set up that was used for quantifying bowing.

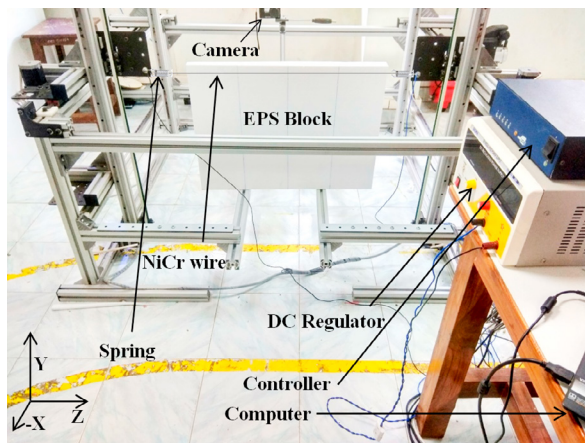


Fig. 3. Experimental Set up

A computer using MultiCNC software converted CAD files (in .dxf format) to G codes for the microcontroller. The microcontroller, through stepper motor and drives, moves the wire using linear motion guides through the timer belt-pulley mechanism. A two-axis configuration was used to analyse bowing during straight cutting of EPS foam by hot wire. In this configuration, the motors on the left and the right were synchronized for motion along the X-direction.

2.2. Work material

A low density (16 kg/m^3) EPS block was used for all the experiments. A NiCr wire of length 800 mm and diameter 0.32 mm (28 gauge) was used as the hot cutting element. Straight cuts of rectangular prismatic EPS blocks were carried out along the positive X-direction. The bowing was recorded for 50 mm, 100 mm and 150 mm of travel distance. It was observed that around 150 mm the wire attained a steady state of bowing. Hence, a block size 700 mm width (along Z-direction), 200 mm in height (along Y-direction) and 150 mm length (along wire feed, X direction) was selected for carrying out the experiments.

2.2.1. Properties of EPS

To build an understanding of the bowing phenomenon it is important to understand the behavior of foam at high temperature. Hence, Differential Scanning Calorimetry (DSC) and Thermogravimetric analysis (TGA) were performed for the EPS sample. The material properties were examined and compared with literature [7].

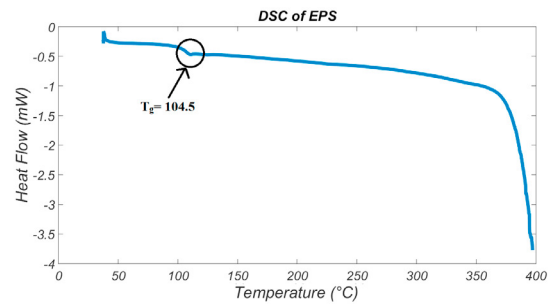


Fig. 4. DSC of EPS

The DSC results gives the glass transition temperature as 104.5°C as shown in Figure 4. Glass transition temperature signifies the start of change in mechanical property of a material. From the results of [10] the glass transition temperature and the melting point temperature appear to be related by equation 1

$$\frac{T_g}{T_m} = 0.87 \quad (1)$$

Hence, the melting point was calculated as 161°C which was found to be similar to literature [7]

The TGA result gives the weight percentage vs. temperature plot as shown in Figure 5. The

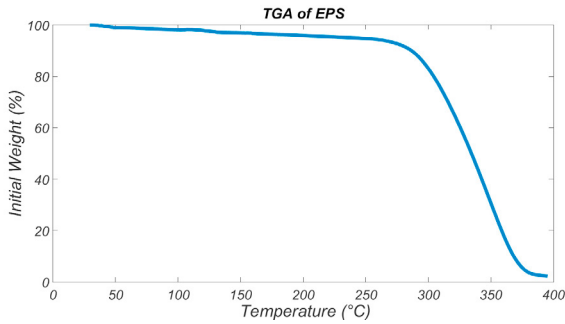


Fig. 5. TGA of EPS

weight percentage starts falling after 248°C (5% mass reduction) and a cut-off observed around 370°C to 400°C, signifying complete volatilization beyond this point. Thus 370° - 400°C could be taken as the upper volatilization temperature and 248°C is the beginning of volatilization for the present sample of EPS foam.

2.2.2. Properties of Nichrome

At high temperature, the knowledge of wire temperature becomes important. This parameter controls the thermal field in the foam cutting region. The wire temperature was recorded using a K type thermocouple (range 0-1200°C) for various currents levels. The measured temperature of wire in air is different from what is likely to be the actual temperature of the wire in the cutting region for the same current. However, it can be argued that given that the wire is thin (ϕ 0.32 mm) and the thermal conductivity of EPS is low (approximately $0.03 \text{ Wm}^{-1}\text{K}^{-1}$), the difference is probably not significant. Figure 6 shows the temperature vs current plot for the present experimental set up. The measured temperatures as well the expected temperatures as provided by the supplier of the NiCr wire are shown.

2.3. Methodology for quantifying bowing

Multiple experiments were performed to quantify the magnitude of bowing. EPS foam blocks were placed and straight cuts were made at four different feed rates and three different current values which were chosen to conform to values common in industrial usage. The operating parameters are summarized in Table 1.

Due to bowing, different regions of the wire exit the block at different times, with the two ends of the wire exiting first and the centre of the wire exiting last. A common point and shoot camera (Sony DSC-H55) was

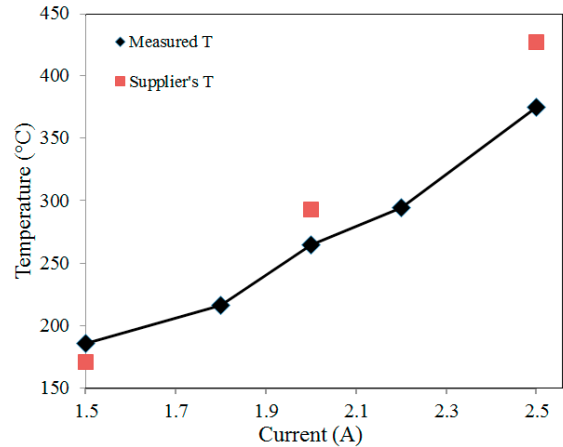


Fig. 6. Temperature versus Current

Table 1. Experimental Conditions

| S.No. | Current (A) | Feed (mm/min) |
|-------|-------------|---------------------|
| 1 | 1.5 | 250, 500, 750, 1000 |
| 2 | 2 | 250, 500, 750, 1000 |
| 3 | 2.5 | 250, 500, 750, 1000 |

used to record the exit of the wire. The camera was calibrated using a pendulum to confirm the frame rates. From the video, frames were extracted by MATLAB for analysis. The time of exit of five representative points of the wire were calculated. These data combined with the feed rate of the wire were used to extract the bent shape of the wire. Bowing was taken to be the difference in deflections between the maximum deflected point and the exit point of the wire as shown in Figure 7.

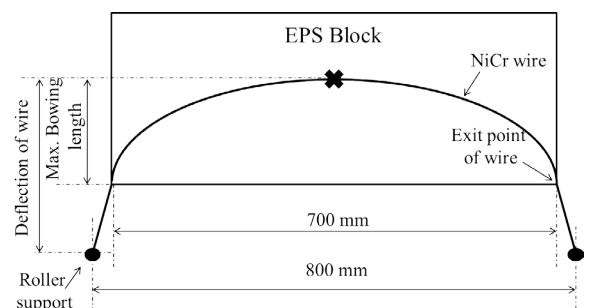


Fig. 7. Schematic of Bowing

3. Results and discussion

3.1. Bowing of the hot wire

We infer that bowing is experienced by the wire when there is ‘physical’ contact between the wire and the foam. As the foam touches the wire - which is moving at a greater feed than the ablation rate and is at very high temperature - it starts degrading. This degradation process was inferred from the TGA and DSC results along with the observations under high speed camera (Photron FastCam Mini ux100) at 1000 fps as shown in Figure 8. Here, it was seen that the cells lose its boundary and wrap the wire beyond the glass transition temperature (105 °C - 160 °C) and with the temperature being further raised, the EPS starts becoming viscous and molten beyond its melting point (161 °C). At higher temperature, volatilization starts (248°C) and finally ablates releasing gases (370 °C - 400 °C).

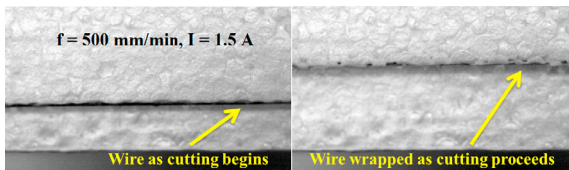


Fig. 8. Skim cut of EPS under high speed camera

The bowing of the wire at very high feed and low current was prominent enough to be easily visible to the naked eye just before the wire completely exits the EPS block as shown in the Figure 9.

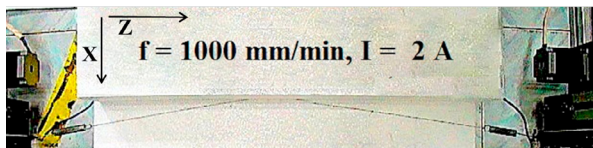


Fig. 9. Top view of bowing

The wire profile generated from the experimental data is shown in Figure 10 and Figure 11. Bowing was quantified and plotted as a quadratic fit at 95% significance level. At a current of 1.5 A (Figure 10), bowing is noticeable even at a feed rate of 500 mm/min with 1000 mm/min producing very large bowing effect compared to that at wire current of 2 A (Figure 11).

At a current of 2 A, feed rates of 250 mm/min and 500 mm/min (Figure 11) result in hardly any bowing. However at a feed rate of 1000 mm/min, significant bowing can be seen.

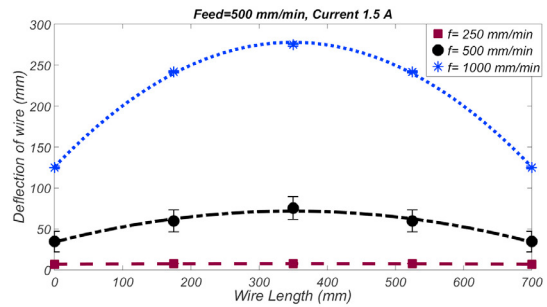


Fig. 10. Wire profile at 1.5 A

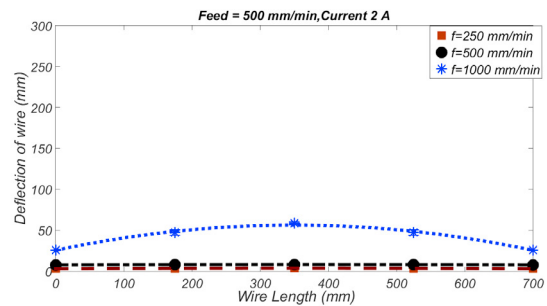


Fig. 11. Wire profile at 2.0 A

3.2. Bowing Regime

To create a predictor of whether bowing will occur or not, a graph was formulated using the experimental data as shown in Figure 12 at various feed rates and currents. The ‘crosses’ represent ‘no bowing’ and ‘diamonds’ represent observable ‘bowing’. Regression was carried to check the contribution of each of the dependent variables in formation of bow. It was seen the contribution of current is dominant and effect of feed is somewhat lower. Increasing the model complexity by introducing square terms in fact reduces the F value and thus were not included.

In practical cutting applications bowing of the wire is unavoidable. Bowing has no severe effect while producing straight cuts in EPS. But while producing

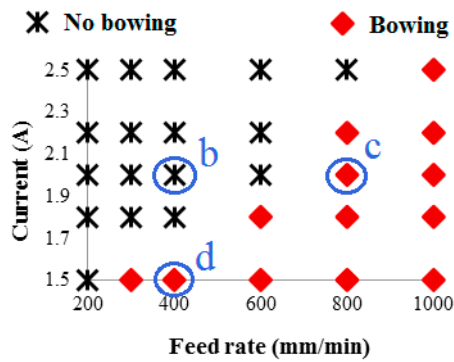


Fig. 12. Bowing Regime

simple shapes such as shown in Figure 13 (a), errors in form are observed. The step shape was cut at various feeds and currents for validating the bowing regime. Due to the bowing of the wire in transverse direction, when the wire has to shift its motion from transverse direction (along X axis) and move along Y- direction, a curved step is generated instead of a sharp edge. This was because, as the wire traverses and the bowing of the wire begins, the central part of the wire lags behind. This laggard part of the wire is unable to make a straight turn as the direction changes sharply.

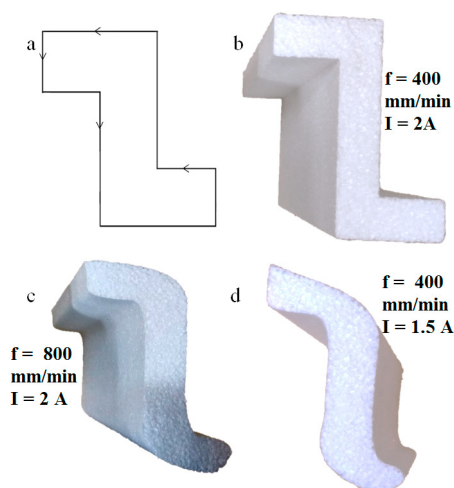


Fig. 13. Step shape and its effect due to bowing

Figure 13 (b) shows how the step profile looked like when the cutting parameters were chosen from the ‘no bowing’ regime i.e. at 400 mm/min and 2 A (point b in Figure 12). When the parameters were chosen from ‘Bowing’ regime i.e. at higher feed (800 mm/min) and the current constant (2 A) (point c in Figure 12), curved edges would be produced as shown in Figure 13 (c). Also when the current was reduced (1.5 A) at a constant feed rate (400 mm/min) (point d in Figure 12), Figure 13 (d) shows how the step was produced. Here, the difference in curvature that is produced as the feed is increased and the current is decreased can be easily noticed. The curvature in Figure 13 (d) is significantly more as compared to Figure 13 (c). This proves that the effect of current in bowing is more prominent, providing validation to the regression analysis. Hence it is advisable for a process planner to choose the parameters from the ‘No bowing’ regime for building a more precise prototype at minimum build time.

3.3. Discussion

Ahn et. al. [4] has discussed that a minimum gap of 0.28 mm must be maintained between the foam and the wire to restrict the wire from entering the melt zone of EPS prematurely. But as the feed rate is increased the wire travels towards the foam faster than the rate at which it ablates. Hence, the hot wire interacts with the molten foam and as the temperature rises due to prolonged contact, volatilization of the polystyrene occurs releasing gaseous by-products. These gases evolving from the degradation of foam remain trapped between the wire and molten foam. The gases from the edges of the foam have a scope to effuse through the sides of the block (at $z=0$ and $z=700$ mm). But the gases trapped in the center along with the molten layer of foam contributes to further reduction of the wire temperature and slowing down of the cutting process. These gases and the molten EPS combine to provide a resistance to the motion of wire. All these factors contribute in bowing of the wire.

Simply put, bowing can be avoided with low feed and high current. But at these conditions kerfwidth is another factor which becomes predominant. So as we move towards ideal cutting conditions the area of influence of the cut increases which affects the precision of the final product. Intricate features cannot be produced when the kerfwidth of EPS is high. Hence, the kerfwidth was experimentally found as suggested by [11]. At a feed rate of 250 mm/min Figure 14 shows

the variation of kerfwidth with current. Thus conflicting requirements must be balanced based on the demands of specific workpiece shapes.

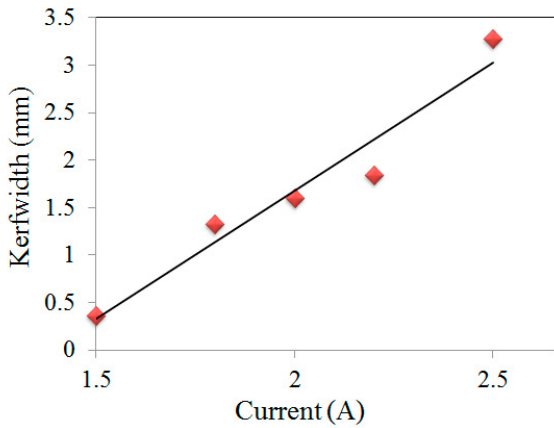


Fig. 14. Kerfwidth vs Current

3.4. Analytical Formulation

When the wire enters the foam block it is straight. But the high heat energy melts the EPS in direct contact with the wire. Thereafter the wire is in contact with this semi-melt only. As the wire starts to advance against the melt, it starts experiencing a drag force which is assumed to be constant per unit length of wire as the temperature of the wire is constant throughout the wire. After some transition time, dynamic equilibrium is attained and a bow shape trajectory of wire is generated. In this steady state condition the free-body of the wire is shown in Figure 15.

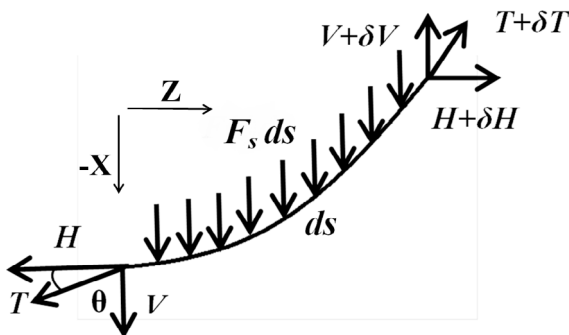


Fig. 15. Free body diagram of the wire

By doing a force balance in the vertical direction:

$$V + \delta V - F(z)dz - V = 0 \tag{2a}$$

$$\Rightarrow \frac{dV}{dz} = F(z) \tag{2b}$$

Also from Figure 15, we have,

$$\tan(\theta) = \frac{V}{H} = -\frac{dx}{dz} \tag{3a}$$

$$\Rightarrow V = -H\frac{dx}{dz} \tag{3b}$$

By substituting equation 2b in 3b and differentiating, we get 4,

$$\Rightarrow -F(z) = H\frac{d^2x}{dz^2} \tag{4}$$

Consider the arc length we get,

$$ds = \sqrt{dz^2 + dx^2} \tag{5a}$$

$$\Rightarrow \frac{ds}{dz} = \sqrt{1 + \left(\frac{dx}{dz}\right)^2} \tag{5b}$$

$$F(z) = F_s \sqrt{1 + \left(\frac{dx}{dz}\right)^2} \tag{6a}$$

$$\Rightarrow F(z)dz = F_s ds \tag{6b}$$

By substituting equation 4 and 6a in equation 6b we get 7,

$$H\frac{d^2z}{dz^2} = -F_s \sqrt{1 + \left(\frac{dx}{dz}\right)^2} \tag{7}$$

Integrating equation 7 we get,

$$x = -\frac{H}{F_s} \cosh\left(\frac{F_s}{H}z + C_1\right) + C_2$$

To find C_1 and C_2 , the following boundary conditions are applied:

$$x = 0 \text{ for } z=0, L$$

$$\frac{dx}{dz} = 0 \text{ for } x=L/2$$

By substituting the constants we get,

$$x = -\frac{H}{F_s} \left(\cosh\left(-\frac{F_s}{H}\left(z - \frac{L}{2}\right)\right) - \cosh\left(\frac{F_s L}{2H}\right) \right) \tag{8}$$

The equation 8 gives the trajectory of the bowing when a steady state has been attained. It shows that bowing will be higher with higher values of drag force and will reduce as the tension in the wire increases. The wire takes the shape of a catenary. Tension in the wire reduces to a smaller fixed value as the current is turned on.

3.5. Force Analysis

From literature it can be found that some researchers [9] have used load cells to measure the force experienced by the hot wire during cutting of EPS. Further they formulated an empirical relation by data fitting of experimental results using linear regression. The erstwhile force model depends on the effective heat (Q_{eff}). Effective heat is a function of effective length (l_e), current and feed rate. The empirical force relation is given by,

$$F = C_1 e^{-C_2 Q_{eff}} + C_3 \quad (9)$$

The present analytical model is used to predict the drag force that the wire experiences during the straight cut of EPS by fitting the experimentally found bow shape and tension to the analytical expression of bow shape (equation 8).

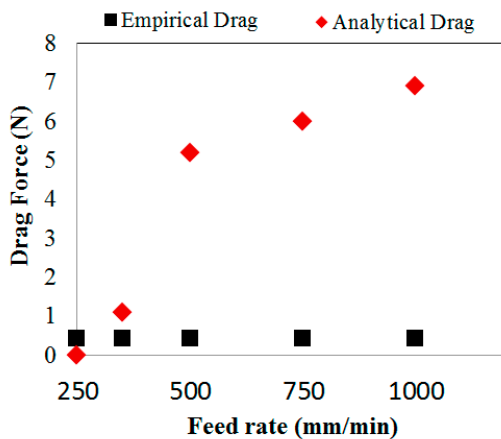


Fig. 16. Empirical and Analytical Force vs Feed

The present analysis agrees with the erstwhile empirical model for no/low bowing conditions - for instance at feeds less than 350 mm/min and 1.5 A. However, in the present study, the absolute forces were found to be much higher than those of the earlier researchers in low current and high feed situations as shown in Figure 16. This could be due to:

- The present analytical model explicitly accounts for bowing while the erstwhile empirical model does not account for bowing.
- Since l_e is much larger in the current study, the cut surface area is also much larger resulting in

significantly more heat loss. This results in a reduction of the effective heat generation from the nominal I^2R value, and hence an increased force is expected in the present case.

- Roller support: Springs were used in the present experiment instead of pneumatic cylinders in the erstwhile work as roller supports. Springs allow more flexibility, resulting in higher bowing of the wire. While pneumatic cylinders as roller support tend to snap off the wire at greater force values.
- Workpiece size: The erstwhile model was formulated by working with small workpieces (30 mm - 50 mm) while the present analysis has been made on large scale prototypes (700 mm). The present model predicts force per unit length and the absolute forces could be impacted by a size effect.

3.6. Can gaseous by-products influence bowing?

Researchers have found that when EPS is heated above volatilization temperature, mostly oligomers of styrene, such as dimer and trimer, toluene, benzene, ethyl-benzene and methyl-styrene are released as gaseous by-products [7]. And the volume of gaseous degradation products produced per unit mass of the foamed polymer is a strong function of temperature. Hence the amount of gases increase as the temperature rises. Suspecting that the entrapment of these gaseous by-products hinders the heat transfer from the wire to the EPS, a novel experiment was performed. Vent holes (Figure 18) were made on the top surface of the EPS block to allow passage for the by-products. Straight cut was carried out at a depth ' d ' as shown in equation 10. The depth of the holes depend on the ' d ' and the expected kerfwidth at a particular current. This ' d ' as shown in Figure 17 is just sufficient for the vent holes to act as risers and yet not effect the cutting process. It was believed that the vent holes will reduce bowing of the wire. This would also quantify the contribution of gaseous by products in bowing

$$d = \text{hole depth} + \frac{\text{kerfwidth}}{2} \quad (10)$$

Figure 19 shows the effect of vent holes when the cuts were made at a depth ' d '. It is seen that there is significant reduction in bowing with the vent holes. Around 13% of reduction in bowing was achieved with 0.3% reduction in area in case of small holes ($\phi=3$ mm). Whereas, for larger holes ($\phi=6$ mm) with 1.2% reduction in area the bowing reduces by 19%. Hence, it can be said that there is a major contribution of gases in bowing. Therefore, the effect of such gaseous

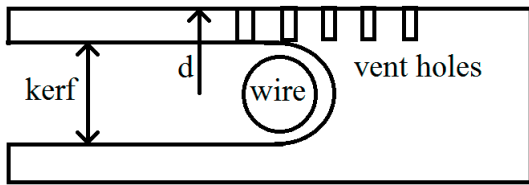


Fig. 17. Vent hole location

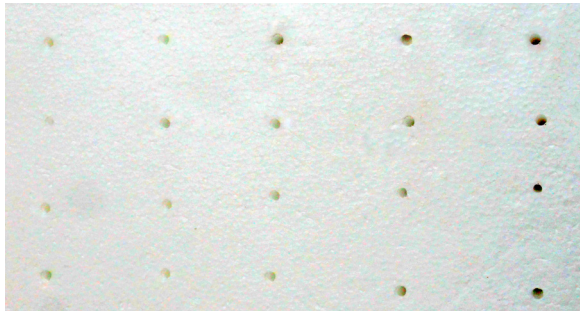


Fig. 18. Vent holes on EPS

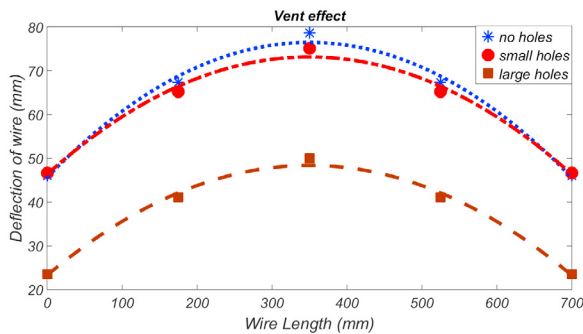


Fig. 19. Effect of vents on bowing

by-products needs to be accounted for in analytical or numerical modeling techniques. This work is currently in progress and will be reported in the near future.

4. Conclusions

The paper discusses a novel problem faced by the hot wire industry. The present work makes the following contributions:

- The bowing of the wire is an unfavourable phenomenon present in all practical cutting situations. The present work has reported a quantification of this phenomenon.
- The role of different operating parameters on bowing has been reported. It is seen that increased current has a dominant effect in reducing bowing. Lower feeds also reduce bowing somewhat. But increased currents also increase kerfwidth and limit the shapes attainable in practice.
- A bowing regime is presented for selecting mitigating operating conditions.
- An analytical-experimental method has been developed to find the exact catenary shape of the bow.
- A force analysis has been presented which will be helpful in predicting the life of the hot wire. High bowing results in higher forces in the wire and hence reduces the tool life. If the wire experiences high forces for a long time, it will tend to snap off resulting in a short service life.
- A novel analysis was made on the impact of gases on bowing. Presence of vent holes was found to significantly reduce the degree of bow. A 13% reduction in bowing with small holes and 19% with larger holes was observed.

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