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Ground resonance : nonlinear modeling and analysis

Salini S. Nair * **Haradev G. S.** ** **Ranjith Mohan** ***

* *Doctoral Student, Department of Aerospace Engineering, IIT Madras (e-mail: snairsalini@gmail.com).*
** *B.Tech. Student, Department of Aerospace Engineering, IIT Madras (e-mail: haradev05@gmail.com).*
*** *Assistant Professor, Department of Aerospace Engineering, IIT Madras (e-mail: ranjith.m@iitm.ac.in).*

Abstract: Aeromechanical instabilities arising due to frequency coalescence of different modes of a helicopter rotor and fuselage system is investigated. The focus is on removing the small angle approximation typically used for ground resonance analysis and perform simulation of the resulting nonlinear model. The results reveal significantly different characteristics (existence of limit cycles) for the nonlinear model compared to linear model that is commonly used. These differences are primarily (though not necessarily) restricted to regions of frequency coalescence of more than one degrees of freedom present in the system. The paper does numerical simulations of different simplified models of coupled-rotor-fuselage systems used in the past by researchers and show that the nonlinear behavior is fundamental to the system. The results in the paper stand out because, in the past, limit cycle oscillations for coupled-rotor-fuselage systems were shown to exist only with nonlinearity associated with stiffness or damping parameter. The paper clearly shows that nonlinearity inherent in the lead-lag and flap equations will result in limit cycle oscillations and this is also demonstrated through experimental studies.

Keywords: Helicopter dynamics, Modeling, Ground Resonance, Nonlinear analysis, Co-ordinate transformations

1. INTRODUCTION

Ground resonance (GR) is an instability that can occur during landing or take-off conditions when a helicopter is in contact with the ground. The system degrees of freedom responsible for ground resonance are the fuselage rotational degrees of freedom and the blade lead-lag (in plane) degree of freedom. The instability occurs due to frequency coalescence between two modes of the system, namely the regressive lag mode and the fuselage mode, when the damping present in the system is insufficient. It is influenced by aerodynamics as well. Traditionally, broadly termed, aeromechanical instabilities include ground resonance, air resonance, flap-lag instability etc and are often studied from linearized system perspective, using eigenvalue analysis if the system is constant coefficient or using Floquet theory if the system has periodic coefficients. All such analysis are centered around the equilibrium solution with small angle approximation. However, there has been no attempt to explore the solutions when this approximation is removed. This has practical relevance when new designs of highly flexible rotor blades are analyzed. The main theme of this paper is to show that once we relax the small angle approximations, the inherent nonlinearity in the system bring forth dynamics like limit cycles associated with nonlinear systems. It is to be noted that in past limit cycles have been reported for GR simulations, using nonlinear damping or stiffness coefficients. Here we stress that, parameters used for the system are linear and nonlinearity arises from relaxing small angle approximations. A test setup to simulate ground resonance was used and limit cycle oscillations (LCO) were observed, a fact that is in agreement with simulation results. We will show that such nonlinear behavior is present in different models that have been studied in past.

1.1 Literature

The first study that identified the physics of ground resonance (Coleman and Feingold, 1958) brought out the degrees of freedom of the helicopter that are required for its analysis. Ground resonance involve coupled dynamics of the fuselage motion on its landing gear and the blade lead-lag degree of freedom. It can be analyzed in a simplified manner by neglecting aerodynamics, and approximating the rotational motion of fuselage on its landing gear as linear translations of the rotor hub. This approach was followed in the theoretical studies done in Hammond (1974),Sanches et al. (2011). A detailed analysis of GR involves the inclusion of fuselage roll, pitch degrees of freedom and blade lead-lag and flap degrees of freedom, along with aerodynamics (Ormiston (1991)). A seminal work on GR was done in Bousman (1981), that provided a rich database from experiments as well as correlation with model having fuselage rotational degrees of freedom. Above modeling approaches relied on linearized system and focus was on measurement and prediction of regressive lead-lag (RLM) damping. Nonlinear analysis of GR was

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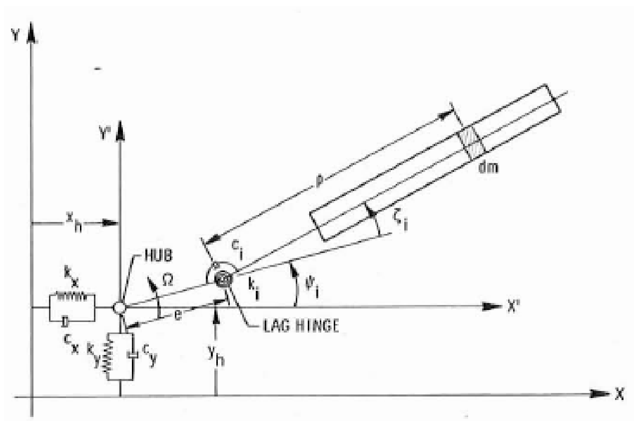

Fig. 1. Coordinate frame used in the simplified analysis(Hammond, 1974)

done by Kunz (2002) wherein the effect of inclusion of nonlinear damping and stiffness terms was analyzed.

2. MOTIVATION

2.1 Ground Resonance and RLM

As the name implies, ground resonance occurs when the helicopter is operating in contact with the ground. However, the term 'resonance' is a misnomer. In the conventional sense, resonance refers to the scenario wherein the frequency of an externally applied force matches the natural frequency of the system. In case of ground resonance, frequency coalescence do occur, however, the coalescence is between two modes of the system itself, specifically the fuselage mode (roll or pitch) and the regressive lag mode (RLM).RLM refers to the whirling motion of the rotor center of gravity (C.G.) about the hub center (Johnson (2013)). The RLM frequency can be roughly expressed as $\Omega - \omega_\zeta$. Here, Ω is the rotor RPM, and ω_ζ is the blade lead-lag frequency in the rotating frame. ω_ζ depends on the system parameters as well as the rotor RPM. The fuselage mode frequency ω_θ for pitch mode and ω_ϕ for roll mode, may also vary with rotor RPM. Frequency coalescence, in case of GR refer to the condition where $\Omega - \omega_\zeta = \omega_\theta$ or ω_ϕ. However, frequency coalescence is a necessary but not sufficient condition for ground resonance (instability) to occur. Ground resonance will occur only if the damping present in the system at the rotor speed where frequency coalescence occurs is not sufficient.

2.2 Modeling

As discussed in the introduction, there are several ways to model the system to study ground resonance. In a simplified model, the fuselage motion on its landing gear is approximated as translational motion of the rotor hub. The blade lead-lag motion is considered, but the blade flap motion as well as aerodynamics is neglected. The reference frame for simplified analysis is shown in Fig. 1 (Hammond, 1974). To do a more complete analysis, the fuselage rotational degrees of freedom, blade lead-lag, flap degrees of freedom and aerodynamics are included in the analysis. The reference frame used in this case is shown in Fig. 2. In all our analysis, we consider blades to be rigid.

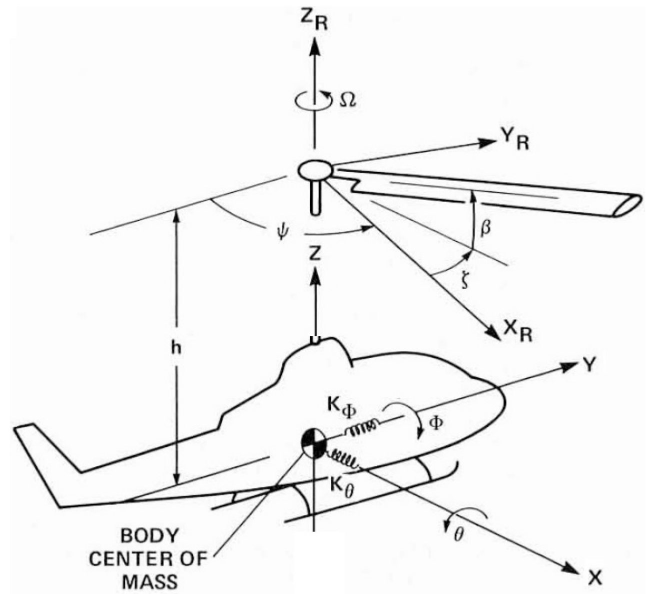

Fig. 2. Coordinate frame used in the complete analysis(Ormiston, 1991)

2.3 Analysis

Conventional GR analysis involve analyzing the stability of the system at different rotor speeds. Here, small angle approximation is used and the derived equations are linearized, so as to do an eigenvalue analysis. The real part of the eigenvalue is taken as the modal damping and the imaginary part as the modal frequency. If any of the modal damping values is greater than zero, the system is unstable at that particular RPM. The corresponding region in the modal frequency plot will show frequency coalescence between RLM and fuselage mode (Sanches et al., 2011).

2.4 Nonlinearity

The effect of nonlinearity on the instability, in general, is assumed to be small and hence neglected. However, there are references in literature to existence of LCO during ground resonance(Kunz, 2002), which point towards nonlinearity. These oscillations were thence attributed to the nonlinear stiffness or damping elements present in the mechanical system. A nonlinear analysis, wherein all the structural elements have linear characteristics, have not been explored so far. In this paper, an experimental and theoretical analysis of a rotor-fuselage system is done to investigate the existence of these periodic solutions.

3. METHODOLOGY

3.1 Experimental analysis

An experimental analysis on scaled down model of a helicopter with fuselage roll, pitch degrees of freedom and blade lead-lag degree of freedom was performed in laboratory environment. The schematic of the setup is shown in Fig. 3 and further details are given in Nair et al. (2019). The primary aim of the paper (Nair et al., 2019) was to understand effect of ground proximity on RLM damping and

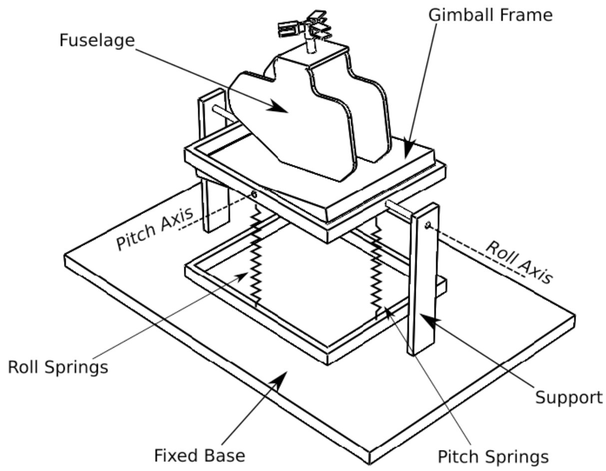

Fig. 3. Schematic of the experiment setup

hence does not refer to observation of LCO. The fuselage motion is measured using an Inertial Measurement Unit (IMU). The RLM damping and frequency is computed from the measured fuselage response of the system using Matrix Pencil Method (Hua and Sarkar, 1990).

3.2 Theoretical analysis

In theoretical analysis, we integrate the nonlinear periodic equations of motion of the coupled rotor-fuselage system and determine the periodic solution, if it exists (this inherently assumes the periodic solution is stable). Initially the results obtained using complete model is discussed. However, due to difficulty in analyzing the equations for the complete model, the simplified model is used for detailed analysis.

4. RESULTS

4.1 Experimental analysis

The RLM damping and frequency is computed from measured fuselage response of the system. The modal damping and frequency results are given in Nair et al. (2019). The expected time response of the system at an unstable RPM is an exponentially increasing behavior. However, in the course of the experiment, it was observed that the system response, after the initial exponential increase, went into LCO in the unstable regime.A sample time response obtained at 1270 RPM for $\theta_0 = 2°$ is shown in Fig. 4. Here, the fuselage roll signal measured using IMU is shown as a function of time. The response initially shows an exponential rise but later settles into constant amplitude oscillations. The exponential and LCO part along with corresponding frequency spectrums are shown separately in Fig. 5. The peak in both frequency spectra(Fig. 5B, 5D) appear at 6.33 Hz, which is quite close to the RLM frequency obtained using modal analysis (6 Hz). A similar observation is obtained at other RPMs and collectives enabling us to conclude that the experimental model is characerised by LCO at RLM frequency in the ground resonance regime. To address the concern of whether this observation is related to any structural element, it was tested and ensured that the structural stiffness and damping elements present in the system are linear.

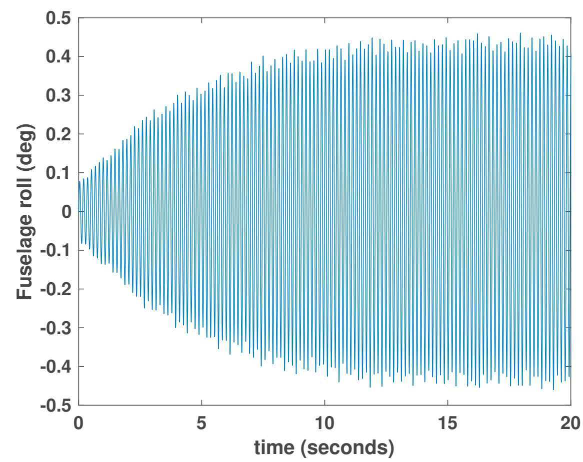

Fig. 4. Fuselage response obtained using IMU

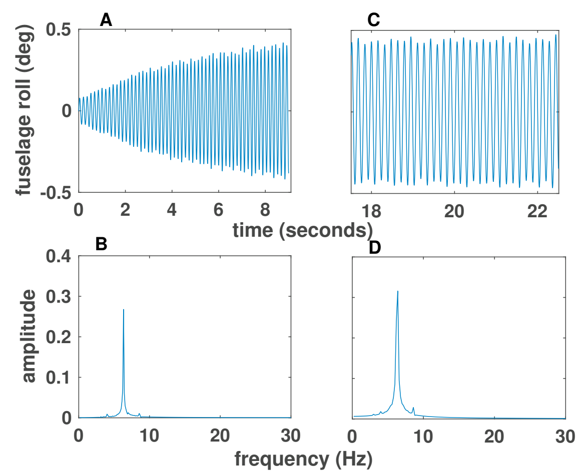

Fig. 5. Different regions of the fuselage response obtained using IMU with the corresponding frequency spectrum

4.2 Theoretical Analysis

All the structural elements of the models considered in theoretical analysis are linear. In the initial study, experimental model from Bousman (1981) is considered. The model has rotational fuselage degrees of freedom, blade lead-lag and flap degrees of freedom and aerodynamics is included. The time response analysis of the nonlinear equations of motion is done at stable and unstable rotor speeds. While the time response in the stable regions show convergence, the response in the unstable region show an initial exponential trend followed by sustained oscillations of frequency equal to the corresponding RLM frequency. The linear model response along with the nonlinear model response at 760 RPM (unstable RPM for this model) and $\theta_0 = 0°$ is shown in Fig. 6. The linear model, as expected shows exponential increase while the nonlinear model settles into constant amplitude oscillations after the initial exponential rise. It can also be observed that in the initial region, the nonlinear and linear system responses agree. The equations of motion considered for this analysis has fuselage roll, pitch degrees of freedom and blade lead-lag and flap degrees of freedom along with aerodynamics. It will be difficult to analyze these set of equations to

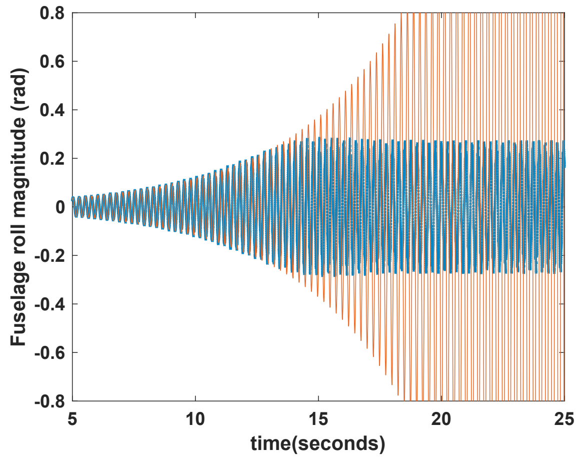

Fig. 6. Fuselage roll time response obtained using linear and nonlinear models at 760 RPM for $\theta_0 = 0°$

understand the nature of the observed oscillations. Hence, we consider the simplified model (Hammond, 1974) to do a detailed analysis.

4.3 Simplified model: Theoretical analysis

In the simplified model, the fuselage rotational motion on the landing gear is approximated as linear translations of the rotor hub. Aerodynamics and blade flap motion is neglected. It may be noted that this simplified model is not physically relevant when large displacements are considered, nevertheless, it serves the purpose of demonstrating the modeling and effects of inherent nonlinearity.

Modeling The reference frame used in the analysis is based on (Hammond, 1974), and is shown in Fig. 1. The states of the system can be represented as,

$$X = [x, y, \zeta_i, \dot{x}, \dot{y}, \dot{\zeta_i}]\ i = 1, 2, ..., N_b$$

Here x,y refer to the linear translations of the rotor hub, ζ_i to the lead-lag degree of freedom of the i^{th} blade. The equations are derived based on the approach used in (Hammond, 1974). The obtained nonlinear equations are periodic with period equal to that of rotor rotation. The system parameters considered in this paper are based on a modified form of the system considered in Sanches et al. (2011). The helicopter system in Sanches et al. (2011) has zero structural damping for fuselage as well as blades ($c_x = c_y = c_i = 0$). This results in certain modes having zero modal damping, hence, time response analysis at stable RPMs show the states going into constant amplitude oscillations(neutral stability) instead of damping out. In our analysis, we have considered non-zero damping, specifically, $c_x = c_y = c_i = 100mNs/rad$. All the other parameters are as given in table 1. The model is isotropic in both fuselage and blade, that is, the fuselage properties in the x and y directions are same and all the blades have identical properties (damping, stiffness, inertia).

Stability Analysis To do stability analysis, small angle approximation is applied to the equations and they are further linearized about the equilibrium point (all states are 0 when aerodynamics is ignored). The obtained equations can be written in the state-space form, $\dot{X} = A(t)X$,

Table 1. Model parameters

Parameter	Value
Number of blades N_b	4
Blade mass m_b	31.9 kg
Hub mass m_x, m_y	2902.9 kg
Hub frequency w_x, w_y	3 Hz
Hub damper c_x, c_y	100 mN-s/rad
Hinge offset e	.2 m
Blade effective length b	2.5 m
Blade moment of inertia about lead-lag hinge I_b	458 kg-m^2
Blade lead-lag frequency w_b	1.5 Hz
Blade damper c_b	100 mN-s/rad

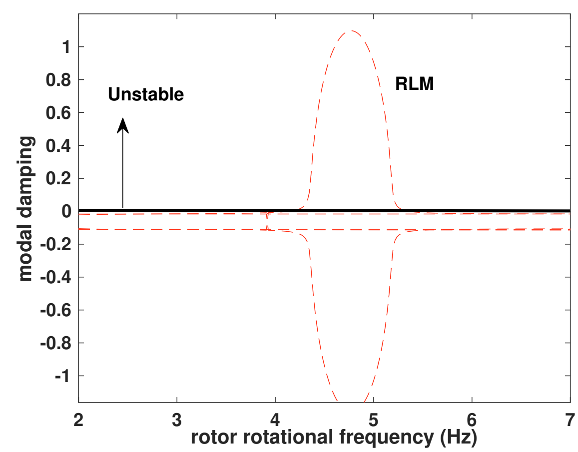

Fig. 7. Variation of modal damping with rotor speed

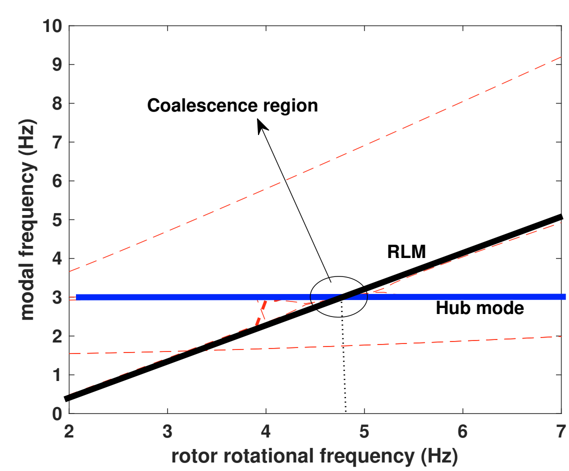

Fig. 8. Variation of modal frequency with rotor speed

where $A(t)$ is the system matrix, which will be periodic in this specific case. Stability analysis basically involves varying the rotor RPM Ω of the system, and computing the eigenvalues (modal damping and modal frequency) at each rotor speed. The eigenvalues of periodic coefficient system can be computed using Floquet method (Hammond, 1974).Alternatively, since all the blades are identical in this case, we can apply a transformation called the multiblade coordinate transformation (Hohenemser and Yin, 1972) to the states and thus obtain the system matrix in constant coefficient form. The results from stability analysis are shown in Figs. 7 and 8. Here, RLM refers to the regressive

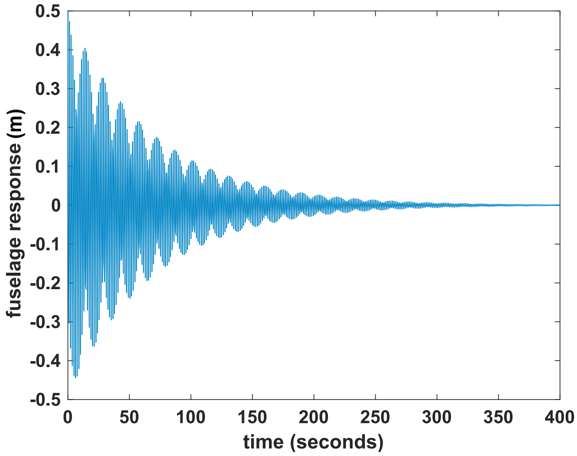

Fig. 9. Fuselage response at 3 Hz

lag mode which usually is the least damped mode of the system. If any of the modal damping values lie above the black line (modal damping = 0) in Fig. 7, the system is unstable at that rotor speed. As seen in Fig. 8, coalescence occurs between hub (fuselage) mode and RLM when rotor frequency lies between 4.5 and 5 Hz, and corresponding region in the modal damping plot (Fig. 7) shows RLM damping to be greater than zero (the mode is unstable).

Nonlinear analysis The time response analysis of the periodic coefficient nonlinear equations is performed for rotational frequencies corresponding to stable and unstable regions. The response obtained by perturbing the system at a stable rotational frequency (3 Hz) is shown in Fig. 9. It is observed that, however large the perturbation, the system states converge to steady state (all states = 0). Here, the fuselage response refers to the translational motion of the rotor hub and hence the response is in metres. The fuselage response on perturbing the system at an unstable frequency (4.5 Hz) is shown in two separate figures, Figs. 10 and 11. As can be observed from the figures, initial part of the response (Fig. 10) shows an exponential increase while final part (Fig. 11) shows that the response settles down to LCO. As expected, a similar trend is observed for all states. Obviously, a linear analysis of the system would not have revealed the existence of such periodic solutions. The phase portrait (x and $\dot{x}$) obtained using the 5 cycles of rotor revolution in the LCO regime is shown in Fig. 12.

Rotor center of gravity From the time response of the states, we can compute rotor C.G. location at each instant using equations 1 (Hammond, 1974).

$$x_c(t) = x(t) + \frac{\rho_c}{N_b}\sum_{i=1}^{N_b}\cos(\psi_i + \zeta_i)$$

$$y_c(t) = y(t) + \frac{\rho_c}{N_b}\sum_{i=1}^{N_b}\sin(\psi_i + \zeta_i) \tag{1}$$

The C.G. motion as obtained from last few cycles of the time response of the nonlinear system is shown in Fig. 13 as $x_c(t)$ versus $y_c(t)$ plot. The spiral divergence obtained from linearized system equations is also shown for reference. Obviously, the values reached by C.G. position for the

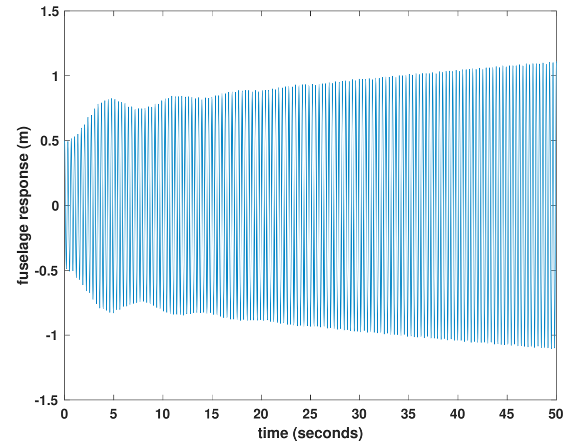

Fig. 10. Fuselage response at 4.5 Hz (initial part)

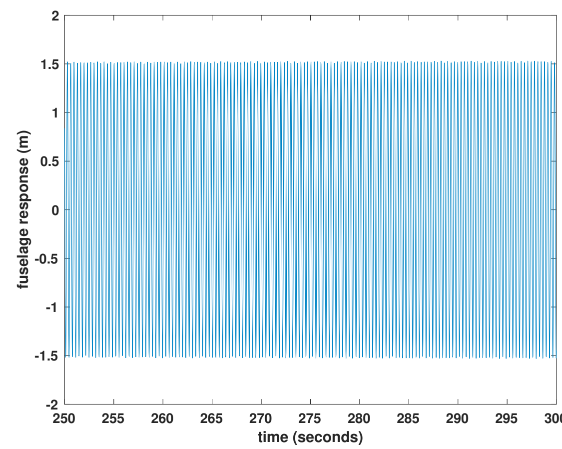

Fig. 11. Fuselage response at 4.5 Hz (final part)

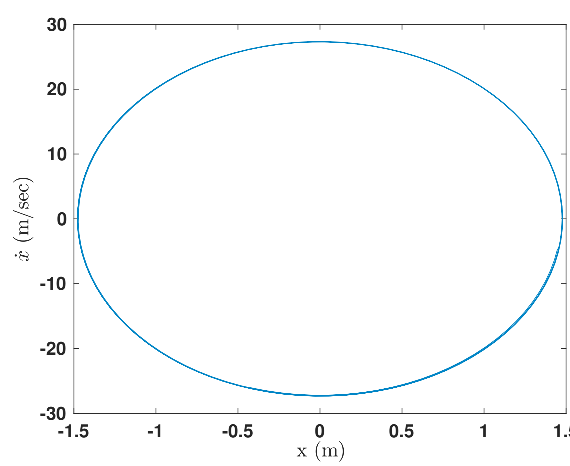

Fig. 12. Phase portrait using data from 5 cycles(LCO part)

periodic solution is very high and of no practical relevance. However, it is shown here to demonstrate that limit cycles are inherent in the system. One could possibly tell that these systems are akin to Duffing equations (coupled) and such LCO are expected.

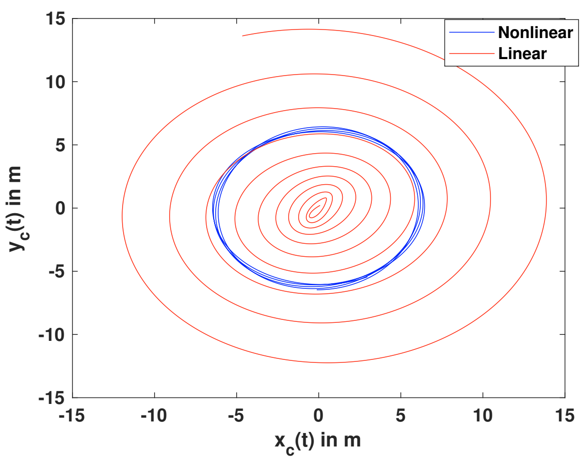

Fig. 13. Rotor center of mass motion

Table 2. Eigenvalues at 4.5 Hz

Eigenvalue	Modal frequency (Hz)	Mode
$-.1132 \pm 39.9107i$	6.352	Progressive lag mode
$-.9403 \pm 17.9700i$	2.86	Fuselage mode
$.8167 \pm 18.0546i$	2.8735	Regressive lag mode
$-.0166 \pm 18.8642i$	3.0023	Fuselage mode
$-.1091 \pm 10.7997i$	1.7188	Collective lag mode
$-.1091 \pm 10.7997i$	1.7188	Differential lag mode

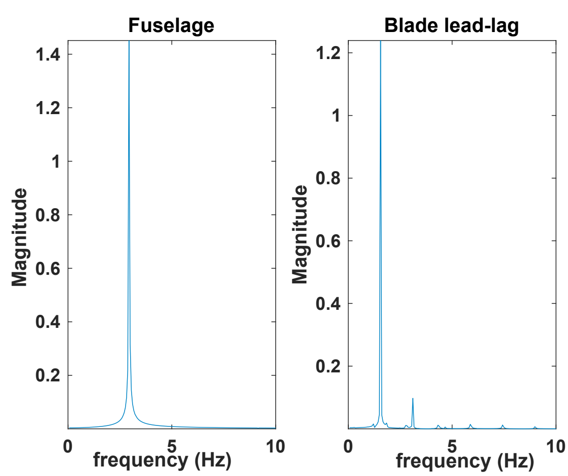

Fig. 14. Frequency spectrum of final part of fuselage and blade response (oscillations)

Eigenvalues The eigenvalues obtained at 4.5Hz from the linearized system equations are shown in table 2. The table also shows the corresponding modes, identified using eigenvector amplitude and modal frequency analysis. The unstable mode (modal damping > 0) corresponding to RLM has modal frequency 18.0546 rad $=$ 2.87 Hz. The RLM frequency, as discussed before, can be written as $\Omega - \omega_\zeta$, where ω_ζ is the lead-lag frequency. For the particular case, we have $\Omega = 4.5Hz$ and RLM frequency $2.87Hz$, hence, $\omega_\zeta = 1.62Hz$.

Frequency spectrum The frequency spectrum of the oscillations observed in the fuselage and blade response is shown in Fig. 14. The fuselage response spectrum shows the peak at 2.937 Hz (close to RLM frequency obtained from eigenvalue analysis) while the blade response spectrum shows the prominent peak at $1.555Hz$, which is equal to $\Omega - 2.937$, thus corresponding to ω_ζ. The blade response spectrum shows additional peaks at multiples of ω_ζ.

4.4 Flap-lag system

Studies on an isolated rotor system, with flap and lag degrees of freedom, show the existence of such oscillatory behaviour in unstable regime (flap-lag instability). This implies that the oscillatory behaviour does not originate from the presence of fuselage but is inherent in the lead-lag dynamics getting coupled with another mode, flap or fuselage.

5. CONCLUSION

A helicopter rotor and fuselage system was considered without small angle approximation and was shown to exhibit limit cycle oscillations. Two different modeling approximations with respect to fuselage degrees of freedom are considered and periodic solutions are obtained. Similar results from experiments are also reported. The frequency of periodic solution is that of regressive lag mode and occurs in regions of instability. It is possible that limit cycles may exist in the stable regions. A bifurcation study using continuation tools (like Matcont) is not included in the scope of current paper.

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