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IFAC-PapersOnLine 49-1 (2016) 807–812

Harvesting Energy from Vibration Absorber under Random Excitations

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Abstract: A dynamic vibration absorber is a passive device used often to reduce the vibrations in host structures. The potential of harvesting energy out of the motion of vibration absorber has shown promise under harmonic excitations. To achieve this goal, vibration absorber is supplemented with a piezoelectric stack for both vibration confinement and energy harvesting. The primary goal is to control the vibration of the host structure and the secondary goal is to harvest energy out of the dynamic vibration absorber simultaneously. The harvested energy can be used to power low-powered wireless sensor systems used for infrastructural health monitoring. In field applications the excitation frequency may not be harmonic and may have randomness. This manuscript looks into the power harvested by these devices under random excitations. Analytical studies are carried out considering stationary Gaussian white noise excitation within the frequency band of interest. The analytical results show the promise of broadband energy harvesting using the device combined with vibration reduction in the primary structure.

Keywords: Passive control, Piezoelectric Energy harvesting, Random vibrations, Dynamic vibration absorber.

1. INTRODUCTION

The need for smart cities and smart structures with inbuilt health monitoring systems has necessitated the use of millions of wireless sensors to connect them with a network. Powering these is a problem from both technological and environmental aspect.

This motivated to drive the technology to reliable and green alternatives like wind,light,heat,strain and vibrations. The required energy is in micro-Watt scale and since most of these sensors are used in infrastructures to monitor their vibrations, Vibration source can be used for harvesting energy.

The two main vibration based energy harvesting technologies are electromagnetic and piezoelectric. The electromagnetic harvester generates power from changes in magnetic field induced due to motion of the host structure (Williams and Yates, 1996; Kulkarni et al., 2008). Piezoelectric energy harvesters generate power from the strain induced in piezoelectric materials while the host structure undergoes vibrations (Sodano et al., 2005; Halvorsen, 2008; Adhikari et al., 2009).

Research on energy harvesting and simultaneous vibration control devices are less reported (Nakano et al., 2003; Choi and Wereley, 2009; Chtiba et al., 2010; Tang and Zuo, 2010) since one needs to find a trade-off between the vibration reduction and the power harvested. In this regard a suitable choice would be a dynamic vibration absorber (DVA), also known as tuned mass dampers (TMDs).

The energy harvesting DVA (EHDVA) consists of a spring, a damping and a piezoelectric element to harvest energy. The piezoelectric stack is attached with the electric circuit to store the scavenged energy (Ali and Adhikari, 2013). The piezoelectric stack performs dual role, first it increases the damping of the DVA thereby reducing the high damping requirement of optimal DVAs and secondly it acts as an energy scavenger.

The previous research (Ali and Adhikari, 2013) on EHDVA has focused on harmonic excitations and determining the optimal parameters for the EHDVA. In this paper the feasibility of using this concept when the structure is under random input is studied. The study is feasible under earthquake excitations, which are random in nature and are modeled as stationary Gaussian process. Analytical study is presented using probabilistic linear random vibration theory. The base excitation to the system is considered as a random process. Effects of randomness to the mean and variance of harvested power are presented.

2. DYNAMIC VIBRATION ABSORBER AND THE FIXED-POINT THEORY

A dynamic vibration absorber is a widely used passive vibration control device. A DVA consisting of only a mass and a spring has a narrow operation region and its performance deteriorates significantly when the exciting frequency varies. The performance robustness can be improved by using a damped DVA. The key design parameters of a damped DVA are its tuning parameter (also known as the frequency ratio) and the damping ratio.

10.1016/j.ifacol.2016.03.156

The first mathematical theory on the damped DVA was presented by Ormondroyd and Hartog (1928). Den Hartog (Hartog, 1985) introduced the optimum solution of a damped DVA that is attached to a damping-free primary system. His *fixed-point* theory states the existence of two fixed points, P and Q in the frequency response curves of the displacement of the primary structure. The points P and Q are independent of the damping in the absorber as shown in Fig. 1. This reduces the optimization parameters only to the frequency ratio. Based on the *fixed-point* theory, Den Hartog found the optimum tuning parameter and defined the optimality for the optimum absorber damping.

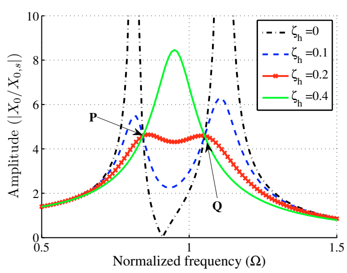

Fig. 1. Frequency response for an undamped structure with Dynamic vibration absorber. Existence of fixed points is evident.

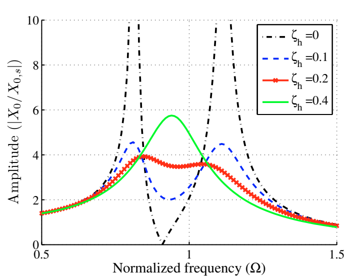

Fig. 2. Frequency response for a damped structure with Dynamic vibration absorber, Exact fixed point are unavailable.

As shown in Fig. 2, when a primary system is damped, the usefulness of the *fixed-points* feature is no longer strictly valid. Thus, obtaining an exact closed-form solution for the optimum tuning parameter or optimum damping ratio becomes very difficult. A number of studies have focused on the approximate and numerical solutions. These include numerical optimization schemes proposed by Randall et al. (1981); Zhu et al. (2004); nonlinear programming techniques by Liu and Coppola (2010); frequency locus method by Thompson (1980) and min-max Chebyshev's criterion by Pennestri (1998) to name a few. Numerical studies based on minimax optimization are reported in Chtiba et al. (2010); Brown and Singh (2011). In spite of being exact, these analyses are usually problem specific and may not give physical insights to the phenomenon for a general case.

Ghosh and Basu (2007) identified that for small damping in the primary system, although there exist no points like $P$and Q in a strict sense, the fixed-point theory can be applied. Ghosh and Basu (2007) derived an approximate analytical solution for the optimum tuning parameter based on the assumption that the *fixed-points* theory approximately holds when a damped DVA is attached to a lightly or moderately damped primary system (see Fig. 2). Motivated by this, a fixed-point theory for the coupled electromechanical system was developed in Ali and Adhikari (2013).

In this paper, we aim to provide an analytical derivation of the power harvested by an energy harvesting DVA under random excitation. The optimal DVA parameters which reduce the primary structure vibrations as well as harvest maximum power are discussed.

3. ENERGY HARVESTING DYNAMIC VIBRATION ABSORBER

In this section a description of the energy harvesting vibration absorber is given. An energy harvesting dynamic vibration absorber (EHDVA) consist of a vibration absorber with piezoelectric material and an electric circuit to harvest energy out of the vibration absorber.

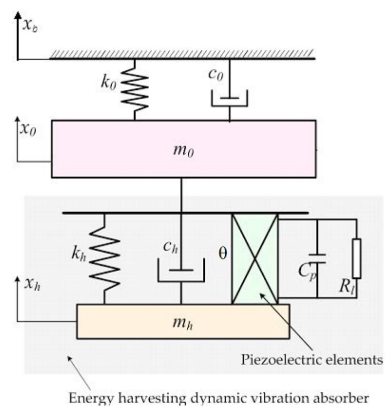

Fig. 3. Energy harvesting dynamic vibration absorber

The coupled two degree of freedom electromechanical system representing the EHDVA as shown in the Fig 3. A DVA is added to primary mass and consists of spring, damping, piezoelectric element and an electric circuit for harvesting energy from dynamic vibration absorber (Ali and Adhikari, 2013). The electromechanical model is subjected to base excitation and the output voltage is obtained across the load resistance. Linear random vibration theory is considered for the analytical work.

The coupled dynamics of system is expressed by three ordinary differential equations. Apart from the parameters considered in the (Ali and Adhikari, 2013), the random base excitation $x_b(t)$ is considered here.

$$\begin{gathered}
m_0\ddot{x_0} + k_0(x_0 - x_b) + c_0(\dot{x_0} - \dot{x_b}) + k_h(x_0 - x_h) \\
+c_h(\dot{x_0} - \dot{x_h}) = 0 \\
m_h\ddot{x_h} + k_h(x_h - x_0) + c_h(\dot{x_h} - \dot{x_0}) - \theta v = 0 \\
C_p\dot{v} + \frac{v}{R} + \theta(\dot{x_h} - \dot{x_0}) = 0
\end{gathered} \tag{1}$$

Where, m_0, c_0 and k_0 are the mass, damping and stiffness of the primary structure respectively. m_h, c_h and k_h represent the mass, damping and stiffness of the DVA respectively. The mechanical equations in (1) are coupled with the electrical equation through the electro-mechanical coupling parameter θ, C_p is the capacitance and R is the load across the electrical circuit with voltage, v.

Converting in to frequency domain and normalizing with the resonant frequency of primary mass, we have,

$$\begin{gathered}(-\Omega^2 + 2(\zeta_0 + \zeta_h \mu\beta)i\Omega + 1 + \mu\beta^2)X_0 \\ -\mu(2\zeta_h\beta i\Omega + \beta^2)X_h = (1 + 2\zeta_0 i\Omega)X_b \\ -(2\zeta_h\beta i\Omega + \beta^2)X_0 + (-\Omega^2 + 2\zeta_h\beta i\Omega + \beta^2)X_h \\ -\frac{\theta\beta^2 V}{k_h} = 0 \\ \frac{-\theta\alpha i\Omega X_0}{C_p} + \frac{\theta\alpha i\Omega X_h}{C_p} + (\beta + \alpha i\Omega)V = 0\end{gathered} \tag{2}$$

where the dimensionless variables are defined as

$$\mu = \frac{m_h}{m_0}, \beta = \frac{\omega_h}{\omega_0}, \alpha = \omega_0 C_p R, \zeta_h = \frac{c_h}{2m_h\omega_h}, K^2 = \frac{\theta^2}{k_h C_p}$$

The equations in 2 can be written in matrix form as $AX = B$. On solving, we get

$$X_0 = H(\omega)X_b, V = K(\omega)X_b$$

$$H(\omega) = \left[\frac{\begin{gathered}(-\Omega^2\beta - i\Omega^3\alpha + 2\,i\zeta_h\beta^2\Omega - 2\,\zeta_h\beta\,\Omega^2\alpha \\ +\beta^3 + i\beta^2\alpha\,\Omega + iK^2\beta^2\alpha\,\Omega)(1 + 2\,i\zeta_0\Omega)\end{gathered}}{\Delta}\right] \tag{3}$$

$$K(\omega) = \frac{-i\theta\,\alpha\,\Omega^3\,(1 + 2\,i\zeta_0\Omega)}{\Delta\,C_p} \tag{4}$$

Here Δ is the determinant of matrix A.

4. ANALYSIS USING STOCHASTIC THEORY

Consider a stochastic process with input $x(t)$, and output $y(t)$, then the mean square of the input is given by

$$< x(t)^2 >= \int_{-\infty}^{\infty} S_{xx} d\omega = \int_{-\infty}^{\infty} \lim_{T\to\infty} \frac{X(\omega)X^*(\omega)}{T}\, d\omega \tag{5}$$

where, S_{xx} is the power spectral density (PSD) of the input, $x(t)$. $X(\omega)$ is the Fourier transform of the input variable. $(.)^*$ represents the complex conjugate.

The Fourier transform of the output $(y(t))$ represented by $Y(\omega)$ is related to the input using transfer function, $H(\omega)$.

$$Y.Y^* = |H(\omega)|^2 X.X^* \tag{6}$$

This follows the following relation in time domain.

$$< y(t)^2 > = \int_{-\infty}^{\infty} |H(\omega)^2| S_{xx}(\omega) d\omega \tag{7}$$

Following Eq 5 and Eq 7, we get

$$S_{yy}(\omega) = |H(\omega)^2| S_{xx}(\omega) \tag{8}$$

The estimation of mean square value of the primary mass displacement $(< {x_0}^2 >)$ and mean value of power$(< p >)$ generated are carried out for two cases (i) where the base displacement is a white noise processes and (ii) the base acceleration is a white noise process.

4.1 Case 1: base displacement as white noise

A white noise has a constant PSD. Therefore, the mean square value of primary mass displacement $(< x_0^2(t) >)$ and mean square value of voltage $(< v^2(t) >)$ generated are given as

$$< x_0^2 > = c\omega_0 \int_{-\infty}^{\infty} |H(\Omega)|^2 d\Omega \tag{9}$$

$$< v^2 > = c\omega_0 \int_{-\infty}^{\infty} |K(\Omega)|^2 d\Omega$$

and mean value of power is given as

$$< p > = \frac{c\omega_0}{R} \int_{-\infty}^{\infty} |K(\Omega)|^2 d\Omega \tag{10}$$

where 'c' is PSD of base displacement $(S_{x_b x_b})$. PSD of primary mass displacement and voltage generated are given by

$$S_{x_0 x_0}(\omega) = |H(\Omega)|^2 S_{x_b x_b}(\omega) \tag{11}$$

$$S_{vv}(\omega) = |K(\Omega)|^2 S_{x_b x_b}(\omega) \tag{12}$$

Since we are interested in the power parameter

$$S_{pp}(\omega) = \frac{S_{vv}(\omega)^2}{R^2} \tag{13}$$

$H(\Omega)$ and $K(\Omega)$ are given by Eq 3 and Eq 4 , respectively.

4.2 Case 2: base acceleration as white noise

In many real life cases the host structure may undergo random acceleration, like in earthquake excitations or strong wind. It is easier to formulate the problem in terms of acceleration variables in the forcing function. The equations in this case are given as

$$< x_1^2 > = \frac{c_1}{\omega_0^3} \int_{-\infty}^{\infty} \frac{|H(\Omega)|^2}{\Omega^4} d\Omega \tag{14}$$

$$< v^2 > = c_1\omega_0 \int_{-\infty}^{\infty} |K(\Omega)|^2 d\Omega$$

and mean value of power is given as

$$< p > = \frac{c_1\omega_0}{R} \int_{-\infty}^{\infty} |K(\Omega)|^2 d\Omega \tag{15}$$

where $c_1 = S_{\ddot{x_b}\ddot{x_b}}(\omega)$, hence $S_{x_b x_b}(\omega) = \frac{c_1}{\omega^4}$

The integrals in Eq 9, Eq 10, Eq 14 and Eq 15 are difficult to have exact solutions. An approximate way to find solution is given in Roberts and Spanos (1990). The solution to an integral of the form

$$I_m = \int_{-\infty}^{\infty} \frac{\Xi_m(\omega)}{\Delta(\omega)\Delta^*(\omega)} d\omega$$

where

$$\Xi_m(\omega) = \zeta_{m-1}\omega^{2m-2} + \zeta_{m-2}\omega^{2m-4} + \ldots + \zeta_0$$

$$\Delta(\omega) = \lambda_m(i\omega)^m + \lambda_{m-1}(i\omega)^{m-1} + \ldots + \lambda_0$$

is given as

$$I_m = \frac{\pi}{\lambda_m}\frac{det[N]}{det[D]} \tag{16}$$

where the numerator and the denominator are given as

$$[N] = \begin{bmatrix} \zeta_{m-1} & \zeta_{m-2} & \cdots & \cdots & \cdots & \zeta_0 \\ -\lambda_m & \lambda_{m-2} & -\lambda_{m-4} & \lambda_{m-6} & \cdots & \cdots \\ 0 & -\lambda_{m-1} & \lambda_{m-3} & -\lambda_{m-5} & \cdots & \cdots \\ 0 & \lambda_m & -\lambda_{m-2} & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & -\lambda_2 & \lambda_0 \end{bmatrix}$$

$$[D] = \begin{bmatrix} \lambda_{m-1} & -\lambda_{m-3} & \lambda_{m-5} & -\lambda_{m-7} & \cdots & \cdots \\ -\lambda_m & \lambda_{m-2} & -\lambda_{m-4} & \lambda_{m-6} & \cdots & \cdots \\ 0 & -\lambda_{m-1} & \lambda_{m-3} & -\lambda_{m-5} & \cdots & \cdots \\ 0 & \lambda_m & -\lambda_{m-2} & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & -\lambda_2 & \lambda_0 \end{bmatrix}$$

5. RESULTS AND DISCUSSIONS

This section discusses the parametric variation in the power harvested with respect to the mechanical (absorber damping coefficient, ζ_h) and electrical parameters (nondimensional time constant, α, nondimensinal coupling coefficient, K). Other parameters that are left constant in the analysis are

Mass of frame	527.9 Kg
Length	186 mm
Resonant frequency	1.078 Hz
Mass of DVA	17.6 Kg
Damping	1.0 %
Frequency	1.06Hz

5.1 Base Displacement as white noise

The response for the case 1 are shown in Fig. 4 to Fig. 11. Figure 4 shows the variation of mean power generated *w.r.t* the mechanical damping in the DVA. It is observed that the mean value of power generated increases by reducing the damping in the absorber. As damping increases the vibration of the DVA reduces and thereby less power is generated.

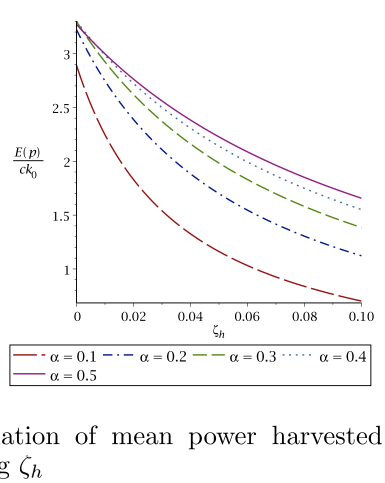

Fig. 4. Variation of mean power harvested *w.r.t* DVA damping ζ_h

From Fig 5 and Fig 6 it can be observed that the mean value of power generated attains a maximum for particular values of electrical parameters α and K. Hence by designing these values in the range close to these peaks one can get maximum returns.

The spectral density plot of power generated from Fig 7 shows that the maximum power generated is when the frequency of vibration are close to natural frequency of the primary mass system. This is obvious as there is more motion in the primary and the secondary structures near the resonant frequencies.

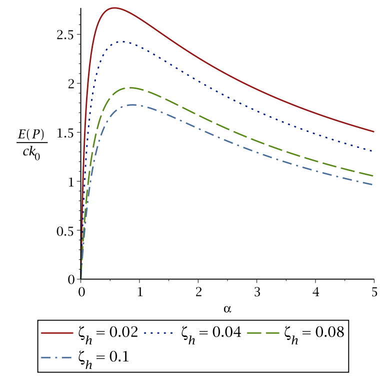

Fig. 5. Variation of mean power harvested *w.r.t* α

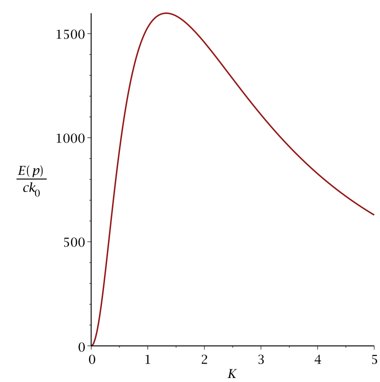

Fig. 6. Variation of mean power harvested *w.r.t* K

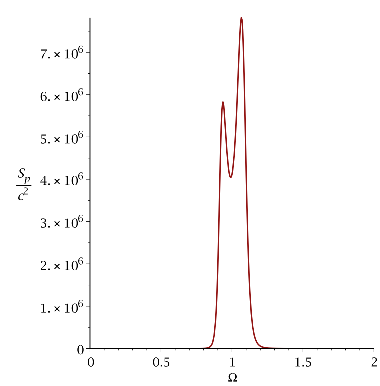

Fig. 7. PSD of power harvested

Fig 8 shows the variation of the mean square primary mass displacement. There exists a particular value of α beyond which increase in α increases the variance in displacement.This particular α value depends on the existing damping.Similar information can be obtained from Fig 9.From Fig 10 we can see that the primary mass displacemnet is minimum for a particular value of K.

Fig 11 shows that the maximum spectral density of primary mass displacement is for Ω close to 1, this is true because maximum displacement is observed near the frequencies close to natural frequency.

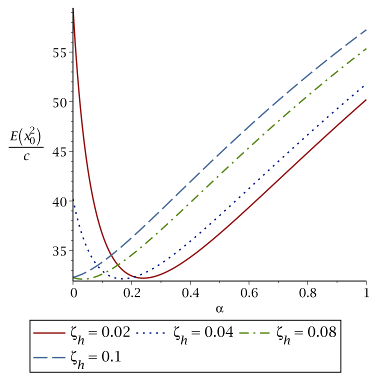

Fig. 8. Variation of mean square primary mass displacement $w.r.t$ α

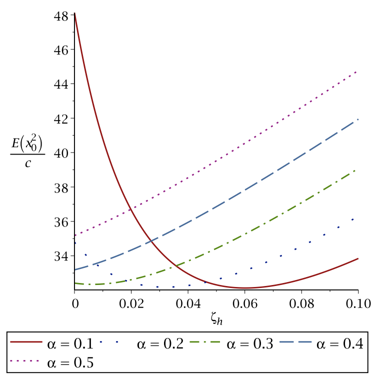

Fig. 9. Variation of mean square primary mass displacement $w.r.t$ ζ_h

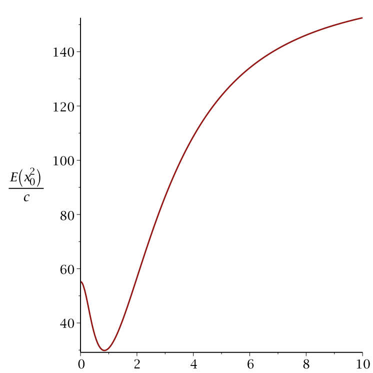

Fig. 10. Variation of mean square primary mass displacement $w.r.t$ K

Hence by suitably choosing the values of mechanical and electrical coefficients one can achieve minimum displacement in the primary mass while maximizing the power harvested.

5.2 *Base acceleration as white noise*

Variation in the power harvested for the case 2 are shown from Fig 12 to Fig 16. By observing these plots of variation of mean power generated one can make similar conclusions as in the case 1.

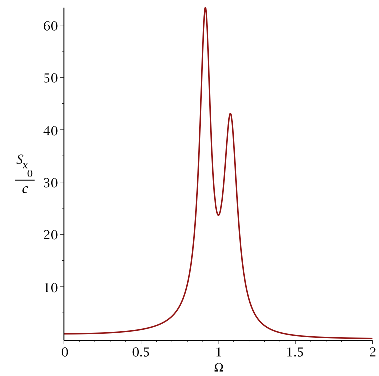

Fig. 11. PSD of primary mass displacement

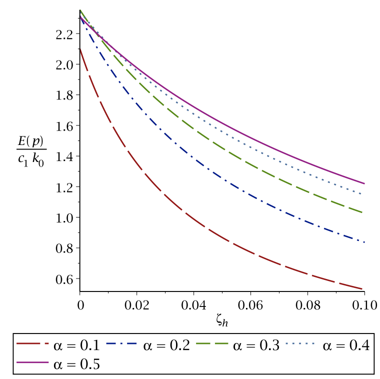

Fig. 12. Variation of mean power harvested $w.r.t$ vs ζ_h

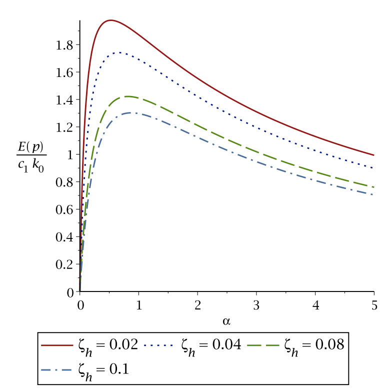

Fig. 13. Variation of mean power harvested $w.r.t$ α

6. CONCLUSION

It has been shown that the vibrations of structures can be controlled using the EHDVA when the base excitation is random. Further it has been shown that the unwanted vibrations can be used to generate energy. The order of generated energy increases with the mass of structure. Hence for sufficiently large bodies , like bridges, EHDVA can be used to run low power sensor devices and hence can work autonomously.

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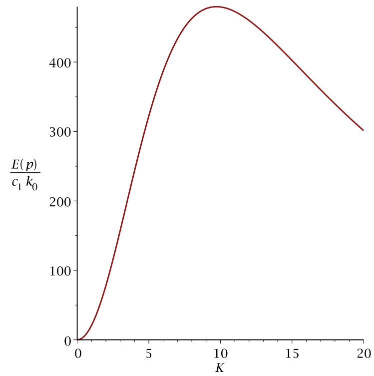

Fig. 14. Variation of mean power harvested *w.r.t* coefficient(K)

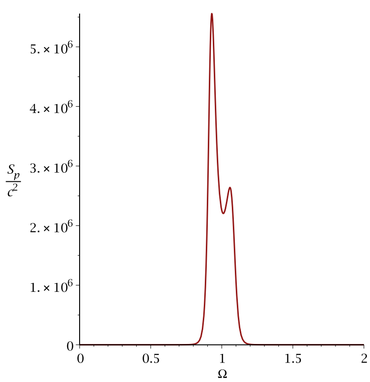

Fig. 15. PSD of power harvested

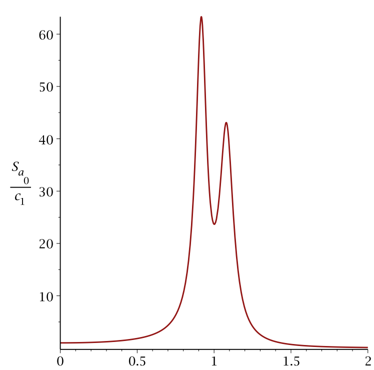

Fig. 16. PSD of primary mass acceleration

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