Fuzzy fault tree analysis of oil and gas leakage in subsea production systems

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Abstract

This paper presents a probabilistic failure analysis of leakage of the oil and gas in a subsea production system using fault tree analysis (FTA). A fault tree was constructed by considering four major areas where the leakages can be initiated. These are: gas and oil wells, pipelines, key facilities and third party damage. Conventional FTA requires precise values for the probability of failure of the basic events. However, since the failure data are uncertain, a fuzzy approach to these data is taken which leads to the so-called fuzzy fault tree analysis (FFTA), a method that employs expert elicitation and fuzzy set theories to calculate the failure probabilities of the intermediate events and the top event through identification of the minimal cut sets of the fault tree. A number of importance measures for minimal cut sets and the basic events have been obtained which helps to identify the nature of dependence of the top event on the basic events and thereby can identify the weakest links that may cause leakage in the subsea production system.
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Keywords: Subsea production system; Fuzzy fault tree analysis; Failure probability; Importance measures.

1. Introduction

Subsea oil production systems consist of wells, flow-lines, manifolds, X-trees, risers, pipelines etc. Their operation involves many challenges such as pressure containment, maintenance and flow assurance in hostile sea surface and subsea environment. Low temperature and high pressure are the common characteristics of this environment that lead to high costs of maintenance and repair of such systems. Due to the large geographical spread of the subsea oil production systems, the task of risk assessment for gas or oil leakage becomes an extensive task. One of the most powerful risk analysis methods is the fault tree analysis, which is a quantitative method for the computation of failure probabilities of the components of a particular system.

Fault tree analysis (FTA) is a top-down deductive failure analysis for estimating the system reliability using Boolean logic. In FTA, a top event is defined and is further resolved into intermediate and basic events that are interrelated by logic gates. The fault tree is analysed using the rules of Boolean algebra with which the tree is represented by an equivalent set of Boolean equations.

In conventional FTA, the failure probabilities of the basic events (*BE*) are exact values. However, the precise estimation of failure probabilities of the *BE*s is impractical due to insufficient data [19]. Hence, it is often necessary to work with approximate estimates of probabilities in the absence of precise data. In such cases, it is appropriate to use 'possibility' instead of 'probability' [21]. Fuzzy approach provides a way to determine failure probability values when little quantitative information is available ([21,23] and [31]) wherein the *BE* probabilities are treated as fuzzy numbers. In this scenario, the conventional FTA is replaced by fuzzy fault tree analysis (FFTA).

Lin and Wang [20] combined fuzzy set theories with expert elicitation to evaluate the failure probability of the *BE*s of a robot drilling system. Shu et al. [29] used fuzzy methods to analyse fault trees for a printed circuit board assembly. Khan and Abbasi [15] developed computer based tool for

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determining the reliability of chemical process industries. Research had been carried out to evaluate the importance of the BEs in fuzzy fault tree analysis (FFTA). Furuta and Shiraishi [6] represented the values of fuzzy membership functions to find the importance of the BEs. Suresh et al. [31] followed Euclidean distance to evaluate the importance and uncertainty of fuzzy basis set. FFTA had been extensively applied to various engineering problems.

The various aspects of safety and reliability of components of subsea systems have received attention, e.g. Hui et al. [11] on reliability estimates of safety instrumented systems and Wanvik [33] on reliability of subsea X-trees etc. However, application of risk based methodologies, specifically the FTA methodology in a subsea system had been rare. The FTA approach, however, had long been extensively adopted in aerospace and nuclear industries. The oil and gas transportation pipelines and wells represent an area where one can find many applications of FTA, FFTA and Bayesian network. Some recent works in this area are by Shan et al. [28], Guo et al. [8], Lavasani et al. [17] and Li et al. [18]. These works deal with oil and gas pipelines and abandoned oil and gas wells. Specifically, Lavasani et al. [16] and Shahriar et al. [27] used FFTA in studying offshore oil and gas pipelines. Application to deepwater drilling operation is treated in Rathnayaka et al. [25] and Khakzad et al. [13]. Actual accidents in the oil and gas sector have been studied using a variety of risk assessment methodologies in Khakzad et al. [14] and Kalantarina et al. [12].

The subsea production systems are complex, operate in harsh environment, and inaccessible, making the repair and maintenance activities costly and time consuming. Therefore, it is imperative that quantitative risk estimation methodologies be adopted in their design and operation. Hu et al. [10] made an attempt to use the FFTA approach to the problem of leakage of oil and gas in a subsea production system, but the details had been sketchy and analysis preliminary. In this paper, this problem has been adopted and an exhaustive FFTA has been carried out. Several importance measures reflecting the sensitivity of the TE on the BEs are computed and discussed. The methodology presented in this paper can be readily extended to the subsea systems that include production platforms in an oil and gas field and incorporate the pipeline export system to shore.

It should be noted that FTA, FFTA and several other approaches to quantitative risk analysis (QRA) such as ETA, combined FTA and ETA or bow tie analysis, Bayesian networks etc. have been adopted in many application oriented papers for assessing risk of oil and gas pipelines (both onshore and offshore), submarine pipelines, oil and gas wells (both onshore and offshore), deep water drilling operations, blow out prevention and oil and gas leakage in offshore platforms. In comparison, there had been practically no work (except a preliminary study by [10]) reported in the literature on application of QRA techniques (specifically FTA and FFTA) in subsea production systems. Whereas in other applications a variety of fault trees have been considered, for subsea production systems, which constitute the mainstay of

deepwater production, no generic or detailed fault tree structure is either available or studied. This paper presents, for the first time, a comprehensive analysis of a generic fault tree of a subsea production system. In doing so, a variety of well known and widely used probabilistic risk assessment measures are computed and in addition, a measure called 'fuzzy weighted index', which has rarely been used in the literature for systems in the oil and gas sector, is also computed for the chosen subsea production system. The spreads of fuzzy failure probabilities, arising out of uncertainties that are always present in any expert elicitation model that is used in FFTA, both towards left (lower probability) and right (higher probability), affect the top or critical event in an asymmetric manner. This aspect, rarely used and reported in the literature for FFTA studies of oil and gas sector problems, has also been considered in the present work for the subsea production system.

The problem statement, which follows in the next section, considers a generic fault tree model of a subsea system. Typically, FTA of such systems are carried out. However, FFTA approach to this problem, that has not been reported in the literature in a comprehensive way, yields new insight governed by the fuzzy weighted index (FWI) of BEs that contribute to failure, and such an importance measure can only be used if the fault tree probabilities are fuzzy in nature. Also, the spread of fuzzy probabilities, both to the left and to the right, affects the system failure in a fashion that has not been investigated for a system of this nature in the literature.

2. Definition of the problem

The fault tree of the subsea pipeline system with leakage as the top event (TE), built up through deduction, is shown in Fig. 1.

The leakage could be from

(a) gas or oil well, OR
(b) leakage in pipe, OR
(c) leakage in key facilities such as connector, X-tree, manifold, pipeline end manifold (PLEM) and pipeline end termination (PLET), OR
(d) third party damage.

Leakage in gas or oil well requires

(a) overpressure in well AND
(b) failure of control in well.

Leakage in pipe requires

(a) defect in pipe AND
(b) failure of leakage control in pipe.

Leakage in key facilities could be from

(a) connector leakage, OR
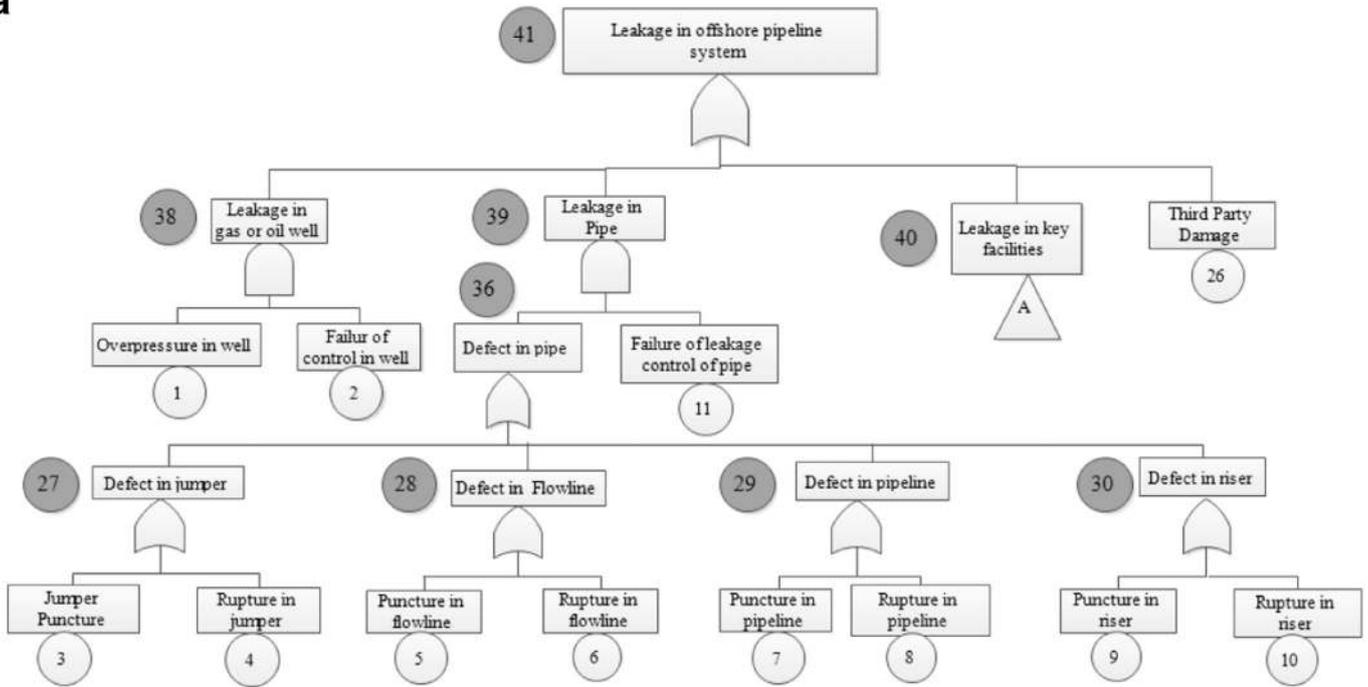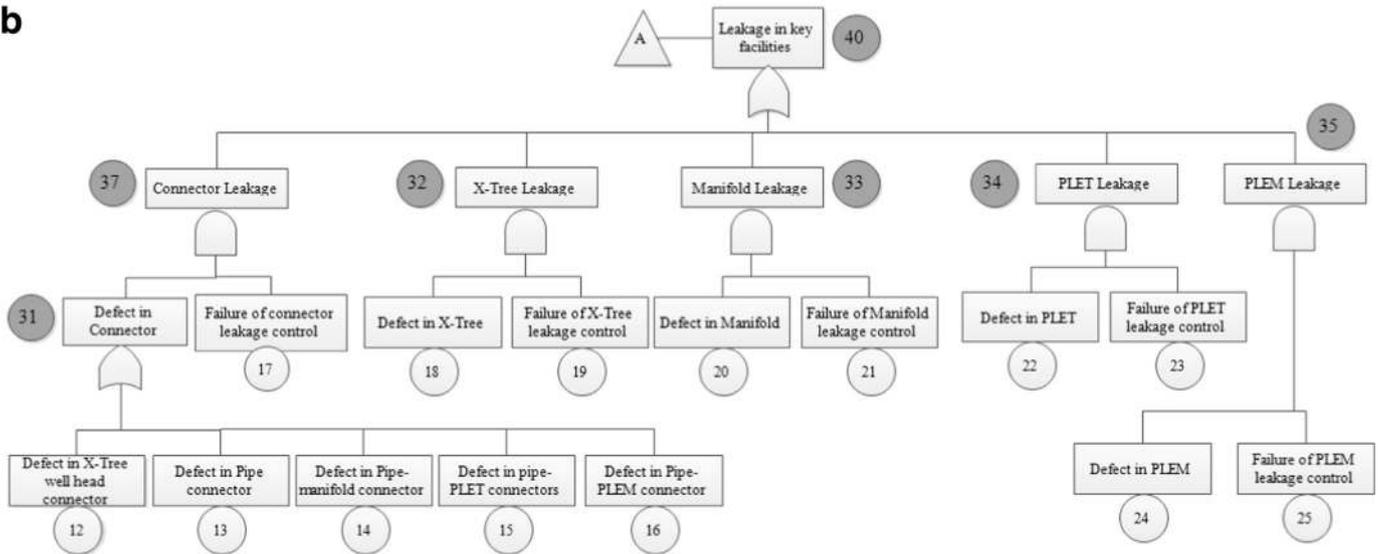
Fig. 1. Fault tree for leakage in subsea production system.

(b) X-tree leakage, OR
(c) manifold leakage, OR
(d) PLEM leakage, OR
(e) PLET leakage.

Defect in pipe could be due to

(a) defect in jumper, OR
(b) defect in flow-line, OR
(c) defect in pipeline, OR
(d) defect in riser.

Each one of these could be due to (a) puncture, OR (b) rupture.

Connector leakage requires

(a) defect in connector, AND
(b) failure of connector leakage control.

Similar argument holds for X-tree, manifold, PLEM and PLET leakages.

Defect in connector could be due to

(a) defect in X-tree wellhead connector, OR
(b) defect in pipe connector, OR
(c) defect in pipe manifold connector, OR
(d) defect in pipe PLEM connector, OR
(e) defect in pipe PLET connector.

From the above description, it is seen that the problem is a generic one and the deductive structure using OR and AND logic gates lends itself to adaptation for any practical leakage problem of a subsea oil and gas production system. The task is to quantify the failure probability of the leakage in the subsea pipeline system, which is the top event. The BEs that lead to the failure of the TE do not have well quantified probabilities. This, therefore, requires a fuzzy treatment of the FTA. Furthermore, the events that have major contribution to the failure of the TE are of interest.

The fault tree in Fig. 1 has 26 basic events (BEs) numbered X_1, X_2, …,X_{26}; 14 intermediate events numbered X_{27}, X_{28}, …, X_{40}; all leading to the top event (TE) which is numbered X_{41}. The BEs are numbered first, the intermediate events next and the TE last. The intermediate events are numbered in such a way that the event number of an intermediate event is connected (downward) to events with smaller event numbers.

3. FTA basics

The fault tree presented in Fig. 1 is a combination of 'AND' and 'OR' gates. The 'AND' gate implies that the output event will occur if all the input events occur, while the 'OR' gate implies that the output event will occur if any one of the input events occur. Let a top event (TE), designated by T, be connected to the basic events (BEs) X_1 to X_N, where N is the number of BEs. The probability of failure of the event X_i is $P(X_i)$. Then, the probability of failure of the TE, denoted $P(T)$, for the cases when the connecting gate is either AND or OR is given by:

$$P(T) = \begin{cases} \prod_{i=1}^{N} P(X_i) & \text{for AND gate} \\ 1 - \prod_{i=1}^{N} \{1 - P(X_i)\} & \text{for OR gate} \end{cases} \quad (1)$$

The failure probability of the TE in a fault tree (as in Fig. 1) is assessed by obtaining its minimal cut sets (MCS) which are minimal, necessary and sufficient conditions for the occurrence of the TE. Once all the MCSs, denoted C_i ($i = 1$, 2, …,N_c) where N_c is the number of MCS, are determined, the failure probability of the TE, denoted $P(T)$, is given by

$$P(T) = 1 - \prod_{i=1}^{N_c} \{1 - P(C_i)\} \quad (2)$$

where $P(C_i)$ is the probability of failure of C_i. As long as the failure probabilities of the BEs are small, (i.e. $P(X_i) << 1$), the above estimate of $P(T)$ can be approximated well by

$$P(T) \approx \sum_{i=1}^{N_c} P(C_i) \quad (3)$$

A typical MCS is a collection of the BEs and may be denoted as

$$C_j = \bar{X}_1, \bar{X}_2, ..., \bar{X}_{N_j-1}, \bar{X}_{N_j} \text{ where } \bar{X}_i \in (X_1, X_2, ..., X_N) \quad (4)$$

where N_j is the number of BEs in C_j, also called the order of the MCS.

The determination of MCSs from the fault tree follows a well-established procedure, a detailed account of which may be found in Vesely et al. [32]. The cut sets are the unique combinations of component (i.e. BE) failures that can cause system (i.e. TE) failure. A cut set is minimal when any BE is removed from the set, the remaining BEs collectively are no longer a cut set. A longer MCS (i.e. consisting of more number of BEs, also called the order of the cut set) makes the TE less vulnerable to that MCS. In other words, a cut set of order one is more critical than a cut set of order two or higher. When a cut set has only one BE, the TE will occur as soon as this BE occurs. When a cut set has two BEs, both of these have to occur at the time to cause the TE to occur. Also, higher N_c implies higher vulnerability.

4. System data and analysis procedure

4.1. Failure probabilities of the basic events

In subsea oil production systems, there is a lack of observed data and yet, to carry out FTA, the occurrence probabilities of the BEs must be known. Expert elicitation or expert opinion provides information to compute the failure probability of the BEs which includes interview, Delphi method, ranking and scaling, method of paired comparison [9] etc. Indirect interaction/interview is adopted in the present study. A definite advisory table is made and given to the experts. The experts give the opinions based on their experience. Experts apply natural linguistic expressions such as 'very low', 'low', 'medium', 'high' and 'very high' to describe the probability of the BEs. Fuzzy set theory is used to handle these linguistic expressions using fuzzy set theory.

Expert judgement models play a crucial role in quantitative risk assessment. Several models are available in the literature [26] such as classical expert model and Bayesian expert model. Expert judgements can suffer from bias, dispersion and independency [2,4]. There are statistical tests that can restrict the judgement data from multiple experts from these deficiencies. Checks on variance and correlation may also be necessary. The choice of a model, however, must satisfy the need of the risk analyst and this is achieved, and in practice dictated, by the protocol followed by a particular industry. The expert judgement evaluation protocol followed by industry is achieved by 'past experience' and hence is usually reliable. In the present work, such a protocol has been used for analysis.

4.2. Expert elicitation

Experts from different fields are selected to give their assessment of the failure probabilities of the BEs in the form of

Table 1
Weights for attributes.

Attributes				Weight
Title/Designation	Experience in years	Education level	Age in years	
Professor/Senior Manager	>30	Doctoral	>50	5
Associate Professor Manager	20–30	Masters	40–50	4
Asst. Professor Asst. Manager	10–20	Bachelors	30–40	3
Lecturer/Sr. Officer	5–10	Technical	25–30	2
Worker/Officer	<5	Graduate	<25	1

Table 2
Experts and their weighting factors.

Expert No.	Title	Experience (years)	Education level	Age (years)	Weighting score	Weighting factor
1	Professor	>30	Masters	>50	19	0.1202
2	Manager	10–20	Masters	40–50	15	0.0949
3	Associate Professor	10–20	Masters	30–40	14	0.0886
4	Sr. Officer	20–30	Bachelors	40–50	13	0.0822
5	Sr. Manager	20–30	Masters	>50	18	0.1139
6	Worker	>30	Technical	>50	13	0.0822
7	Professor	10–20	Doctoral	30–40	16	0.1012
8	Manager	5–10	Masters	30–40	13	0.0822
9	Sr. Manager	>30	Masters	>50	19	0.1202
10	Professor	>30	Masters	40–50	18	0.1139
Sum					158	1.0

linguistic expressions based on their experience and knowledge about leakage in subsea pipeline systems. This has been carried out by designing an appropriate questionnaire. The experts' opinions are evaluated based on their respective weighting factors since experts differ in opinion and their levels of expertise. The weights are assigned for attributes such as (a) title or designation, (b) length of experience, (c) education level and (d) age in a scale of 1 to 5 as shown in Table 1. The weighting score of an expert is the summation of these weights for attributes. The weighting factor for each expert is then computed by

Weighting factor of the expert =

$$\frac{\text{Weighting score of the expert}}{\text{Sum of weighting scores of all experts}} \qquad (5)$$

Table 2 gives the weighting scores and weighting factors computed for each of the ten experts consulted in this study. The data in Tables 1 and 2 closely follow the industry protocol of an offshore oil and gas industry.

4.3. Linguistic expressions to fuzzy numbers

A numerical approximation system was proposed by Chen and Hwang [1] to convert linguistic expressions to fuzzy numbers. The linguistic expressions 'very high', 'high', 'medium', 'low' and 'very low' are represented as VH, H, M, L and VL, respectively. These are related to the fuzzy membership functions as shown in Fig. 2 which consist of both triangular and trapezoidal fuzzy numbers. The triangular fuzzy numbers are converted into trapezoidal fuzzy numbers for the ease of computation. Each trapezoidal fuzzy number is represented by four values (a, b, c, d) for the five membership functions

Table 3
Expert opinions on failure of the basic events.

BE	Expert no.									
	1	2	3	4	5	6	7	8	9	10
1	VH	VH	H	VH	H	VH	VH	H	H	VH
2	VH	VH	M	H	VH	VH	M	H	VH	VH
3	M	M	M	H	H	H	M	H	M	H
4	M	H	M	H	M	H	M	H	M	H
5	H	H	H	M	M	M	L	M	M	M
6	M	M	M	H	M	H	H	H	M	H
7	M	M	M	H	H	H	M	M	M	M
8	H	M	H	M	M	L	H	H	H	H
9	VH	H	H	M	H	M	H	H	VH	VH
10	VH	H	H	H	H	M	H	H	VH	VH
11	VH	VH	H	H	VH	VH	H	VH	VH	VH
12	VH	VH	M	VH	H	VH	M	H	M	H
13	H	H	H	H	H	H	H	H	H	H
14	VH	VH	M	VH	H	VH	M	H	M	H
15	VH	H	H	H	H	M	H	H	H	M
16	H	H	H	H	H	M	H	H	H	H
17	VH	H	VH	VH	H	VH	VH	VH	H	VH
18	M	H	M	VH	M	VH	VH	H	VH	H
19	H	VH	H	VH	H	VH	H	H	H	VH
20	VH	VH	M	H	H	VH	M	VH	M	H
21	VH	VH	M	H	H	M	M	VH	VH	H
22	M	VH	M	VH	M	H	H	H	H	VH
23	VH	H	H	H	H	M	H	H	H	H
24	M	VH	M	VH	M	H	H	H	H	VH
25	VH	H	H	H	H	M	H	H	H	H
26	L	M	M	M	H	M	M	L	M	M

(VH, H, M, L and VL) of Fig. 2. The opinions of ten experts for all BEs are summarised in Table 3.
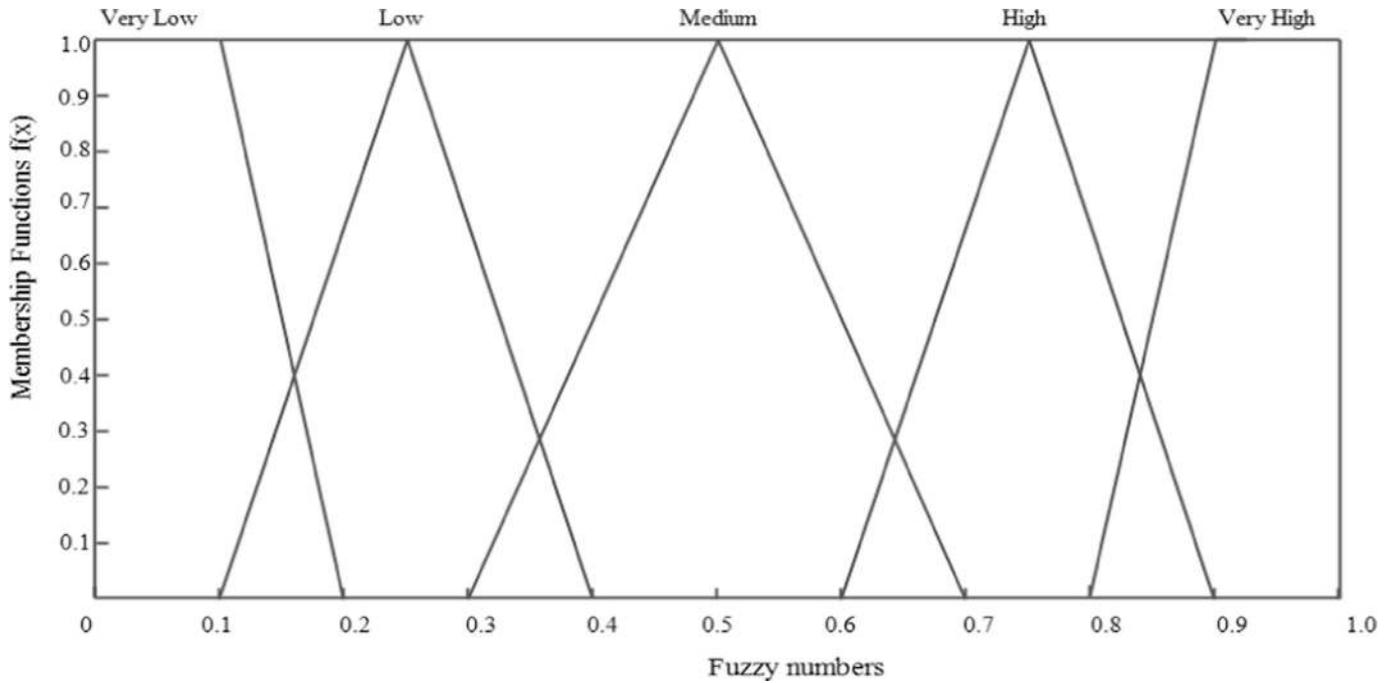
Fig. 2. Fuzzy membership function for fuzzy numbers.

4.4. Aggregation for deriving the estimates of the basic events

Every *BE* is given a rating by the experts as shown in Table 3. All the ratings for a single *BE* must be combined or aggregated to obtain a single opinion. One of the methods to obtain the aggregate is the linear opinion pool [3] given by

$$M_i = \sum_{j=1}^{N_e} A_{ij} w_j \quad (i = 1, 2, ..., N) \tag{6}$$

where N is the number of *BEs* (=26 in Fig. 1), N_e is the number of experts (=10, see Table 4), w_j is the weighting factor of the expert j (given in the last column of Table 2), A_{ij} is the linguistic expression (either a, or b or c or d) of the ith *BE* given by the expert j as given by Table 3 and M_i is aggregated (resultant) trapezoidal fuzzy number of the *BE* X_i. The values of M_i are shown in the second column of Table 4 for all *BEs*.

4.5. Defuzzification process

The defuzzification process converts a fuzzy number to a single point value called fuzzy possibility score (*FPS*) which represents the possibility of the *BE*. The defuzzification process that has been used here is the left and right fuzzy ranking method proposed by Chen and Hwang [1]. The left and right utility score of a fuzzy number can be obtained with the help of Fig. 3 and the corresponding expressions are:

$$\mu_L = \frac{1-a}{1+b-a}; \quad \mu_R = \frac{d}{1+d-c} \tag{7}$$

Table 4
Data of the basic events.

BE (X_i)	Aggregated fuzzy numbers (M) a, b, c, d	FPS	$P(X_i)$	Rank
X_1	0.7190, 0.8392, 0.8987, 0.9595	0.8268	0.04303	3
X_2	0.6722, 0.7994, 0.8639, 0.9266	0.7905	0.03329	4
X_3	0.4424, 0.6187, 0.6187, 0.7949	0.6009	0.00983	17
X_4	0.4367, 0.6139, 0.6139, 0.7911	0.5968	0.00957	19
X_5	0.3709, 0.5506, 0.5506, 0.7304	0.5430	0.00672	21
X_6	0.4386, 0.6155, 0.6155, 0.7924	0.5982	0.00966	18
X_7	0.3835, 0.5696, 0.5696, 0.7557	0.5587	0.00747	20
X_8	0.4715, 0.6361, 0.6361, 0.8006	0.6169	0.01089	16
X_9	0.6215, 0.7620, 0.7975, 0.9025	0.7425	0.02418	7
X_{10}	0.6462, 0.7826, 0.8180, 0.9190	0.7618	0.02742	6
X_{11}	0.7456, 0.8592, 0.932, 0.9728	0.8532	0.05249	1
X_{12}	0.5829, 0.7294, 0.7674, 0.8759	0.7133	0.02003	11
X_{13}	0.6000, 0.7500, 0.7500, 0.9000	0.7175	0.02058	10
X_{14}	0.5829, 0.7294, 0.7674, 0.8759	0.7133	0.02003	11
X_{15}	0.5652, 0.7190, 0.7310, 0.8728	0.6939	0.01771	15
X_{16}	0.5753, 0.7294,0.7294, 0.8835	0.6989	0.01828	13
X_{17}	0.7342, 0.8506,0.9177, 0.9671	0.8418	0.04810	2
X_{18}	0.5804, 0.7272, 0.7658, 0.8741	0.7115	0.01981	12
X_{19}	0.6747, 0.8060,0.8434, 0.9373	0.7847	0.03197	5
X_{20}	0.5829, 0.7294, 0.7674, 0.8759	0.7133	0.02003	11
X_{21}	0.6019, 0.7446, 0.7864, 0.8873	0.7289	0.02214	8
X_{22}	0.5614, 0.7130, 0.7421, 0.8646	0.6947	0.01781	14
X_{23}	0.5994, 0.7475, 0.7595, 0.8956	0.7197	0.02088	9
X_{24}	0.5614, 0.7130, 0.7421, 0.8646	0.6947	0.01781	14
X_{25}	0.5994, 0.7475, 0.7595, 0.8956	0.7197	0.02088	9
X_{26}	0.2937, 0.4778, 0.4778, 0.6620	0.4813	0.00437	22

The *FPS* can be obtained by:

$$FPS = \frac{\mu_R + (1 - \mu_L)}{2} \tag{8}$$
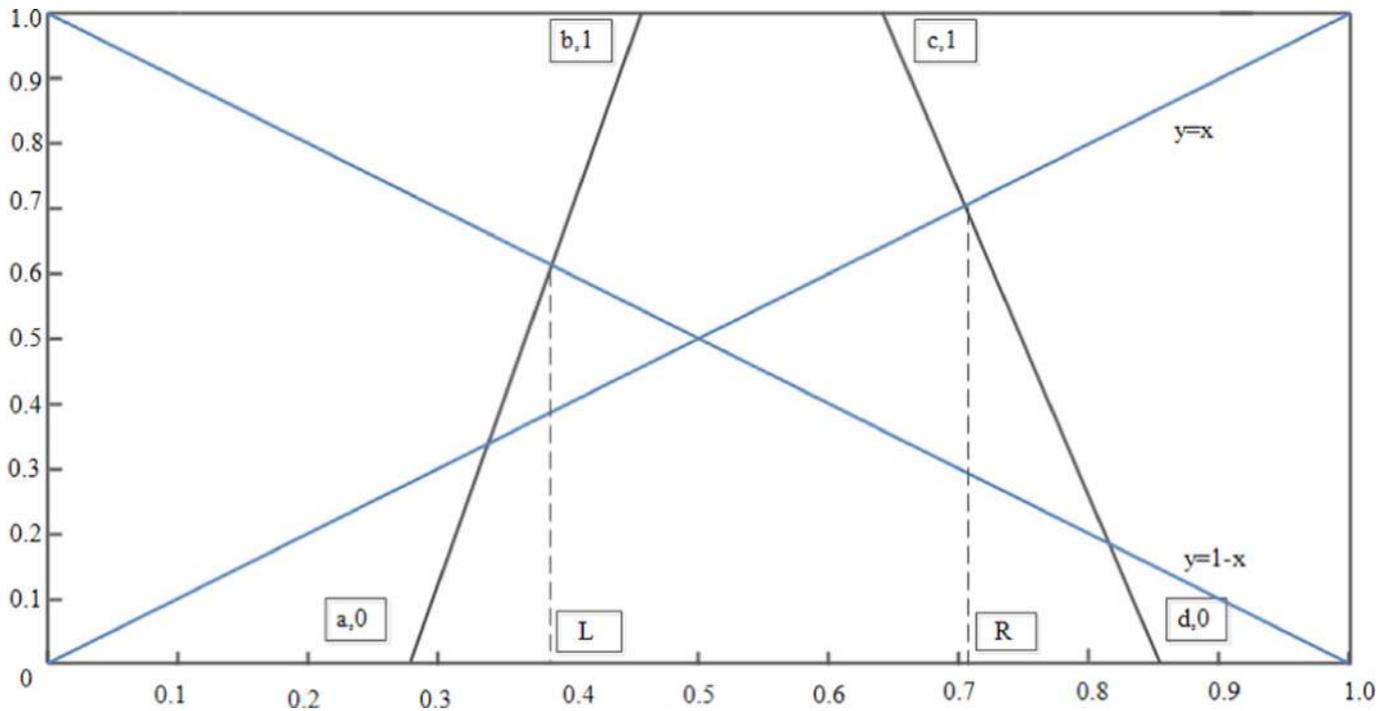
Fig. 3. Representation of fuzzy number and its left and right utility score.

The defuzzification of all *BE*s leads to their *FPS* values and these are given in the third column of Table 4.

4.6. Converting fuzzy possibility score to fuzzy failure probability

The *FPS* of all *BE*s (X_i) needs to be converted to their fuzzy failure probability, $P(X_i)$. The fuzzy failure probability, as defined by Onisawa [24], is given by:

$$P(X) = \begin{cases} \dfrac{1}{10^k} & \text{for } FPS \neq 0 \\ 0 & \text{for } FPS = 0 \end{cases}$$

$$k = 2.301 \left(\frac{1 - FPS}{FPS} \right)^{1/3} \tag{9}$$

The failure probabilities of all *BE*s are given in the fourth column of Table 4.

5. Failure probability of the top event

The failure probability of the top event is given by Eq. (2) and it can also be approximated by Eq. (3). The *MCS*s of the *TE* are determined and these are listed in Table 5. There are 19 minimal cut sets for the *TE* ($N_c = 19$). These sets contain no more than two *BE*s and thus the order of the *MCS*s is 2 ($= \bar{N}_c$). The failure probability of the *TE* (i.e. for the leakage of oil and gas in the system) can be computed either using Eq. (2), which is exact; or using Eq. (3), which is an approximation. The difference between the exact and approximate estimates are of higher order and usually insignificant in practice.

Table 5
Importance measures of basic events and their rankings.

BE (X_i)	PI	Rank (PI)	CI	Rank (CI)	FWI	Rank (FWI)
X_1	0.03329	7	0.08037	5	0.00358	5
X_2	0.04303	6	0.08037	5	0.00358	5
X_3	0.05249	4	0.02894	13	0.00181	11
X_4	0.05249	4	0.02818	15	0.00176	13
X_5	0.05249	4	0.01979	19	0.00123	17
X_6	0.05249	4	0.02844	14	0.00177	12
X_7	0.05249	4	0.02199	17	0.00137	16
X_8	0.05249	4	0.03206	12	0.00143	15
X_9	0.05249	4	0.07118	6	0.00317	6
X_{10}	0.05249	4	0.08075	4	0.00360	4
X_{11}	0.10573	2	0.31133	1	0.01614	1
X_{12}	0.04810	5	0.05405	8	0.00241	8
X_{13}	0.04810	5	0.05552	7	0.00247	7
X_{14}	0.04810	5	0.05405	8	0.00241	8
X_{15}	0.04810	5	0.04779	10	0.00213	10
X_{16}	0.04810	5	0.04933	9	0.00220	9
X_{17}	0.09664	3	0.26075	2	0.01162	2
X_{18}	0.03197	8	0.03553	11	0.00158	14
X_{19}	0.01981	12	0.03553	11	0.00158	14
X_{20}	0.02214	9	0.02488	16	0.00111	18
X_{21}	0.02003	11	0.02488	16	0.00111	18
X_{22}	0.02088	10	0.02086	18	0.00093	19
X_{23}	0.01781	13	0.02086	18	0.00093	19
X_{24}	0.02088	10	0.02086	18	0.00093	19
X_{25}	0.01781	13	0.02086	18	0.00093	19
X_{26}	1.00000	1	0.24542	3	0.00875	3

The failure probabilities of the intermediate events are also calculated by first determining the *MCS*s corresponding to each intermediate event (by considering the intermediate event as the *TE*) and then using Eq. (3).

6. Importance measures

6.1. Rationale

There are various importance measures proposed in the literature that provide information on the contribution of the *BE*s to the occurrence of the *TE*. These measures help identify the weakest link in the system and also help evaluation of alternative design approaches. The measures discussed here are (a) cut sets importance, denoted CSI, (b) probabilistic importance, denoted *PI*, (c) critical importance, denoted *CI* and (d) fuzzy weighted index, denoted *FWI*.

6.2. Probabilistic importance

The probabilistic importance (also called the Birnbaum's measure) is defined as the rate of change in total probability of the system with respect to changes in probabilities of the *BE*s [22]. It is the measure of the functional margin of the system design for the *BE*. The probabilistic importance (*PI*) is given by

$$PI(X_i) = \frac{\partial P(T)}{\partial X_i} \tag{10}$$

The values of *PI* of all *BE*s with their ranking are given in Table 5.

6.3. Critical importance

The critical importance (*CI*) is given by

$$CI(X_i) = \frac{P(X_i)}{P(T)} \frac{\partial P(T)}{\partial X_i} = \frac{P(X_i)}{P(T)} PI(X_i) \tag{11}$$

If it is given that the system has failed, the critical importance of a *BE* X_i is a measure that the failure is caused by this event [34]. In its definition, the probabilistic importance measure is adjusted to the failure probability of the *BE* as a fraction of the total failure probability of the whole system. Therefore, if the probabilistic importance of a *BE* is high, but the failure probability of the *BE* is low with respect to the total failure probability of the system, then the critical importance will automatically adjust its estimate to a lower value [22]. The values of *CI* of all *BE*s with their ranking are given in Table 5.

6.4. Cut sets importance

The relative contribution of the individual *MCS*s to the *TE* is given by a measure called the cut set importance (*CSI*), proposed by Fussel–Vesely [22]. It is given by

$$CSI(C_i) = \frac{P(C_i)}{P(T)} \tag{12}$$

For higher ranked *CSI* (highest rank being 1), it is imperative not to allow the long term average probabilities to increase further. This measure can also be interpreted as the amount of allowed degradation as a function of risk increase. The values and the ranking of CSI of all *MCS*s are given in Table 6.

7. Fuzzy weighted index of basic events

A fuzzy weighted index (*FWI*) is another importance measure where the contribution of each basic event to the failure of the top event is measured in terms of impact or weight [5]. This is estimated by eliminating each basic event from the fault tree and evaluating the total probability of the system. The difference between the total probability of the *TE* with all the basic events and the total probability of the *TE* by eliminating one basic event would be the fuzzy weighted index. This approach can only be followed when the basic events are fuzzy in nature.

The *FWI* is calculated by first converting the deduced crisp probabilities of the *BE*s to fuzzy probabilities. This is done based on the guidelines given in Table 7, and a triangular fuzzy number is obtained with the lower and upper bounds. The fuzzy probability of the *TE* is calculated by following the usual procedure through the minimal cut sets of the fault tree. The *FWI* of a particular *BE* is computed by calculating the *distance* between 'the fuzzy probability of the *TE* with all basic events (A_1)' and 'the fuzzy probability of the *TE* by eliminating the particular basic event (A_2)'. The difference assists in understanding the contribution of the particular basic event to the failure of the top event.

The *FWI* is the *distance* between the two triangular fuzzy numbers A_1 and A_2. It is given by [7]

$$FWI = \delta(A_1, A_2) = 0.5\{\max(|a_1 - a_2|, |c_1 - c_2|) + |b_1 - b_2|\} \tag{13}$$

where $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$ are the triangular fuzzy numbers and δ is the distance between A_1 and A_2. The values of *FWI* of all *BE*s with their ranking are given in Table 5.

8. Spreads of the fuzzy failure probability

The fuzzy failure probability for each basic event is a crisp value, which has been computed from expert opinions. These computed values would have inevitable uncertainties involved in them, as they are purely based on the opinion, judgement and experience of the experts. The extent of uncertainty in the computed values is indicated by the 'spread' of the fuzzy failure probabilities. The width of the membership function is the spread of the crisp value [30]. This function consists of a monotonically increasing function $L(x)$ and a monotonically decreasing function $R(x)$ which intersect each other at the maximum (crisp) value. It has the form (see Fig. 4)

$$\mu(x) = \begin{cases} L(x) \text{ for } x \leq m \text{ (Left spread)} \\ R(x) \text{for } x > m \text{ (Right spread)} \end{cases}$$

where $L(x) = 1 - (m - x)/\alpha, R(x) = 1 - (x - m)/\beta$ with $\alpha > 0$ and $\beta > 0$ \tag{14}

In the above, m is the crisp value of the fuzzy number and α and β are the measures of the left and right spreads, respectively. Thus, a fuzzy number is given by the triple (m, α, β). The two algebraic operations on fuzzy numbers that

Table 6

Minimum cut sets for the top event, their importance measures and ranking.

MCS	Events	$P(C_i)$	CSI	Rank	MCS	Events	$P(C_i)$	CSI	Rank
C_1	$X_1.X_2$	0.00143	0.08037	3	C_{11}	$X_{13}.X_{17}$	0.00099	0.05552	5
C_2	$X_3.X_{11}$	0.00051	0.02894	11	C_{12}	$X_{14}.X_{17}$	0.00096	0.05406	6
C3	$X_4.X_{11}$	0.00050	0.02818	13	C_{13}	$X_{15}.X_{17}$	0.00085	0.04779	8
C_4	$X_5.X_{11}$	0.00038	0.01979	17	C_{14}	$X_{16}.X_{17}$	0.00088	0.04933	7
C_5	$X_6.X_{11}$	0.00051	0.02844	12	C_{15}	$X_{18}.X_{19}$	0.00063	0.03553	9
C_6	$X_7.X_{11}$	0.00039	0.02199	15	C_{16}	$X_{20}.X_{21}$	0.00044	0.02488	14
C_7	$X_8.X_{11}$	0.00057	0.03206	10	C_{17}	$X_{22}.X_{23}$	0.00037	0.02086	16
C_8	$X_9.X_{11}$	0.00127	0.07119	4	C_{18}	$X_{24}.X_{25}$	0.00037	0.02086	16
C_9	$X_{10}.X_{11}$	0.00144	0.08075	2	C_{19}	X_{26}	0.00437	0.24542	1
C_{10}	$X_{12}.X_{17}$	0.00096	0.05406	6					

Table 7

Lower and upper bounds to convert crisp probability to fuzzy probability.

Failure rate	Lower bound	Upper bound
$0.01 < P_i$	$P_i/5$	$2P_i$
$0.001 < P_i < 0.01$	$P_i/3$	$3P_i$
$P_i < 0.001$	$P_i/10$	$10P_i$
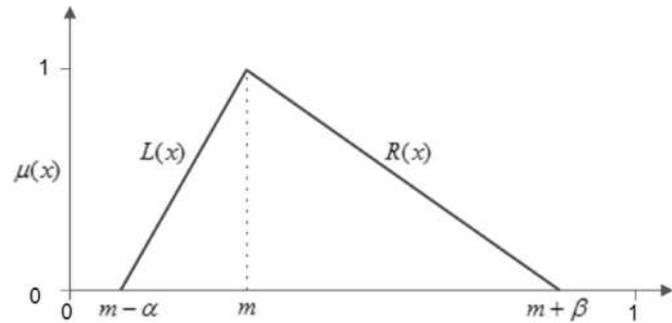
Fig. 4. Left and right spreads of the fuzzy failure probability.

Table 8

Left and right spread (in percentages) of non-basic events including the top event.

Event	MCS	$P(X_i)$	5% spread of BEs		10% spread of BEs	
			$100C_L$	$100C_R$	$100C_L$	$100C_R$
X_{27}	X_3 and X_4	0.01940	4.95	4.95	9.90	9.91
X_{28}	X_5 and X_6	0.01638	4.96	4.96	9.92	9.92
X_{29}	X_7 and X_8	0.01835	4.95	4.96	9.91	9.92
X_{30}	X_9 and X_{10}	0.05160	4.87	4.87	9.73	9.76
X_{31}	X_{12} to X_{16}	0.09664	2.11	2.12	4.20	4.26
X_{32}	X_{18} and X_{19}	0.00063	9.75	10.25	18.96	21.01
X_{33}	X_{20} and X_{21}	0.00044	9.75	10.27	19.03	21.02
X_{34}	X_{22} and X_{23}	0.00037	9.76	10.24	19.01	20.99
X_{35}	C_{18}	0.00037	9.76	10.24	19.01	20.99
X_{36}	X_3 to X_{10}	0.10573	1.04	1.05	2.07	2.11
X_{37}	C_{10} to C_{14}	0.00465	4.30	4.11	8.82	7.98
X_{38}	C_1	0.00143	9.77	10.26	18.98	21.00
X_{39}	C_2 to C_9	0.00555	1.89	1.80	3.87	3.51
X_{40}	C_{10} to C_{18}	0.00620	3.11	2.95	6.35	5.77
X_{41}	C_1 to C_{19}	0.01777	1.16	1.11	2.38	2.17

are required for a fault tree consisting of OR and AND gates are the addition and multiplication of two fuzzy numbers with spreads given by, say, by (m_1, α_1, β_1) and (m_2, α_2, β_2). These operations are given by

$$(m_1, \alpha_1, \beta_1) \oplus (m_2, \alpha_2, \beta_2) = (m_1 + m_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)$$
$$(m_1, \alpha_1, \beta_1) \otimes (m_2, \alpha_2, \beta_2) = (m_1 m_2, m_1 \alpha_2 + m_2 \alpha_1 - \alpha_1 \alpha_2, m_1 \beta_2 + m_2 \alpha_2 + \beta_1 \beta_2) \quad (15)$$

In Eq. (1), the probabilities $P(X_i)$ are the crisp values m_i ($i = 1$ to N), each associated with a left spread measure α_i and a right spread measure β_i. The expansion of $P(T)$ requires addition and multiplication of $P(X_i)$ in various combinations and these are accomplished by the rules of Eq. (15).

Denoting $\alpha = C_L$ m and $\beta = C_R$ m so that $m - \alpha = m(1 - C_L)$ and $m - \beta = m(1 - C_R)$ and as a result $100C_L$ and $100C_R$ are percentage measures of left and right spreads. The fuzzy failure probability of all BEs has been fuzzified by taking a left and right spread of 5% (i.e. $C_L = C_R = 0.05$) and 10% (i.e. $C_L = C_R = 0.1$). Considering these spreads, the top event probability has been calculated. The values obtained for all non-basic events, including the

top event, are listed in Table 8 with their spread measures in percentage. In Table 8, the MCSs for the 14 intermediate events (X27 to X40) of the fault tree of Fig. 1 and their failure probabilities are also shown. The left and right spreads of the TE probability as functions of the equal left and right spreads of the probabilities of all the BEs are plotted in Fig. 5.

At this stage, it may be of interest to evaluate this dependence of the left and right spreads of the TE probability on the spreads of the probabilities of all the BEs in the case of conventional FTA as against fuzzy FTA. These plots for the conventional FTA are shown in Fig. 6, which can be compared with those of Fig. 5 for fuzzy FTA.

9. Discussion of results

The fuzzy set theory has been used in the fault tree analysis of a subsea production system to quantify the 'probability of the leakage', which is the top event (X_{41}) and this probability is computed as 0.017825 using Eq. (2) and 0.01777 (see Table 8) using Eq. (3), showing negligible effect of the higher order terms.
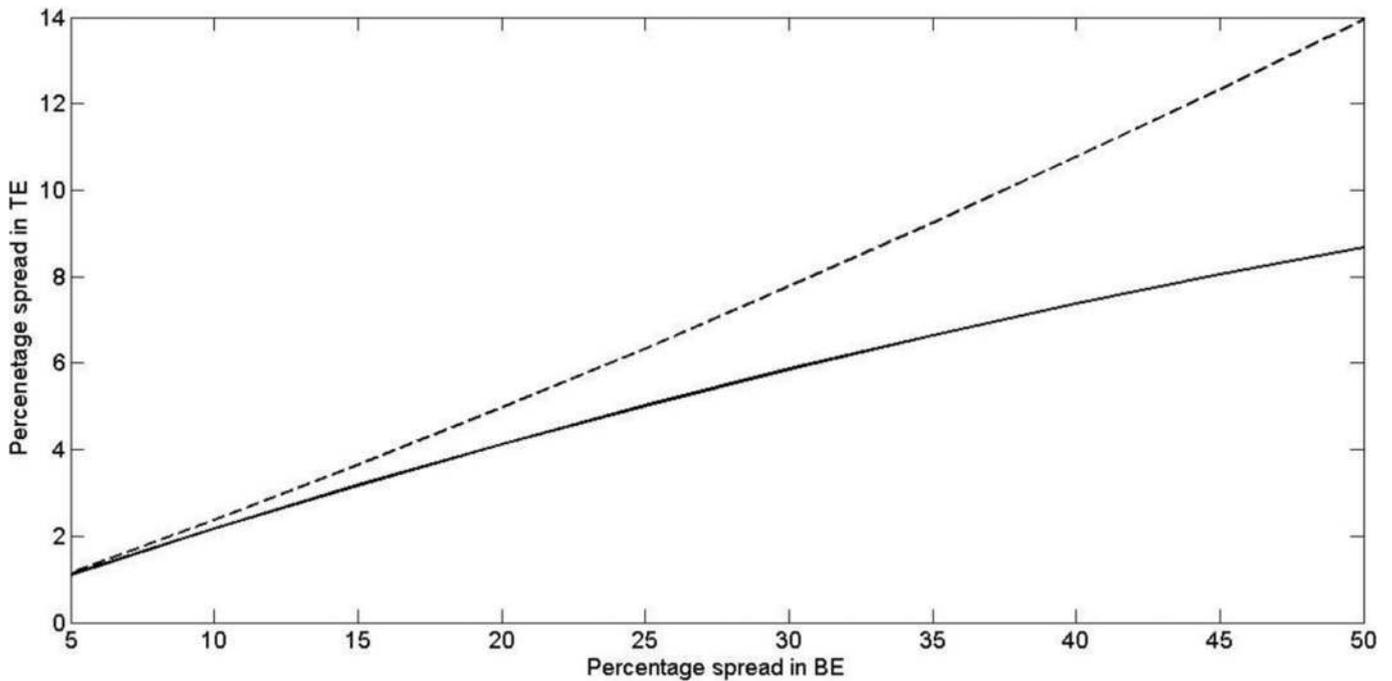
Fig. 5. Spreads of the TE as functions of equal spreads of the *BE*s in FFTA (solid line: right spread; dashed line: left spread).
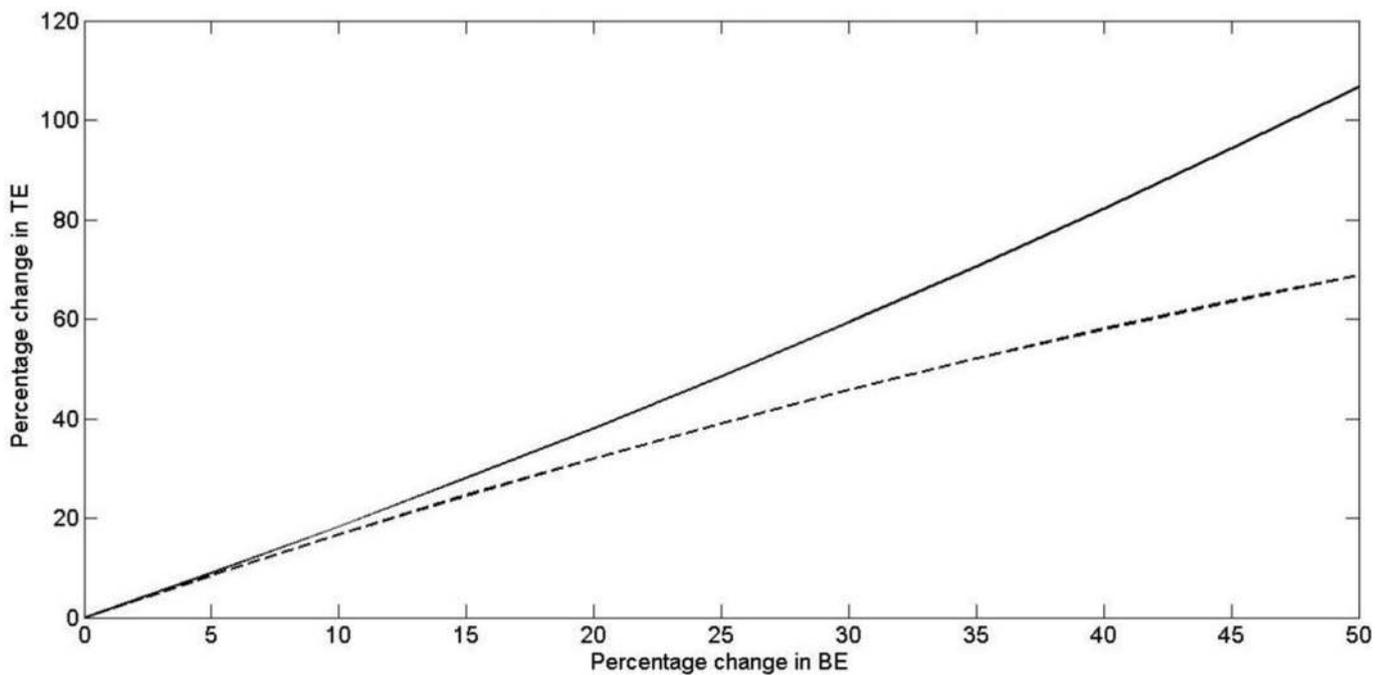
Fig. 6. Spreads of the TE as functions of equal spreads of the *BE*s in conventional FTA (solid line: left spread; dashed line: right spread).

The fault tree has only one first order *MCS* consisting of '*third party damage*' (X_{26}) and the rest *MCS*s are of second order. As a result, X_{26} has the highest rank (i.e. rank 1) of *PI* (see Table 5) and the corresponding *MCS* (i.e. C_{19}) also has the highest rank (see Table 6). Next in the order of importance of the *BE*s, the events '*failure of leakage control in pipe*' (X_{11}) and '*failure of connector leakage control*' (X_{17}) affect the overall system failure significantly since they have

PI ranks of 2 and 3 respectively and *CI* (and *FWI*) ranks of 1 and 2 respectively (see Table 5). Next in the order of importance of the *MCS*s, C_9 and C_1 have *CSI* ranks of 2 and 3 respectively (see Table 6). The *MCS* C_9 consists of '*rupture in riser*' (X_{10}) and '*failure of leakage control in pipe*' (X_{11}), whereas *MCS* C_1 consists of '*overpressure in well*' (X_1) and '*failure of control in well*' (X_2) and therefore these events (X_1, X_2 and X_{10}) are next in order of importance after X_{11} and X_{17}.

From Table 8 and Fig. 5, it is seen that the left and right spreads of the *TE* (i.e. X_{41}) are less than 25% of that of the *BE*s (e.g. 2.38% left spread of *TE* vs. 10% left spread of all *BE*s). This is encouraging for the overall system because it means that the uncertainty of the fuzzy crisp value of the *TE* will be significantly less that the uncertainty of *BE*s. In other words, the crisp value of the *TE* probability is robust. The opposite is true for conventional FTA as can be seen from Fig. 6. The uncertainty of the evaluated *TE* probability is almost 100% more than the uncertainty of *BE*s. In other words, the left and right spreads, which represent the uncertainty band of the probability of the *BE*s, leads to a much larger (about 100% more) uncertainty band of the probability of the *TE* in conventional FTA, whereas it leads to a much smaller (about 75% less) uncertainty band of the probability of the *TE* in fuzzy FTA. This underscores the inherent desirability of a fuzzy description of the *BE* probabilities, and hence the fuzzy route to FTA.

Leakage of oil or gas in any offshore, onshore or subsea production systems may result in serious accidents such as fire and explosion which may involve serious economic as well as personnel losses. This paper deals with FTA of a subsea system. The FTA only evaluates the probability of occurrence of the leakage event. This must be followed up with event tree analysis (ETA) which establishes the variety of consequences that may follow this leakage event established by FTA. A combined FTA-ETA is called 'bow-tie' analysis (e.g. [28]). At the end of ETA, one obtains the risk measures of various consequences and hence the risk profile of the entire system. This risk profile can be used to draw up managerial decisions regarding inspection and monitoring strategies of various system components. The fuzzy ETA does not seem to have been used for any subsea production system and work in that direction is needed to draw up integrity management of such systems based on the fuzzy approach to risk analysis.

10. Conclusion

A probabilistic failure analysis of the leakage of the gas and oil in a subsea oil and gas production system has been carried out using fuzzy fault tree analysis that requires expert elicitation of the failure probabilities of the basic events in a fuzzy framework. A number of importance measures have been evaluated and discussed which helps to identify the weakest links that may cause leakage in the system. Especially, the fuzzy weighted index measure, that is relatively less used in the literature, has been obtained in this work. The analysis is extended to evaluate the uncertainty involved in the failure probability of the top as well as intermediate events due to the assumed uncertainty levels of the basic events using a left and a right spread of errors. It has been found that a large error band of the basic events yields a much smaller error band of the top event failure probability making it robust to the variations of the basic events. It is shown that the opposite is true for conventional fault tree analysis and hence a fuzzy route to the fault tree analysis is more desirable.

The application of FTA to large subsea production systems that include production platforms as well as export pipelines to onshore infrastructure in a complex oil and gas field can be attempted by extending the methodology presented in the present work.

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