Response of finite-time particle detectors in non-inertial frames and curved spacetime

L. Sriramkumar[1] **and T. Padmanabhan**[2]
IUCAA, Post Bag 4, Ganeshkhind,
Pune 411007, INDIA.

Abstract

The response of the Unruh-DeWitt type monopole detectors which were coupled to the quantum field only for a finite proper time interval is studied for inertial and accelerated trajectories, in the Minkowski vacuum in (3+1) dimensions. Such a detector will respond even while on an inertial trajctory due to the transient effects. Further the response will also depend on the manner in which the detector is switched on and off. We consider the response in the case of smooth as well as abrupt switching of the detector. The former case is achieved with the aid of smooth window functions whose width, T, determines the effective time scale for which the detector is coupled to the field. We obtain a general formula for the response of the detector when a window function is specified, and work out the response in detail for the case of gaussian and exponential window functions. A detailed discussion of both $T \to 0$ and $T \to \infty$ limits are given and several subtlities in the limiting procedure are clarified. The analysis is extended for detector responses in Schwarzschild and de-Sitter spacetimes in (1+1) dimensions.

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[1] sriram@iucaa.ernet.in
[2] paddy@iucaa.ernet.in

1. Introduction

In studying the quantum field theory in Minkowski spacetime, we identify the coefficients of the positive frequency component of the field modes to be the annihilation operators and define the state that gets annihilated by these operators to be the vacuum state for the field. This theory being invariant under the Poincare group, the vacuum state defined by this prescription will be the same for all inertial observers. But the vacuum defined by this procedure is not invariant under a general coordinate transformation in flat spacetime. It is well known, for example that quantisation in Minkowski coordinates and Rindler coordinates are not equivalent[1]. This problem araises again while studying quantum fields in a given curved spacetime: the vacuum state and the particle concept are not invariant under general coordinate transformations while the classical field theory is. Concepts like 'vacuum', 'particles' etc., defined through conventional quantum field theoretic methods do not seem to posess any universal significance but rather have an observer dependant quality about them.

The concept of particle detectors was introduced into this subject[2,3], with the goal of improving our understanding of the concept of a particle in an arbitrary curved spacetime. The general philosophy was that: "Particles are what the particle detectors detect"[4]. With this motivation, detectors coupled to quantum fields were designed and their responses were studied. Though this has been done extensively in literature, only a limited amount of insight into field theory in curved spacetime seems to have been acquired in the process.

The response of these detectors is usually studied for their entire history, *viz* from the infinite past to the infinite future in the detector's proper time. But in any realistic situation the detectors can be kept switched on only for a finite period of time and in this context the study of the response of the detector during a finite interval in proper time gains importance. There has been a couple of attempts in literature in the recent past, when the finite time detector response was calculated[5,6]. The authors of these papers, however, have encountered certain divergent results which are difficult to interpret physically. The authors in [5] resort to a complicated 'renormalisation' procedure to remove the divergences while in [6] an attempt is made to eliminate the divergences using a smooth window function.

We reanalyse this question in the present paper. We begin by noting that a detector which is "kept on" only for a finite interval T will be affected by the transients related to the process of switching. This has the consequence that, even an inertial detector in the Minkowski vacuum will register a response for finite T. This effect, as we shall see, needs to be clearly identified before one studies the response in an accelerated trajectory for finite T. Further, we expect the response to vanish when $T \to 0$ for *any* realistic detector on *any* trajectory. This is simply a physical requirement arising from the demand that "a detector which was never switched on should not detect anything". While this demand sounds reasonable, its mathematical implementation turns out to be fairly subtle. We will see that spurious results can arise if one does not implement the limiting procedure with care. When they are done properly no divergences appear and the results turn out to be physically reasonable.

The response of a detector depends, in general, on the following three elements: (1) the state of the quantum field, (2) the trajectory of the detector and (3) the nature of coupling that exists between the field and the detector. In this paper, we assume that the coupling between the detector and the field is of the linear monopole type[2,3]. We consider inertial and accelerated trajectories with the field being in the Minkowski vacuum.

This paper is organised as follows. In section **2** we review the monopole detector theory and comment on certain limiting procedures. In section **3** we study the response of the detector which is operational only for a finite interval of time; the case of a smooth window function as well as that of abrupt switching on is considered. Section **4** dicusses the extension of the finite time detector response theory analysed in the earlier sections to Schwarzschild and de-Sitter spacetimes. Section **5** discusses possible conclusions from our analysis.

2. Response of Unruh-DeWitt detector - revisited

In this section we study the case of a massless, minimally coupled scalar field in (3+1) or (1+1) dimensions with the field being assumed to be in the Minkowski vacuum. The detector-field interaction is described by the interaction lagrangian of the form $c\, m(\tau)\ \Phi[x(\tau)]$, where c is a small coupling constant and $m(\tau)$ is the detector's monopole operator. For a general trajectory, the detector will not remain in its ground state E_0 but will undergo a transition to an excited state E due to its interaction with the scalar field. The amplitude for transition in the first order of perturbation theory is

$$\mathcal{A} \;=\; ic\ < E, \Psi| \int_{-\infty}^{\infty} d\tau\ m(\tau)\ \Phi[x(\tau)]\ |0_M, E_0 > \qquad (1).$$

Using the equation for the time evolution of $m(\tau)$,

$$m(\tau) \;=\; e^{iH_0\tau}\ m(0)\ e^{-iH_0\tau} \qquad (2)$$

where $H_0|E> = \ E|E>$, the above transition amplitude factorises to

$$\mathcal{A} \;=\; \mathcal{M} \int_{-\infty}^{\infty} d\tau\ e^{i(E-E_0)\tau} < \Psi|\Phi(x)|0_M > \qquad (3)$$

where

$$\mathcal{M} = ic < E|m(0)|E_0 > \qquad (4)$$

with $|0_M >$ denoting the Minkowski vacuum and $x(\tau)$ is the location of the detector at proper time τ.

If Φ is expanded in terms of the standard Minkowski plane wave modes, it is clear from equation (3) that the non-zero contribution to the amplitude arises only from the state $|\Psi > = \ |1_k >$. For the case of an inertial trajectory in (1+1) dimensions, with

$$x(\tau) \;=\; x_0 \,+\, vt \;=\; x_0 \,+\, v\gamma\tau \qquad (5)$$

where $\gamma = (1-v^2)^{1/2}$, x_0 and v are constants, $|v| < 1$, the amplitude (3) turns out to be

$$\mathcal{A}_{ine} \;=\; \mathcal{M}\,\frac{e^{-ikx_0}}{\sqrt{4\pi\omega}}\int_{-\infty}^{\infty} d\tau\; e^{i(E-E_0)\tau}\;\; e^{i\gamma\tau(\omega-kv)} \tag{6}$$

with $\omega = |k|$. The integral gives a Dirac delta function and we get

$$\mathcal{A}_{ine} \;=\; \mathcal{M}\,\frac{e^{-ikx_0}}{\sqrt{4\pi\omega}}\,\delta(a) \;=\; 0 \tag{7}$$

where $a \;\equiv\; (E \;-\; E_0 \;+\; \gamma(\omega - kv))$. The last equality follows from noting that since, $k\ v \;\leq\; |k|\ |v| \;<\; \omega$ and $E > E_0$, the argument of the δ- function is always greater than zero. The transition in the detector being essentially forbidden on the grounds of energy conservation.

The following points should be stressed regarding the above - apparently trivial - calculation: (i) the amplitude is being calculated for the system to make a transition from the state $|E_0>$ in the *infinite past*, to the state $|E>$ in the *infinite future*. To do so we need to know the trajectory $x^i(\tau)$ for all τ; *i. e* for $-\infty < \tau < \infty$. No realistic detector can be kept switched on forever. Suppose the detector was kept switched on only during the time interval $-T \leq \tau \leq T$; *then the amplitude will be non-zero*:

$$\begin{aligned}\mathcal{A}_{ine}(T) \;&=\; \mathcal{M}\,\frac{e^{-ikx_0}}{\sqrt{4\pi\omega}}\int_{-T}^{T} d\tau\; e^{i(E-E_0)\tau}\;\; e^{i\gamma\tau(\omega-kv)} &(8)\\ &=\; \mathcal{M}\,\frac{e^{-ikx_0}}{\sqrt{4\pi\omega}}\left\{\frac{2\ \sin(a\,T)}{(a\,T)}\right\} &(9).\end{aligned}$$

The probability for transtition with a fixed ω will be

$$\mathcal{P}_{ine,\omega}(T) \;=\; |\mathcal{A}_{ine}(T)|^2 \;=\; \left\{\frac{|\mathcal{M}|^2}{\pi\omega}\right\}\left\{\frac{\sin(a\,T)}{a}\right\}^2 \tag{10}$$

which is finite for all finite T. For small T, $\mathcal{P}_{ine,\omega} \propto T^2$ and hence vanishes for $T \to 0$; for large T, we use the relations

$$\begin{aligned}\lim_{T\to\infty}\left\{\frac{\sin(a\,T)}{\pi\ a}\right\}^2 \;&=\; \lim_{T\to\infty}\left\{\left(\lim_{T\to\infty}\frac{\sin(a\,T)}{\pi a}\right)\frac{\sin(a\,T)}{\pi a}\right\}\\ &=\; \lim_{T\to\infty}\left\{\frac{\delta(a)\sin(a\,T)}{\pi a}\right\} \;=\; \lim_{T\to\infty}\left\{\frac{T}{\pi}\delta(a)\right\} \qquad (11).\end{aligned}$$

In other words

$$\lim_{T\to\infty}\left\{\frac{\mathcal{P}_{ine,\omega}(T)}{T}\right\} \;=\; \left\{\frac{|\mathcal{M}|^2}{\omega}\right\}\delta(a) \qquad (12).$$

Clearly the rate of transitions $\mathcal{R}_{ine,\omega}(T) \;=\; \mathcal{P}_{ine,\omega}(T)/T$ has the following behaviour: $\mathcal{R}_{ine,\omega} \propto T$ for small T and $\mathcal{R}_{ine,\omega} \propto \delta(a)$ for large T. Hence $\mathcal{R}_{ine,\omega}$ vanishes in both the limits.

The above analysis should teach us three lessons: Firstly, even an inertial detector will "detect particles" if it is switched on and off. This is merely a manifestation of the energy-time uncertainty principle; a detection process lasting for a time $2T$ cannot measure energy differences with an accuracy greater than $(2T)^{-1}$. So for $(a\ 2T) \lesssim 1$, the rate $\mathcal{R}$ will be significantly non-zero. Secondly, the rate $\mathcal{R}$ is a more reliable quantity to compute than $\mathcal{P}$, especially if one is considering the $T \to \infty$ limit. In particular, $\mathcal{P}$ is infinite if we take $T \to \infty$ limit naively in (10). Thirdly, if we want to study the response of accelerated detectors which are switched on only for a finite time, we should subtract out the finite result which is already present in the inertial case. The limits also need to be handled with care to obtain sensible results. We shall say more about it later on.

For the case of an uniformly accelerated trajectory in (1+1) dimensions, the tranformations from the Minkowski to the accelerated frame are

$$x \ = \ g^{-1}\xi \ \cosh(g\tau); \qquad t \ = \ g^{-1}\xi \ \sinh(g\tau); \tag{13}$$

where τ is the proper time of the accelerated observer at ξ. In what follows we shall set $\xi = 1$ without any loss of generality. The transition amplitude for the accelerated trajectory of the detector turns out to be

$$\mathcal{A}_{acc} \ = \ \frac{\mathcal{M}}{\sqrt{4\pi\omega}} \int_{-\infty}^{\infty} d\tau \ e^{i(E-E_0)\tau} \ e^{-ikg^{-1}\cosh g\tau + i\omega g^{-1}\sinh g\tau} \qquad (14).$$

The above integral can be written down in terms of Gamma functions:

$$\mathcal{A}_{acc} \ = \ \frac{\mathcal{M}}{\sqrt{4\pi\omega}} \, g^{-1} \, e^{\frac{-\pi\Omega}{2g}} (\omega g^{-1})^{i\Omega g^{-1}} \ \Gamma(-i\Omega g^{-1}) \tag{15}$$

where $\Omega = E - E_0$. This is clearly non-zero. The probalility for transition $\mathcal{P}_\omega \ = \ |\mathcal{A}|^2$ with a fixed ω will be

$$\mathcal{P}_{acc,\omega} \ = \ |\mathcal{A}_{acc}|^2 = \frac{|\mathcal{M}|^2}{4\pi\omega} \, \frac{1}{g^2} \, e^{-\pi\Omega g^{-1}} |\Gamma(-i\Omega g^{-1})|^2 \ = \ \frac{|\mathcal{M}|^2}{4\pi\omega} \left\{ \frac{2\pi}{\Omega g} \, \frac{1}{(e^{2\pi\Omega g^{-1}} - 1)} \right\} \tag{16}$$

which has a Planckian form in Ω with temperature $\beta^{-1} \ = \ (\frac{g}{2\pi})$.

The finite proper time integral for the transition amplitude for the accelerated trajectory, obtained after substituting for x and t from (13) is

$$\mathcal{A}_{acc}(T) \ = \ \frac{\mathcal{M}}{\sqrt{4\pi\omega}} \, J \tag{17}$$

where

$$J \ = \ \int_{-T}^{T} d\tau \ e^{i\Omega\tau} \ e^{-i\omega g^{-1}(\cosh g\tau - \sinh g\tau)} \qquad (18).$$

This integral for J can be rewritten as

$$J \;=\; \int_{-\infty}^{\infty} d\tau\; e^{i\Omega\tau}\; e^{-i\omega g^{-1} e^{-g\tau}} \;-\; \int_{-\infty}^{T} d\tau\; e^{i\Omega\tau}\; e^{-i\omega g^{-1} e^{-g\tau}} \;-\; \int_{T}^{\infty} d\tau\; e^{i\Omega\tau}\; e^{-i\omega g^{-1} e^{-g\tau}} \qquad (19).$$

Each of the above integrals can be expressed in closed form as

$$J \;=\; \left\{ g^{-1}\, e^{\frac{-\pi\Omega}{2g}}\, (\omega g^{-1})^{-i\Omega g^{-1}} \right\} \left\{ \Gamma(-i\Omega g^{-1}) \;-\; \gamma(-i\Omega g^{-1}\,,\; i\omega g^{-1} s^{-1}) \right.$$
$$\left. -\; \Gamma(-i\Omega g^{-1}\,,\; i\omega g^{-1} s) \right\} \qquad (20)$$

where $s \;=\; e^{gT}$, $\Gamma(m)$ is the complete gamma function and $\Gamma(m,n)$ and $\gamma(m,n)$ are the incomplete gamma functions[7]. Consider now the limit $T \to 0$, ($i.e$ when the detector is not switched on at all). In this limit, $s \to 1$ and the two incomplete gamma functions add upto the complete gamma function thereby giving $J \;=\; 0$. In the other limit, $T \to \infty$, $s \to \infty$, $s^{-1} \to 0$ we get

$$J \;=\; g^{-1}\, e^{-\dfrac{\pi\Omega}{2g}}\, (kg^{-1})^{i\Omega g^{-1}}\; \Gamma(-i\Omega g^{-1}) \qquad (21).$$

Evaluating $|J|^2$ we obtain the thermal spectrum in (16) . Thus we obtain reasonable results for both the limits $T \to 0$ as well as $T \to \infty$.

There is another feature that needs emphasis as regards both (16) and (10) . These are probabilities for transition to fixed final states $|1_k>$ charecterised by a given momentum k. Normally one would like to integrate over all k so as to find the net probability for the detector to have made a transition from $|E>$ to $|E_0>$. This will lead to an integral

$$I_{ine} \;=\; \int_0^{\infty} \frac{d\omega}{\omega} \left\{ \frac{\sin((\Omega+\omega)T)}{(\Omega+\omega)} \right\}^2 \qquad (22)$$

in the case of (10) and to an integral

$$I_{acc} \;=\; \int_0^{\infty} \frac{d\omega}{\omega} \;=\; \lim_{T_1\to\infty} \lim_{T_2\to 0} \left\{ ln\left(\frac{T_1}{T_2}\right) \right\} \qquad (23)$$

in the case of (16) . Both these integrals are formally divergent. However, consider the limit

$$\lim_{T\to\infty} \left\{ \frac{I_{inertial}}{T} \right\} = \int_0^{\infty} \frac{d\omega}{\omega} \left\{ \lim_{T\to\infty} \left(\frac{1}{T} \left(\frac{\sin((\Omega+\omega)T)}{(\Omega+\omega)} \right)^2 \right) \right\}$$
$$= \int_0^{\infty} \frac{d\omega}{\omega} \left\{ \frac{1}{\pi}\delta(\Omega+\omega) \right\} \qquad (24).$$

If $\Omega > 0$, $\omega > 0$ the integrand identically vanishes and we may take this integral to be zero, thereby recovering the earlier result. Exactly the same phenomenon takes place in the case of an accelerated detector.

The probability of transition to all possible E and $|\Psi>$ from E_0 and $|0_M>$, can be expressed in a more formal and concise manner as:

$$\mathcal{P} \;=\; \sum_{E,\,|\Psi>} |\mathcal{A}|^2 \;=\; \sum_{E} |\mathcal{M}|^2\, \mathcal{F}(\Omega) \tag{25}$$

with

$$\mathcal{F}(\Omega) \;=\; \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau'\; e^{-i\Omega(\tau-\tau')}\; G^{+}(x(\tau), x(\tau')) \qquad (26).$$

The detector response function $\mathcal{F}(\Omega)$, is independent of the details of the detector and is determined completely by the positive frequency Wightman function $G^{+}(x(\tau), x(\tau'))$ defined to be

$$G^{+}(x(\tau), x(\tau')) =< 0_M|\Phi(x)\Phi(x')|0_M > \qquad (27).$$

The detector response function $\mathcal{F}(\Omega)$ represents the bath of particles it experiences due to its motion. The remaining factor in (25) represents the selectivity of the detector to this bath and depends on the internal structure of the detector.

For trajectories in Minkowski space, which are integral curves of time-like Killing vector fields (for e.g the inertial and the accelerated trajectories) the Wightman function is invariant under time translations in the reference frame of the detector[8]. Hence

$$G^{+}(x(\tau), x(\tau')) \;=\; G^{+}(\tau - \tau') \;=\; G^{+}(\Delta(\tau)) \tag{28}$$

And the double integration in (26) reduces to a Fourier transform of the two point function multiplied by an infinite time interval. This divergence is usually handled by interpreting the Fourier transform of the two point function to be the transition probability per unit time, *i.e* the rate is given by

$$\mathcal{R}(\Omega) \;=\; \sum_{E} |\mathcal{M}|^2 \int_{-\infty}^{\infty} d\Delta\tau\; e^{-i(E-E_0)\Delta\tau}\; G^{+}(\Delta\tau) \qquad (29).$$

The Wightman function in the (3+1) dimensions for our field is [9]

$$G^{+}(x, x') \;=\; -\frac{1}{4\pi^2\left((t - t' - i\epsilon)^2 - |\mathbf{x} - \mathbf{x}'|^2\right)} \tag{30}$$

which, for the case of an inertial trajectory given by (5) ,reduces to

$$G^{+}_{ine}(\Delta\tau) \;=\; -\frac{1}{4\pi^2(\Delta\tau - i\epsilon)^2} \qquad (31).$$

(We have absorbed a positive factor γ into ϵ). Since $E > E_0$, the integral (29) can be performed by closing the contour in an infinite semi-circle in the lower-half plane. But the pole of the two point function (31) being at $\Delta\tau = i\epsilon$, it does not contribute to the integral and the detector response is zero; in other words the inertial detector does not see any particles in the Minkowski vacuum.

For the case of an accelerated trajectory given by (13) the Wightman function is

$$G^{+}_{acc}(\Delta\tau) \;=\; -\left\{16\pi^2 g^{-2}\sinh^2(\frac{g\Delta\tau}{2} - ig\epsilon)\right\}^{-1} \qquad (32).$$

Using the expansion

$$\csc^2 \pi x \;=\; \pi^{-2}\sum_{n=-\infty}^{\infty}(x \;-\; n)^{-2} \qquad (33)$$

we can express (32) as

$$G^{+}_{acc}(\Delta\tau) = \left(\frac{-1}{4\pi^2}\right)\sum_{n=-\infty}^{\infty}\left(\Delta\tau - 2i\epsilon + 2\pi i g^{-1} n\right)^{-2} \qquad (34).$$

Substituting (34) into (29) and performing the contour integral in the lower-half of the complex plane we obtain the transition probability rate of the detector to be

$$\mathcal{R}_{acc}(\Omega) \;=\; \left(\frac{1}{2\pi}\right)\sum_{\Omega}\frac{|\mathcal{M}|^2\,\Omega}{(e^{2\pi g^{-1}\Omega} - 1)} \qquad (35)$$

which is the well known thermal spectrum.

Having reviewed the Unruh-DeWitt detector theory and calculated the transition probability rate of the detectors for the whole history of the detector trajectory, *i.e* for infinite time intervals, let us now get on with the crux of this paper, *viz* finite time detectors.

3. Detector response with window functions

To understand some of the subtlities mentioned above, we shall begin with a simple example.

Consider an Unruh-DeWitt detector which was moving in a trajectory $x(\tau)$ and was switched on during the interval $\tau = -T$ to $\tau = T$. The response of such a detector is governed by the integral

$$\mathcal{F}(\Omega, T) \ = \ \int_{-T}^{T} d\tau \int_{-T}^{T} d\tau' \ e^{-i\Omega(\tau-\tau')} \ G^{+}(\tau), \tau') \qquad (36).$$

We shall further assume that the trajectory is along the integral curve to a timelike Killing vector field so that $G^{+}(\tau, \tau') \ = \ G^{+}(\tau - \tau')$. *It is clear from this expression that* $\mathcal{F} \to 0$ *as* $T \to 0$ *irrespective of any other details.* Similarly, we should recover the standard results when $T \to \infty$.

We shall now rewrite this expression differently and take the limits $T \to 0$ and $T \to \infty$. Changing the variables to

$$x \ = \ \tau - \tau'; \qquad y \ = \ \tau + \tau' \qquad (37)$$

so that

$$\int_{-T}^{T} d\tau \int_{-T}^{T} d\tau' \ e^{-i\Omega(\tau-\tau')} \ G^{+}(\tau - \tau') \ = \ \left(\frac{1}{2}\right) \int_{-2T}^{2T} dx \int_{-2T+|x|}^{2T-|v|} dy \ e^{-i\Omega x} \ G^{+}(x) \quad (38).$$

The factor 1/2 is the Jacobian of the transformation from the (τ, τ') coordinates to the (x, y) coordinates. Using this, we get

$$\mathcal{F}(\Omega, T) \ = \ \int_{-2T}^{2T} dx \ e^{-i\Omega x} \ G^{+}(x) \ (2T \ - \ |x|) \qquad (39).$$

Let us now consider the limits $T \to \infty$ and $T \to 0$ of this integral. When $T \to \infty$, we get

$$\mathcal{F}(\Omega, T) \ = \ \lim_{T \to \infty} \left\{ (2T) \ \tilde{G}^{+}(\Omega) \ - \ \int_{-2T}^{2T} dx \ e^{-i\Omega x} \ G^{+}(x) \ |x| \right\} \qquad (40)$$

where $\tilde{G}^{+}(\Omega)$ is the Fourier transform of $G^{+}(x)$. Clearly

$$\begin{aligned} \mathcal{R}(\Omega) \ &= \ \lim_{T \to \infty} \left\{ \frac{\mathcal{F}(\Omega, T)}{2T} \right\} \ = \ \lim_{T \to \infty} \left\{ \tilde{G}^{+}(\Omega) \ - \ \left(\frac{1}{2T}\right) \int_{-\infty}^{\infty} dx \ e^{-i\Omega x} \ G^{+}(x) \ |x| \right\} \\ &= \ \tilde{G}^{+}(\Omega) \end{aligned} \qquad (41)$$

provided the second integral is well defined. This expression is finite and represents a constant rate of transition; we have thus recovered the standard result in the necessary limit.

Let us next consider the $T \to 0$ limit which is somewhat tricky. We need to evaluate

$$\mathcal{F}(\Omega, 0) \; = \; \lim_{T\to 0} \int_{-2T}^{2T} dx \; e^{-i\Omega x} \; G^+(x) \; (2T - |x|) \qquad (42).$$

The integral over x is confined to a small range $(-2T, 2T)$ around the origin. This implies that we can expand the integrand in a Taylor series around the origin to obtain the required limit. We write

$$e^{-i\Omega x} \; G^+(x) \; \simeq \; (1 - i\Omega x - \frac{1}{2}\Omega^2 x^2 + ...) \; (G^+(0) + G'^+(0) \; x + \frac{1}{2} \; G''^+(0) \; x^2 + ...) \quad (43).$$

Substituting the above expression into (42) and integrating we obtain

$$\begin{aligned} \mathcal{F}(\Omega, T) \; &\simeq \; 4T^2 G^+(0) \; - \; \frac{4}{3} T^4 \; (G''^+(0) - \Omega^2 G^+(0) - 2i\Omega G'^+(0)) \; + \; O(T^4\omega^4) \\ &\simeq \; 4T^2 \; G^+(0) \qquad (44) \end{aligned}$$

to the lowest order. All derivatives of $G^+(x)$ in (3+1) dimensions behave as ϵ^{-n} at origin and in particular,

$$G^+(0) \; = \; \frac{1}{4\pi^2\epsilon^2} \qquad (45)$$

giving

$$\mathcal{F}(\Omega, T) \; \simeq \; \left(\frac{T^2}{\pi^2 \epsilon^2} \right) \qquad (46).$$

The above expression shows that care should be excercised when the limits $T \to 0$, $\epsilon \to 0$ are incorporated. It is clear from the fundamental definition of the integral in (36) that we must have $\mathcal{F}(\Omega, 0) \; = \; 0$ for all regular integrands. If the integrand has a pole in the real axis (requiring an $i\epsilon$ prescription to give meaning to the integral) then we should *arrange* the limiting procedure in such a way that $\mathcal{F}(\Omega, 0) \; = \; 0$. This can be achieved by using the rule that $\epsilon \to 0$ limit should be taken right at the end, after the limit $T \to 0$ has been taken. Since

$$\lim_{\epsilon\to 0} \left\{ \lim_{T\to 0} \; \left(\frac{T^2}{\epsilon^2} \right) \right\} \; = \; 0; \qquad \lim_{T\to 0} \left\{ \lim_{\epsilon\to 0} \; \left(\frac{T^2}{\epsilon^2} \right) \right\} \; = \; \infty \qquad (47)$$

only the former ordering will provide physically reasonable results. This prescription is also necessary to ensure that $G^+(0), G'^+(0)$...etc exist in the Taylor expansion for $G^+(x)$. For $\epsilon = 0$, this expansion ceases to exist.

In (1+1) dimensions $G^+(x)$ has a logarithmic dependence in x; hence the limit will be modified to the form

$$\mathcal{F}(\Omega, T) \;\propto\; T^2\;\ln(\epsilon^2) \qquad (48).$$

taking $T \to 0$ limit first will give the sensible result $\mathcal{F}(\Omega, 0) \;=\; 0$ while if $\epsilon \to 0$ limit is taken first we will obtain a logarithmic divergence. We shall see explicit examples of such ambiguities (and their resolution) in what follows.

We shall now calculate the response of three different kinds of detectors which have been switched on for a finite time. In selecting these examples, we are motivated by the fact that no realistic detector can be switched on abruptly. Hence, instead of working with (36) , we will consider the integral of the form

$$\mathcal{F}(\Omega, T) \;=\; \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau'\; e^{-i\Omega(\tau-\tau')}\; W(\tau, T)\; W(\tau', T)\; G^+(x(\tau), x(\tau'))$$

where $W(\tau, T)$ is a "window function" with the properties

$$W(\tau, T) \;=\; \begin{cases} 1 & (\text{for } -T \ll \tau \ll T) \\ 0 & (\text{for } |\tau| \gg T) \end{cases} \qquad (49).$$

The abrupt switching corrseponds to $W(\tau, T) \;=\; \Theta(T - \tau) \;+\; \Theta(T + \tau)$. More gradual switching on and off can be mimicked, for *e. g*, by the functions

$$W_1(\tau, T) \;=\; \exp(-\frac{\tau^2}{2T^2})\;; \qquad W_2(\tau, T) \;=\; \exp(-\frac{|\tau|}{T}) \qquad (50)$$

etc. In order to see the effects of smoothness of the window functions on the detector response, we shall discuss the results for all the three cases: W_1, W_2 and W, in that order.

The motivation to study the detector response with smooth window functions W_1 and W_2 is to carefully identify any possible divergence that may arise when a finite time detection is performed. We study the detector response with a gaussian window function (W_1) in **3(a)**, in the presence of a window function with an exponential cut-off (W_2) in **3(b)** and show that no divergences arise in these cases. In the third sub- section **3(c)** we calculate the response for the window function W. All these results remain finite if the limits are handled carefully.

3(a). Gaussian window function:

The detector response integral with the window function W_1 is

$$\mathcal{F}(\Omega, T) \;=\; \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau'\; e^{-i\Omega(\tau-\tau')}\; exp(-\frac{\tau^2}{T})\; exp(-\frac{{\tau'}^2}{T})\; G^+(\tau, \tau') \qquad (51)$$

which can be rewritten as

$$\mathcal{F}(\Omega, T) \;=\; \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau'\; e^{-i\Omega(\tau-\tau')}\; G^+(x(\tau), x(\tau'))\; \exp -\frac{1}{2T^2}\left((\tau+\tau')^2 + (\tau-\tau')^2\right) \qquad (52).$$

Substituting the Wightman function (31) for the inertial trajectory in the above integral and performing the transformations (37) the integral simplifies to

$$\mathcal{F}_{ine}(\Omega,T) = \left(\frac{1}{2}\right)\int_{-\infty}^{\infty} dy\ \exp(-\frac{y^2}{2T^2})\int_{-\infty}^{\infty} dx\ \left\{\frac{-1}{4\pi^2(x-i\epsilon)^2}\right\}\ e^{-i\Omega x}\ \exp(-\frac{x^2}{2T^2})$$
$$= -\frac{T}{8\pi^2}\sqrt{2\pi}\ I \qquad (53)$$

where

$$I = \int_{-\infty}^{\infty}\frac{dx}{(x-i\epsilon)^2}\ e^{-i\Omega x}\ exp(-\frac{x^2}{2T^2}) \qquad (54).$$

Writing the gaussian function in x in the above integral as a Fourier transform using

$$\exp(-\frac{x^2}{2T^2}) = \frac{T}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\ \exp(-\frac{k^2T^2}{2})\ e^{ikx} \qquad (55)$$

and interchanging the order of integration we obtain

$$I = \frac{T}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\ \exp(-\frac{k^2T^2}{2})\int_{-\infty}^{\infty} dx\ \frac{e^{i(k-\Omega)x}}{(x-i\epsilon)^2} \qquad (56).$$

When $k > \Omega$, the x integral can be performed as a contour integral by closing the contour in the upper half of the complex x-plane and the second order pole at $x = i\epsilon$ gives the non-trivial contribution to the integral. But, when $k < \Omega$ the contour has to be closed in the lower-half and since the function is analytic in this half the integral vanishes. Hence the limits of the k-integral may be set to Ω and ∞. After some manipulations and substituting this result in (54) , we get

$$\mathcal{F}_{ine}(\Omega,T) = \left\{\frac{\exp(\epsilon^2/2T^2)\ \exp(\Omega\epsilon)}{2\ \pi}\right\}\int_{r}^{\infty} dp\ e^{-p^2}(p-r) \qquad (57)$$

where

$$p = \frac{kT}{\sqrt{2}} + \frac{\epsilon}{\sqrt{2}T}; \qquad r = \frac{\Omega T}{\sqrt{2}} + \frac{\epsilon}{\sqrt{2}T} \qquad (58).$$

Before proceeding further let us check that the expression (57) gives sensible results for the limits $T \to 0$ and $T \to \infty$. Since this is an inertial detector we must have $\mathcal{F}(\Omega,\infty) = 0$ and for a detector on any trajectory we should have $\mathcal{F}(\Omega, 0) = 0$. These two limits can be obtained from the above result. In the $T \to \infty$ the lower and the upper limits of the above integral coincide the integral vanishes identically, thus reproducing the result appropriate for the inertial detector. (Note that for large r,

$$r\int_{r}^{\infty} dp\ e^{-p^2} \simeq \frac{1}{2}e^{-r^2}\left\{1 + O(\frac{1}{r^2})\right\} \qquad (59)$$

vanishes exponentially). Hence, there is no ambiguity in this result.

Studying the limit $T \to 0$ of (57) , when the window function is sharply peaked at the origin, has to be done more carefully as follows. In this case, because it matters whether the limit $T \to 0$ is taken first and the condition $\epsilon \to 0$ is incorporated later or vice-versa. The earlier alternative is to be adopted (as has been mentioned earlier) for the reason that the ϵ term helps us to identify the poles in the contour integrals; hence unless and until all the other limits in the problem have already been taken care of, the limit on ϵ should not be incorporated. Keeping this point in mind, we consider the limit $T \to 0$, $r \to (\epsilon/\sqrt{2}T)$ and rewrite $\mathcal{F'}_{ine}(\Omega, T))$ as

$$\mathcal{F}_{ine}(\Omega, T) \; = \; \left\{ \frac{\exp(\epsilon^2/2T^2)\; \exp(\Omega\epsilon)}{2\,\pi} \right\} \; B$$

where

$$B \; = \; \left\{ \int_{\frac{\epsilon}{\sqrt{2}T}}^{\infty} dp\; e^{-p^2} p - \frac{\epsilon}{\sqrt{2}T} \left(\int_0^{\infty} dp\; e^{-p^2} - \int_0^{\frac{\epsilon}{\sqrt{2}T}} dp\; e^{-p^2} \right) \right\} \qquad (60).$$

The last term in the above expression is the error function and its asymptotic form for large arguments is given to be

$$\frac{2}{\sqrt{\pi}} \int_0^x dv\; e^{-v^2} \; = \; 1 - \; \frac{e^{-x^2}}{\sqrt{\pi}} \; \left\{ \frac{1}{x} - \frac{1}{2x^3} + \frac{3}{4x^5} \ldots\ldots \right\} \qquad (61).$$

Substituting the above expression in (60) , we obtain the detector response when $T \to 0$ to be

$$\mathcal{F}_{ine}(\Omega, 0) \; = \; \left(\frac{e^{\Omega\epsilon}}{4\pi} \; \frac{T^2}{\epsilon^2} \right) \to 0 \qquad (62)$$

for finite ϵ. This expression has the same form as (46) and clearly illustrates the need to keep $\epsilon \neq 0$ till the end. Note that the detector response function as well the rate of transition $\mathcal{R}_{ine}(\Omega, T) \propto \mathcal{F}_{ine}(\Omega, T)/T$ vanish when $T \to 0$. The non-commutativity of the limiting procedure as regards $T \to 0$, $\epsilon \to 0$ in the detector response functions is evident due to the presence of factors like ϵ/T. If the condition $\epsilon \to 0$ is incorporated first in (57) it factorises to

$$\mathcal{F'}_{ine}(\Omega, 0) \; = \; \left(\frac{1}{\pi} \right) \int_{\frac{\Omega T}{\sqrt{2}}}^{\infty} dp\; e^{-p^2} \; \left(p - \frac{\Omega T}{\sqrt{2}} \right) \qquad (63).$$

If we now take the limit $T \to 0$ we obtain

$$\mathcal{F'}_{ine}(\Omega, 0) \; = \; (\frac{1}{\pi}) \int_0^{\infty} dp\; e^{-p^2} \; p \; = \; \frac{1}{2\pi} \qquad (64).$$

On the other hand, $\mathcal{F}_{ine}(\Omega, T)$ vanishes if we take the limit $T \to 0$ before we set $\epsilon \; = \; 0$. We stress again that the procedure of letting ϵ to zero only after the $T \to 0$ limit is taken is the proper one.

If we are only interested in finite, non-zero values of T, then we can set $\epsilon = 0$ in the integral (57) can be written in a closed form as

$$\mathcal{F}_{ine}(\Omega, T) = \frac{1}{4\pi} \left\{ \exp -(\frac{\Omega^2 T^2}{2}) - \left(\frac{\Omega T}{\sqrt{2}}\right) \Gamma\left(\frac{1}{2}, \frac{\Omega^2 T^2}{2}\right) \right\} \qquad (65).$$

For $\Omega T \gg 1$, this expression has the asymptotic form

$$\mathcal{F}_{ine}(\Omega, T) \simeq \frac{1}{4\pi} \frac{\exp(-\Omega^2 T^2/2)}{\Omega^2 T^2} \qquad (66).$$

This shows that an inertial detector, switched on for a finite period of time, does give a non-zero response which goes to zero exponentially as $T \to \infty$.

Let us now carry out the same analysis for the case of an accelerated detector. For this case, the Wightman function given by (34) , when substituted into (52) and the transfomations (37) when incorporated, the result is

$$\mathcal{F}_{acc}(\Omega, T) = -\left(\frac{1}{8\pi^2}\right) \int_{-\infty}^{\infty} dy \; \exp(-\frac{y^2}{2T^2}) \int_{-\infty}^{\infty} dx \left\{ \sum_{n=-\infty}^{\infty} \frac{e^{-i\Omega x} \; \exp(-x^2/2T^2)}{(x - ib_n)^2} \right\} \qquad (67)$$

where $b_n = \epsilon - 2\pi g^{-1} n$. With the aid of (55) , the above integral can be simplified to the form

$$\mathcal{F}_{acc}(\Omega, T) = -\frac{T}{8\pi^2} \sqrt{2\pi} \sum_{n=-\infty}^{\infty} I_n \qquad (68)$$

where

$$I_n = \frac{T}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \; \exp(\frac{-k^2 T^2}{2}) \int_{-\infty}^{\infty} dx \; \frac{e^{i(k-\Omega)x}}{(x - ib_n)^2} \qquad (69).$$

When $k > \Omega$ the x integration has to be performed by closing the contour in the upper-half of the complex x-plane and the poles corresponding to the values of n between $-\infty$ and zero contribute nontrivially to $\mathcal{F}_{acc}(\Omega, T)$ giving,

$$\mathcal{F}_{acc1}(\Omega, T) = \sum_{n=-\infty}^{0} \left\{ \frac{\exp({b_n}^2/2T^2) \; \exp(\Omega b_n)}{2\,\pi} \right\} \int_{r'}^{\infty} dp' \; e^{-p'^2} \; (p' - r') \qquad (70)$$

where

$$p' = \frac{kT}{\sqrt{2}} + \frac{b_n}{\sqrt{2}T}; \qquad r' = \frac{\Omega T}{\sqrt{2}} + \frac{b_n}{\sqrt{2}T} \qquad (71).$$

When $k < \Omega$ the contour has to be closed in the lower half plane with the contributions arising from the poles corresponding to the values of $n = 1, 2, 3, ...$:

$$\mathcal{F}_{acc2}(\Omega, T) \;=\; \sum_{n=1}^{\infty} \left(\frac{\exp(b_n^2/2T^2)\ \exp(\Omega b_n)}{2\,\pi} \right) \int_{-r'}^{\infty} dp'\ e^{-p'^2}\ (p' + r') \qquad (72).$$

The complete result is $\mathcal{F}_{acc}(\Omega, T) \;=\; \mathcal{F}_{acc1}(\Omega, T) \;+\; \mathcal{F}_{acc2}(\Omega, T)$, *i.e*

$$\mathcal{F}_{acc}(\Omega, T) \;=\; \sum_{n=-\infty}^{0} \left\{ \frac{\exp({b_n}^2/2T^2)\ \exp(\Omega b_n)}{2\,\pi} \right\} \int_{r'}^{\infty} dp'\ e^{-p'^2} (p' - r')$$

$$+ \sum_{n=1}^{\infty} \left(\frac{e^{\frac{{b_n}^2}{2T^2}}\ e^{\Omega b_n}}{2\,\pi} \right\} \int_{-r'}^{\infty} dp'\ e^{-p'^2}\ (p' + r') \qquad (73).$$

Let us again check the two relevant limits. In the limit $T \to \infty$ the lower limit of the above integrals reduce to ∞ and $-\infty$ respectively, so that only ${\mathcal{F}'}_{acc2}(\Omega)$ contributes to the detector response. Evaluating this and imposing the condition $\epsilon \to 0$, we get the standard result:

$$\mathcal{F}_{acc}(\Omega) \;=\; \frac{T}{2\,\sqrt{2\pi}}\ \frac{\Omega}{(e^{2\pi g^{-1}\Omega} - 1)} \qquad (74).$$

In this case the ratio $\mathcal{R}(acc) \;=\; \mathcal{F}_{acc}(\Omega)/T$ should be intrepreted as the transition probability rate.

When $T \to 0$, we can perform the analysis as in the case of inertial detector, since only the $n = 0$ term in the series (73) contributes nontrivially; we obtain the result to be

$$\mathcal{F}_{acc}(\Omega, 0) \;=\; \frac{e^{\Omega\epsilon}}{2\pi}\ \frac{T^2}{\epsilon^2} \qquad (75).$$

This is identical to the inertial detector result and shows that the transition probablity (as well the rate) will go to zero as $T \to 0$.

The fact that both accelerated and inertial detectors give identical results for the $T \to 0$ limit is to be expected on physical grounds. The curvature of the trajectory cannot make its presence felt for infinitesimal intervals and the response of the detector cannot depend on parameters like g which charecterise the detector trajectory.

(Note that, for any T, the detection is now due to two effects: (i) The trajectory being non-inertial and (ii)the detector being kept switched on only for a finite time. The second effect is present even for inertial trajectory. It may be physically more useful to subtract the inertial response from the accelerated detector response to obtain the effects that are uniquely due to (i). In this case $\mathcal{F}_{net} \;=\; \mathcal{F}_{acc} \;-\; \mathcal{F}_{ine}$ vanishes trivially for $T \to 0$. This is, of course, not mandatory to obtain sensible results.)

It is possible to state some of these results in greater generality for this window function. Note that for a detector moving along any trajectory for which $G^+(x, x') \;=\; G^+(\tau - \tau')$ the response function is

$$\begin{aligned}\mathcal{F}(\Omega,T) &= \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \exp -\frac{1}{2T^2}(\tau^2+\tau'^2)\, e^{-i\Omega(\tau-\tau')} G^{+}(\tau-\tau') \\ &= \left(\frac{1}{2}\right) \int_{-\infty}^{\infty} dy\ \exp(-\frac{y^2}{2T^2}) \int_{-\infty}^{\infty} dx\ \exp(-\frac{x^2}{2T^2})\, e^{-i\Omega x}\, G^{+}(x) \\ &= T\sqrt{\frac{\pi}{2}} \int_{-\infty}^{\infty} dx\ \exp(-\frac{x^2}{2T^2})\, e^{-i\Omega x}\, G^{+}(x) \qquad (76).\end{aligned}$$

We can write

$$f(x)\ \left(e^{-i\Omega x}\ G^{+}(x)\right) = f\left(i\frac{\partial}{\partial\Omega}\right)\left[e^{-i\Omega x}\ G^{+}(x)\right] \qquad (77)$$

for any function $f(x)$ which has a power series expansion around $x = 0$. Hence we can write

$$\begin{aligned}\mathcal{F}(\Omega,T) &= T\sqrt{\frac{\pi}{2}} \int_{-\infty}^{\infty} dx \left\{\exp\left(\frac{1}{T}\frac{\partial^2}{\partial\Omega^2}\right)\right\}\left[e^{-i\Omega x}\ G^{+}(x)\right] \\ &= \exp\left(\frac{1}{2T^2}\frac{\partial^2}{\partial\Omega^2}\right)\ \left[\mathcal{F}(\Omega,\infty)\right] \qquad (78).\end{aligned}$$

The expression in the square brackets is the result for the infinite time detector. The corresponding rates are

$$\mathcal{R}(\Omega,T) = \exp\left(\frac{1}{2T^2}\frac{\partial^2}{\partial\Omega^2}\right)\ \left[\mathcal{R}(\Omega,\infty)\right] \qquad (79).$$

This formula allows us to systematically calculate finite time corrections as a series in T^{-1}. To the lowest order, the correction is

$$\mathcal{R}(\Omega,T) = \mathcal{R}(\Omega,\infty) + (\frac{1}{2T^2})\frac{\partial^2}{\partial\Omega^2}\mathcal{R}(\Omega,\infty) + O(T^{-4}) \qquad (80).$$

In the case of uniformly accelerated detector, we get

$$\mathcal{R}(\Omega,T) \simeq \mathcal{R}(\Omega,\infty)\left\{1 - \frac{1}{T^2}\frac{2\pi}{g\Omega}\frac{e^{2\pi\Omega g^{-1}}}{\left(e^{2\pi\Omega g^{-1}}-1\right)^2}\left(e^{2\pi\Omega g^{-1}} - 1 - \pi\Omega g^{-1}(e^{2\pi\Omega g^{-1}}+1)\right)\right\} \qquad (81).$$

3(b). Window function with an exponential cut-off

Having studied the detector response with a gaussian window function, we now study the same with the window function of the type W_2. In this case the response function turns out to be

$$\mathcal{F}(\Omega,T) \;=\; \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \; e^{-i\Omega(\tau-\tau')} \;\exp -\frac{1}{T}(|\tau| \;+\; |\tau'|)\; G^{+}(x(\tau),x(\tau')) \qquad (82).$$

Introducing the window functions as Fourier transforms

$$\exp -(\frac{|\tau|}{T}) \;=\; \int_{-\infty}^{\infty} dk\; f(k)\; e^{ik\tau}; \qquad f(k) \;=\; \frac{T}{\pi}\,\frac{1}{(1+k^2T^2)} \qquad (83)$$

and performing the tranformations (37) we obtain the detector response for the case of an inertial detector to be

$$\mathcal{F}_{ine}(\Omega,T) \;=\; (-\frac{1}{8\pi^2}) \int_{-\infty}^{\infty} dk\; f(k) \int_{-\infty}^{\infty} dq\; f(q) \int_{-\infty}^{\infty} dy\; e^{i\frac{y}{2}(k+q)} \int_{-\infty}^{\infty} dx\; \frac{e^{ix(\frac{k-q}{2}-\Omega)}}{(x-i\epsilon)^2} \qquad (84).$$

When the y and the q integrals in the above expression are performed in that order, the result is

$$\mathcal{F}_{ine}(\Omega,T) \;=\; (-\frac{1}{2\pi}) \int_{-\infty}^{\infty} dk\; f(k)\; f(-k) \int_{-\infty}^{\infty} dx\; \frac{e^{i(k-\Omega)x}}{(x-i\epsilon)^2} \qquad (85).$$

Performing the contour integral after substituting for $f(k)$, the detector response function reduces to

$$\mathcal{F}_{ine}(\Omega,T) \;=\; (\frac{1}{\pi^2})\; e^{\Omega\epsilon} \int_{\Omega T}^{\infty} dp\; \frac{\exp(-p\epsilon/T)}{(1+p^2)^2}\; (p-\Omega T) \qquad (86)$$

where $p = kT$. When $T \to \infty$ the limits of the above integral coincide giving a null result as expected. When $T \to 0$ the lower limit of the above integral goes to zero and so does the second term in the integrand with the result

$$\mathcal{F}_{ine}(\Omega,0) \;=\; (\frac{1}{\pi^2})\; e^{\Omega\epsilon} \int_{0}^{\infty} dp\; \frac{p\;\exp(-p\epsilon/T)}{(1+p^2)^2} \qquad (87)$$

which reduces to zero in the limit $T \to 0$ being exponentially damped out by the $\exp(-p\epsilon/T)$ factor. We again note the crucial role played by the ϵ factor. The limits $\epsilon \to 0$, $T \to 0$ do not(again!) commute in the function $\exp(-p\epsilon/T)$:

$$\lim_{T\to 0}\left\{\lim_{\epsilon\to 0} \exp(-p\,\epsilon/T)\right\} \;=\; 1; \qquad \lim_{\epsilon\to 0}\left\{\lim_{T\to 0}\; \exp(-p\,\epsilon/T)\right\} \;=\; 0 \qquad (88).$$

Sensible result for the inertial detector is obtained with the latter sequence, as we have emphasised several times by now.

If we are interested only in the $T \neq 0$ case, then we can set $\epsilon = 0$ in (86) ; this integral with $\epsilon = 0$ can be expressed in closed form:

$$\mathcal{F}_{ine}(\Omega,T) \;=\; \left(\frac{1}{2\pi^2}\right)\,\left\{\frac{1}{(1+\Omega^2T^2)} \;-\; \frac{\Omega T}{2}\left\{\pi \;-\; 2\tan^{-1}(\Omega T) \;-\; \sin 2(\tan^{-1}(\Omega T))\right\}\right\} \tag{89}$$

For $\Omega T \gg 1$, this function behaves as

$$\mathcal{F}_{ine}(\Omega,T) \;\simeq\; \frac{1}{6\pi^2}\,\frac{1}{\Omega^2T^2} \tag{90}$$

We once again see that the inertial detector will respond in the Minkoski vacuum if it is kept switched on only for a finite T. As $T\to\infty$, this response dies as T^{-2}. For the case of an accelerated detector the response function, the corresponding integral factorises to be

$$\mathcal{F}_{acc}(\Omega,T) = (-\frac{1}{8\pi^2})\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}dk\;f(k)\int_{-\infty}^{\infty}dq\;f(q)\int_{-\infty}^{\infty}dy\;e^{i\frac{y}{2}(k+q)}\int_{-\infty}^{\infty}dx\;\frac{e^{i(\frac{k-q}{2}-\Omega)x}}{(x-ib_n)^2} \tag{91}$$

where $b_n = \epsilon - 2\pi g^{-1}n$. Performing the y and the q integrals in that order the detector response function reduces to

$$\mathcal{F}_{acc}(\Omega,T) \;=\; (-\frac{1}{2\pi})\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}dk\;f(k)\;f(-k)\int_{-\infty}^{\infty}dx\;\frac{e^{i(k-\Omega)}}{(x-ib_n)^2} \tag{92}$$

The above contour integral can be performed as before to give the following result:

$$\mathcal{F}_{acc}(\Omega,T) \;=\; (\frac{1}{\pi^2})\left\{\sum_{n=-\infty}^{0}e^{\Omega b_n}\int_{\Omega T}^{\infty}dp\;\frac{e^{\frac{-pb_n}{T}}}{(1+p^2)^2}\,(p-\Omega T)\right.$$
$$\left. +\sum_{n=1}^{\infty}e^{\Omega b_n}\int_{-\Omega T}^{\infty}dp\;\frac{e^{\frac{pb_n}{T}}}{(1+p^2)^2}\,(p+\Omega T)\right\} \tag{93}$$

where $p = kT$. When $T\to\infty$, the $\exp(-p\,b_n/T)$ factors in the integrand reduce to unity and the lower limit of the integrals are ∞ and $-\infty$ respectively. As the limits coincide the first integral vanishes. In the second integral only the second term contributes, the first term being an odd function reducing to zero in the symmetric limits. Thus, in the $T\to\infty$ limit we recover the Fulling- Unruh-Davies thermal spectrum after $\epsilon\to 0$:

$$\mathcal{F}_{acc}(\Omega) \;=\; \frac{1}{\pi^2}\,\Omega T\left\{\int_{-\infty}^{\infty}\frac{dp}{(1+p^2)^2}\right\}\left\{\sum_{n=1}^{\infty}e^{-2\pi g^{-1}\Omega n}\right\} = \frac{T}{2\pi}\,\frac{\Omega}{(e^{2\pi\Omega g^{-1}}-1)} \tag{94}$$

In this case, $\mathcal{F}_{acc}(\Omega)/T$ is to be interpreted as the rate of transition probablity of the detector.

When $T\to 0$ only the $n = 0$ term contributes non-trivially so that the response function factorises to

$$\mathcal{F}_{acc}(\Omega,0) \;=\; \left(\frac{1}{\pi^2}\right)\, e^{\Omega\epsilon} \int_0^{\infty} dp\, \frac{p\, \exp(-p\epsilon/T)}{\left(1+p^2\right)^2} \tag{95}$$

which vanishes when $T \to 0$ because of the exponential damping factor in the integrand.

3(c). A rectangular window function (sum of two step- functions)

In this section we study the detector response for explicit finite time limits without introducing smooth window functions. The detector response integral for this case is given by (36) and when the transformations (37) are performed it reduces to (39) , that is

$$\mathcal{F}(\Omega,T) \;=\; \int_{-2T}^{2T} dx\; e^{-i\Omega x}\, \left(2T - |x|\right) G^{+}(x) \qquad (96).$$

For the case of an inertial detector the integrals to be evaluated are

$$\mathcal{F}_{ine1}(\Omega,T) \;=\; (-\frac{2T}{4\pi^2}) \int_{-2T}^{2T} dx\, \frac{e^{-i\Omega x}}{\left(x - i\epsilon\right)^2} \tag{97}$$

and

$$\mathcal{F}_{ine2}(\Omega,T) \;=\; (\frac{1}{4\pi^2}) \int_{-2T}^{2T} dx\, \frac{e^{-i\Omega x}}{\left(x - i\epsilon\right)^2}\, |x| \tag{98}$$

so that $\mathcal{F}_{ine}(\Omega,T) \;=\; \mathcal{F}_{ine1}(\Omega,T) \,+\, \mathcal{F}_{ine2}(\Omega,T)$. This finite time response of the inertial detector can be obtained by evaluating the above integrals. The detailed calculations are given in Appendix A. The result is

$$\mathcal{F}_{ine}(\Omega,T) \;=\; \frac{1}{4\pi^2}\Bigg\{ -e^{2i\Omega T} \int_0^{\infty} dv\, \frac{e^{-\Omega v} v}{\left(v+\epsilon-2iT\right)^2} - e^{-2i\Omega T} \int_0^{\infty} dv\, \frac{e^{-\Omega v} v}{\left(v+\epsilon+2iT\right)^2} + 2\int_0^{\infty} dv \frac{e^{-\Omega v} v}{\left(v+\epsilon\right)^2} \Bigg\} \qquad (99).$$

The two limits again give sensible results. When $T \to 0$, the first two integrals exactly cancel the third giving $\mathcal{F}_{ine} \;=\; 0$, provided we keep $\epsilon \neq 0$. If we set $\epsilon \;=\; 0$ before we set $T \;=\; 0$, then the limit $T \to 0$ will produce logarithmic divergences at the lower limit of integration. For large T, the rate $\mathcal{R}_{ine} \;=\; \mathcal{F}_{ine}/T$ vanishes because $\mathcal{F}_{ine}$ is bounded and well defined in this limit while $T \to \infty$.

For the accelerated detector case, the evaluation of the response integrals is similar but a bit more involved. The response function is

$$\mathcal{F}_{acc}(\Omega,T) \;=\; \frac{-1}{4\pi^2} \sum_{n=-\infty}^{\infty} \int_{-T}^{T} d\tau' \int_{-T}^{T} d\tau\, \frac{e^{-i\Omega(\tau-\tau')}}{\left(\tau-\tau'-ib_n\right)^2} \tag{100}$$

where $b_n \;=\; \epsilon - 2\pi g^{-1} n$. Performing the transformations (37) we obtain the response function to be

$$\mathcal{F}_{acc}(\Omega,T) \; = \; \sum_{n=-\infty}^{\infty} \{\mathcal{F}_{a1n}(\Omega,T) + \mathcal{F}_{a2n}(\Omega,T)\} \tag{101}$$

where

$$\mathcal{F}_{acc1n}(\Omega,T) \; = \; \frac{-2T}{4\pi^2} \int_{-2T}^{2T} dx \; \frac{e^{-i\Omega x}}{(x-ib_n)^2} \tag{102}$$

and

$$\mathcal{F}_{acc2n}(\Omega,T) \; = \; \frac{1}{4\pi^2} \int_{-2T}^{2T} dx \; \frac{e^{-i\Omega x}\,|x|}{(x-ib_n)^2} \qquad (103).$$

The finite time response of an accelerated detector can be obtained by evaluating the above integrals. The calculation is given in Appendix B; the result is

$$\begin{aligned}\mathcal{F}_{acc}(\Omega,T) \; &= \; \frac{1}{4\pi^2} \sum_{n=-\infty}^{\infty} \Bigg\{ 2\pi \; 2T \; \Omega \; \Theta(n) \; e^{\Omega b_n} - e^{2i\Omega T} \int_0^{\infty} dv \; \frac{e^{-\Omega v} v}{(v+b_n-2iT)^2} \\ &-e^{-2i\Omega T} \int_0^{\infty} dv \; \frac{e^{-\Omega v} v}{(v+b_n+2iT)^2} + 2 \int_0^{\infty} dv \; \frac{e^{-\Omega v} v}{(v+b_n)^2} \Bigg\} \qquad (104).\end{aligned}$$

In the limit $T \to 0$ the above quantity reduces to zero, the first term identically zero being proportional to T; the second and the third terms being cancelled by the fourth one. Whereas in the infinite time limit, concentrating on the transition probablity rate we obtain

$$\mathcal{R}_{acc}(\Omega \; = \; \frac{\mathcal{F}_{acc}(\Omega)}{(2T)} \; = \; \frac{1}{4\pi^2} \sum_{n=-\infty}^{\infty} 2\pi \; \Omega \; \Theta(n) \; e^{\Omega b_n} \; = \; \frac{1}{2\pi} \frac{\Omega}{(e^{2\pi\Omega g^{-1}}-1)} \tag{105}$$

a thermal spectrum, the other terms in (104) vanishing when divided by the infinite time interval.

Finally we shall provide an asymptotic formula for the detector response with an arbitrary window function of the form $W(\tau/T)$. This is a direct generalisation of the results in (76) to (80) . For a general window function we can write

$$\begin{aligned}\mathcal{F}(\Omega,T) \; &= \; \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \; W(\tau,T) \; W(\tau',T) \; e^{-i\Omega(\tau-\tau')} \; G^+(\tau-\tau') \\ &= \; W\left(i\frac{\partial}{\partial\Omega},T\right) \; W\left(-i\frac{\partial}{\partial\Omega},T\right) \; \mathcal{F}(\Omega,T) \qquad (106).\end{aligned}$$

Assuming that $W(\tau,T) \; = \; W(\frac{\tau}{T})$ has the Taylor expansion

$$W(\frac{\tau}{T}) \simeq W(0) + W'(0)\,(\frac{\tau}{T}) + \frac{1}{2}\,W''(0)\,(\frac{\tau}{T})^2,$$
$$\simeq 1 + \frac{1}{2}\,W''(0)\,(\frac{\tau}{T})^2 \qquad (107)$$

and that $W(0) = 1$, $W'(0) = 0$, we get

$$\mathcal{F}(\Omega, T) \simeq \left(1 - \frac{W''(0)}{2\,T^2}\frac{\partial^2}{\partial\Omega^2}\right)^2 \mathcal{F}(\Omega,\infty)$$
$$\simeq \mathcal{F}(\Omega,\infty) - \frac{W''(0)}{T^2}\frac{\partial^2}{\partial\Omega^2}\left[\mathcal{F}(\Omega,\infty)\right] \qquad (108).$$

This gives the rate

$$\mathcal{R}(\Omega, T) = \mathcal{R}(\Omega,\infty) - \frac{W''(0)}{T^2}\frac{\partial^2}{\partial\Omega^2}\,\left[\mathcal{R}(\Omega,\infty)\right] + O\left(\frac{1}{T^4}\right) \qquad (109)$$

for any window function and trajectory.

4. Detector response in Schwarzschild and de-Sitter coordinate systems

In this section we shall indicate how the above results can be generalised to obtain the detector response in Schwarzschild and de-Sitter spacetimes for observers who are stationed at a constant 'radius', in (1+1) dimensions. The Schwarzschild metric in (1+1)dimensions is

$$ds^2 = (1 - \frac{2M}{r})\,dt^2 - \frac{dr^2}{(1-\frac{2M}{r})} \qquad (110)$$

and under the transformation $r^* = r + 2M\,\ln(r/2M - 1)$ the Schwarzschild metric goes over to the Regge-Wheeler metric

$$ds^2 = (1 - \frac{2M}{r})\,(dt^2 - dr^{*2}) \qquad (111).$$

The Kruskal-Szekeres (K-S, hereafter) coordinate system is related to the Regge-Wheeler(R-W, hereafter) system by the following transformation

$$u = \exp(r^*/4M)\,\cosh(\frac{t}{4M}); \qquad v = \exp(r^*/4M)\,\sinh(\frac{t}{4M}) \qquad (112)$$

so that the metric in this coordinate system is

$$ds^2 = \left(\frac{32M^3}{r}\right)\,\exp(-r/2M)\,(dv^2 - du^2) \qquad (113).$$

Thus the metrics in the K-S and the R-W coordinate systems are conformally flat. Since the lagrangian for the massless scalar field in (1+1) dimensions is conformally invariant, we can take the mode functions to be plane waves.

We define a vacuum state with respect to the normal modes of the Kruskal- Szekeres system and study the response of a detector stationed at a constant r^* in the tortoise coordinate system. The curves of constant r^* are hyperbolae in the u-v plane of the K-S coordinates and are similar to the accelerated trajectories in the Minkowski spacetime. It turns out that a particle detector stationed at constant r^* responds in the K-S vacuum in a manner similar to an accelerated detector in the Minkowski vacuum. This well known result can be obtained as follows. The Wightman function in the (1+1)dimensional case for plane wave normal modes is

$$D^+(x,x') \;=\; (-\frac{1}{4\pi})\; \left\{\ln |((t - -t' - i\epsilon)^2 - |x - x'|^2)|\right\} \tag{114}$$

which for the case of a constant r^* in the K-S system becomes

$$D^+(x,x') \;=\; (\frac{-1}{2\pi})\; \left\{\ln |(2\; \sinh(\frac{\Delta t}{8M} - \; i\epsilon)|\right\} \qquad (115).$$

Since this Green's function is invariant with respect to translations in the t-coordinate, the transition probability rate for the detector at constant r^* to get excited to the energy level E from E_0 is

$$\mathcal{R}_{K-S} \;=\; \sum_E |\mathcal{M}|^2 \int_{-\infty}^{\infty} d\Delta t\; e^{-i(E-E_0)\Delta t}\; D^+(\Delta t) \tag{116}$$

where t is the time coordinate in the R-W system. Substituting the K-S Wightman function for an observer at constant r^* in the above integral we obtain

$$\mathcal{R}_{K-S} \;=\; -\sum_E \; |\mathcal{M}|^2 \int_{-\infty}^{\infty} d\Delta t\; e^{-i\Omega\Delta t} \;\; \left\{\frac{1}{2\pi} \ln(2\; \sinh(\frac{\Delta t}{8M} \; - \; i\epsilon)\right\} \qquad (117).$$

Integrating twice by parts, we get

$$\mathcal{R}_{K-S} \;=\; -\sum_E |\mathcal{M}|^2 \int_{-\infty}^{\infty} d\Delta t\; e^{-i\Omega\Delta t} \{\frac{1}{2\pi}(8M\;\Omega\; \sinh(\frac{\Delta t}{8M} \; - \; i\epsilon)\}^{-2} \tag{118}$$

which is the familiar integral we have already dealt with. The result is a Planckian spectrum:

$$\mathcal{R}_{K-S} \;=\; \sum_E \frac{|\mathcal{M}|^2}{\Omega} \frac{1}{(e^{8\pi M\Omega} - 1)} \qquad (119).$$

A similar analysis can be carried out for the case of the de-Sitter metric

$$ds^2 \; = \; (1 - H^2 r'^2) \; dt'^2 \; - \; \frac{dr'^2}{(1 - H^2 r'^2)} \tag{120}$$

where H is a constant. Defining a new coordinate $r^{'*}$ related to the de-Sitter r' as

$$r^{'*} \; = \; \frac{1}{2H} \; \ln \left\{ \left| \frac{1 + Hr'}{1 - Hr'} \right| \right\} \tag{121}$$

we get the metric in this coordinate system $(t', r^{'*})$ to be conformal to the flat space metric, with

$$ds^2 \; = \; (1 - H^2 r'^2) \; (dt^2 - dr^{'*2}) \qquad (122).$$

The following transformations

$$u' \; = \; e^{Hr^{'*}} \; \cosh(Ht); \qquad v' \; = \; e^{Hr^{'*}} \; \sinh(Ht) \tag{123}$$

when performed yields the flat space metric

$$ds^2 \; = \; H^{-2} \, (1 - Hr)^2 \; (dv'^2 \; - \; du'^2) \qquad (124).$$

Just as constant r^* trajectories in K-S sysytem and the accelerated trajectories in the Minkowski (x, t) plane are hyperbolae, the constant $r' *$ trajectories in the (u', v') are also hyperbolae. The study of the response of a detector stationed at constant $r^{'*}$ in a vacuum defined with respect to the normal modes in the (u', v') system is hence similar to the study of the detector response in Schwarzschild as done above and we get a Planckian response with a temperature $T = H/2\pi$.

This analysis can be extended to other trajectories in these spacetimes. Also the finite time detector response with window functions discussed in sections **3(a)**, **3(b)** and **3(c)** for the case of inertial and accelerated frames can be trivially extended to these two spacetimes for the case of detectors in various trajectories.

4. Conclusions

The specific conclusions related to various detector models have been discussed in the earlier sections wherever appropriate. In this section we shall touch upon the relevance of the present work in a somewhat broader context.

In bringing together the principles of quantum theory and general relativity one notices a major issue of conflict: General relativity is inherently loval in its description while the conventional formulation of field theory uses global structures to define even the most primitive concepts like the vacuum state. This point has been repeatedly made in the literature related to quantum gravity. However, it should also be noted that there is another, operational angle to the quantum theory as well. Quantum mechanics emphasises the role of operational definition of physical quantities including that of the quantum state. As a matter of principle the same philosophy should be applicable to the field theory as well. In other words, one would like to define concepts like vacuum state etc in field theory using purely operational procedures similar to the ones used, for example in defining the spin of an electron by using a magnetic field selector.

It is, however, well known that such procedures are exceedingly difficult to formulate in the case of a relativistic field. The role of particle detectors assumes special importance in this context. The work by Unruh and DeWitt comes closest to the operational definition of quantum states in field theory. In a simplified sense this detector model captures the essence of the actual particle detection which takes place in the laboratory. There is, however, one difficulty in the original Unruh-DeWitt model. This model uses the definition for particle detection which is based on asymptotic states. The calculations are done to estimate the transition probability from past infinity to the future infinity. In any laboratory context, particle detection is local in both space and time.

The analysis in this paper makes a first attempt in investigating the possibilty of an inherently local definition of particle detection both in space and time. We have resolved the difficulties which arise in such a definition and we have provided general formulas to calculate the response of detectors which have been coupled to the field only for a finite interval of time. In a future publication we plan to investigate how these detectors respond in (3+1) curved spacetime while on geodesic and non- geodesic trajectories. Since these toy-models mimic the physical situation as regards locality in space and time, we expect the results to shed some light on the operational definition of quantum processes in curved spacetime.

Appendix A

The finite time detector response integral for the rectangular window function is

$$\mathcal{F}(\Omega, T) \;=\; \int_{-2T}^{2T} dx\; e^{-i\Omega x}\; (2T - |x|)\; G^{+}(x) \qquad (125).$$

For the case of an inertial detector the integrals to be evaluated are

$$\mathcal{F}_{ine1}(\Omega, T) \;=\; (\frac{-2T}{4\pi^2}) \int_{-2T}^{2T} dx\; \frac{e^{-i\Omega x}}{(x - i\epsilon)^2} \qquad (126)$$

and

$$\mathcal{F}_{ine2}(\Omega, T) \;=\; (\frac{1}{4\pi^2}) \int_{-2T}^{2T} dx\; \frac{e^{-i\Omega x}}{(x - i\epsilon)^2}\; |x| \qquad (127)$$

so that $\mathcal{F}_{ine}(\Omega, T) = \mathcal{F}_{ine1}(\Omega, T) + \mathcal{F}_{ine2}(\Omega, T)$. The integral for $\mathcal{F}_{ine1}$ can be evaluated with the aid of a rectangular contour (refer figure 1, $\mathcal{F}_{ine1}$) in the lower half of the complex x-plane with the vertices given by $A_{i1}(-2T, 0)$, $B_{i1}(2T, 0)$, $C_{i1}(2T, -i\infty)$ and $D_{i1}(-2T, -i\infty)$. Since this contour does not enclose the pole, by Cauchy's theorem the integral around this closed contour is identically zero. The value of the integral over the edge $A_{i1}B_{i1}$ can be expressed in terms of the integrals over the other edges $B_{i1}C_{i1}$ and $D_{i1}A_{i1}$; the contribution from $C_{i1}D_{i1}$ being zero because of the vanishing integrand on this edge. Thus

$$\mathcal{F}_{ine1}(\Omega, T) \;=\; (\frac{2T}{4\pi^2}) \left\{ \int_{2T}^{2T - i\infty} dx\; \frac{e^{-i\Omega x}}{(x - i\epsilon)^2} \;+\; \int_{-2T - i\infty}^{-2T} dx\; \frac{e^{-i\Omega x}}{(x - i\epsilon)^2} \right\} \qquad (128)$$

which after some simple manipulations can be expressed as

$$\begin{aligned} \mathcal{F}_{ine1}(\Omega, T) \;=\; (\frac{2T}{4\pi^2}) \Big\{ & (ie^{-2i\Omega T}) \int_0^{\infty} dv\; \frac{e^{-\Omega v}}{(v + \epsilon + 2iT)^2} \\ & - (ie^{2i\Omega T}) \int_0^{\infty} dv\; \frac{e^{-\Omega v}}{(v + \epsilon - 2iT)^2} \Big\} \qquad (129). \end{aligned}$$

The term $\mathcal{F}_{ine2}$ in the inertial detector response function has a $|x|$ term in the integrand and hence has to expressed as a sum of the integrals over limits $(-2T, 0)$ and $(0, 2T)$ for evaluation. Incorporating this result and after some manipulations we obtain

$$\mathcal{F}_{ine2}(\Omega, T) \;=\; (\frac{1}{4\pi^2}) \left\{ \int_0^{2T} dx\; \frac{e^{i\Omega x} x}{(x + i\epsilon)^2} \;+\; \int_0^{2T} dx\; \frac{e^{-i\Omega x} x}{(x - i\epsilon)^2} \right\} \qquad (130).$$

The first of these integrals can be evaluated on a rectangular contour (refer figure 2, $\mathcal{F}_{ine2A}$) in the upper-half of the complex x-plane with the vertices at $A_{i2}(0, 0)$, $B_{i2}(2T, 0)$,

$C_{i2}(2T, i\infty)$ and $D_{i2}(0, i\infty)$. Similarly the second integral can be performed with the aid of another rectangular contour(refer figure 3, $\mathcal{F}_{ine2B}$), this time in the lower-half of the complex x-plane with the vertices at $A_{i2*}(0,0)$, $B_{i2*}(2T,0)$, $C_{i2*}(2T,-i\infty)$ and $D_{i2*}(0,-i\infty)$. Since neither of these contours enclose any poles the integral of consequence can be expressed in terms of integrals over the edges $B_{i2/2*}C_{i2/2*}$ and $D_{i2/2*}A_{i2/2*}$ alone, the integrand vanishing on the edge $C_{i2/2*}D_{i2/2*}$. After some simple algebra we obtain

$$\mathcal{F}_{ine2}(\Omega,T) \ = \ (\frac{1}{4\pi^2})\Big\{(2iT\, e^{2i\Omega T})\int_0^\infty dv\, \frac{e^{-\Omega v}}{(v+\epsilon-2iT)^2}$$

$$-\,(2iT\, e^{-2i\Omega T})\int_0^\infty dv\, \frac{e^{-\Omega v}}{(v+\epsilon+2iT)^2} \ - \ e^{2i\Omega T}\int_0^\infty dv\, \frac{e^{-\Omega v}v}{(v+\epsilon-2iT)^2}$$

$$-\,e^{-2i\Omega T}\int_0^\infty dv\, \frac{e^{-\Omega v}v}{(v+\epsilon+2iT)^2} \ + \ 2\int_0^\infty dv\, \frac{e^{-\Omega v}v}{(v+\epsilon)^2}\Big\} \qquad (131)$$

so that the finite time inertial detector response is

$$\mathcal{F}_{ine}(\Omega,T) \ = \ \frac{1}{4\pi^2}\Big\{-e^{2i\Omega T}\int_0^\infty dv\, \frac{e^{-\Omega v}v}{(v+\epsilon-2iT)^2} - e^{-2i\Omega T}\int_0^\infty dv\, \frac{e^{-\Omega v}v}{(v+\epsilon+2iT)^2}$$

$$+2\int_0^\infty dv\, \frac{e^{-\Omega v}v}{(v+\epsilon)^2}\Big\} \qquad (132).$$

This is the result quoted earlier in the text.

Appendix B

For the case of an accelerated trajectory the finite time detector integral with the rectangular window function is

$$\mathcal{F}_{acc}(\Omega,T) \;=\; \sum_{n=-\infty}^{\infty}\left\{\mathcal{F}_{acc1n}(\Omega,T)+\mathcal{F}_{acc2n}(\Omega,T)\right\} \tag{133}$$

where

$$\mathcal{F}_{acc1n}(\Omega,T) \;=\; \frac{-2T}{4\pi^2}\int_{-2T}^{2T} dx\,\frac{e^{-i\Omega x}}{(x-ib_n)^2} \tag{134}$$

and

$$\mathcal{F}_{acc2n}(\Omega,T) \;=\; \frac{1}{4\pi^2}\int_{-2T}^{2T} dx\,\frac{e^{-i\Omega x}\,|x|}{(x-ib_n)^2} \qquad (135).$$

$\mathcal{F}_{acc1n}(\Omega,T)$ can be evaluated with the aid of a rectangular contour (refer figure 4, $\mathcal{F}_{acc1}$) with the vertices at $A_{a1}(-2T,0)$, $B_{a1}(2T,0)$, $C_{a1}(2T,-i\infty)$ and $D_{a1}(-2T,-i\infty)$. This contour encloses the poles corresponding to the values of n between one and infinity and the integral for $\mathcal{F}_{acc1n}(\Omega,T)$ can be expressed in terms of the integrals over the edges $B_{a1}C_{a1}$ and $D_{a1}A_{a1}$ and the residues corresponding to the enclosed poles. After some manipulations we obtain

$$\mathcal{F}_{acc1n}(\Omega,T) = \frac{2T}{4\pi^2}\Bigg\{2\pi\Omega\Theta(n)e^{\Omega b_n} + (ie^{-2i\Omega T})\int_0^\infty dv\frac{e^{-\Omega v}}{(v+b_n+2iT)^2} - (ie^{2i\Omega T})\int_0^\infty dv\frac{e^{-\Omega v}}{(v+b_n-2iT)^2}\Bigg\} \tag{136}$$

where $\Theta(n) \;=\; 1$ for $n \;>\; 0$ and zero otherwise.

$\mathcal{F}_{acc2n}(\Omega,T)$, after having been split into two integrals with the limits $(-2T,0)$ and $(0,2T)$ reduces to

$$\mathcal{F}_{acc2n}(\Omega,T) = \frac{1}{4\pi^2}\left\{\int_0^{2T} dx\frac{e^{i\Omega x}x}{(x+ib_n)^2} + \int_0^{2T} dx\frac{e^{-i\Omega x}x}{(x-ib_n)^2}\right\} \qquad (137).$$

The first of these integrals can be performed with the help of a rectangular contour (refer figure 5, $\mathcal{F}_{acc2nA}$) on upper-half of the complex x-plane with the vertices $A_{a2}(0,0)$, $B_{a2}(2T,0)$, $C_{a2}(2T,i\infty)$ and $D_{a2}(0,i\infty)$; but for the cases when $n>0$ the pole in the integrand sits right on the edge $D_{a2}A_{a2}$ and to avoid it we indent the contour in such a way so that the pole is left outside. Similarly for evaluating the second integral in (137) a contour (refer figure 6, $\mathcal{F}_{acc2nB}$) with vertices $A_{a2*}(0,0)$, $B_{a2*}(2T,0)$, $C_{a2*}(2T,-i\infty)$ and $D_{a2*}(0,-i\infty)$ can be chosen and the poles sprouting on the edge $D_{a2*}A_{a2*}$ for the values of n between one and infinity can be avoided with an indentation as to leave them outside. The indentation on the contours contribute a residue corresponding to the infinitesimal semicircle around the pole and only the principal value of the integral over the edges $D_{a2}A_{a2}$ and $D_{a2*}A_{a2*}$ can be defined with the result

$$\mathcal{F}_{acc2n}(\Omega,T) = \frac{1}{4\pi^2}\Bigg\{(2iTe^{2i\Omega T})\int_0^\infty dv\frac{e^{-\Omega v}}{(v+b_n-2iT)^2}$$

$$-e^{2i\Omega T}\int_0^\infty dv\frac{e^{-\Omega v}v}{(v+b_n-2iT)^2} - (2iTe^{-2i\Omega T})\int_0^\infty dv\frac{e^{-\Omega v}}{(v+b_n+2iT)^2}$$

$$-e^{-2i\Omega T}\int_0^\infty dv\frac{e^{-\Omega v}v}{(v+b_n-2iT)^2} + 2\int_0^\infty dv\frac{e^{-\Omega v}v}{(v+b_n)^2}\Bigg\} \qquad (138).$$

It is assumed that when the pole happens to settle right on the axis of integration the integral over the axis is taken to be its principal value. The complete accelerated detector response is then given by

$$\mathcal{F}_{acc}(\Omega,T) = \frac{1}{4\pi^2}\sum_{n=-\infty}^{\infty}\Bigg\{2\pi 2T\Omega\Theta(n)e^{\Omega b_n} - e^{2i\Omega T}\int_0^\infty dv\frac{e^{-\Omega v}v}{(v+b_n-2iT)^2}$$

$$-e^{-2i\Omega T}\int_0^\infty dv\frac{e^{-\Omega v}v}{(v+b_n+2iT)^2} + 2\int_0^\infty dv\frac{e^{-\Omega v}v}{(v+b_n)^2}\Bigg\} \qquad (139)$$

as quoted earlier.

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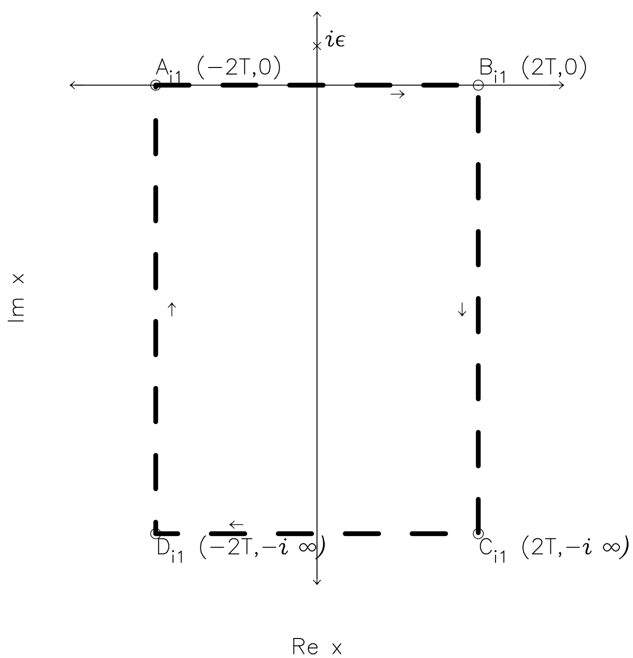

Figure 1, Contour for $\mathcal{F}_{ine1}$

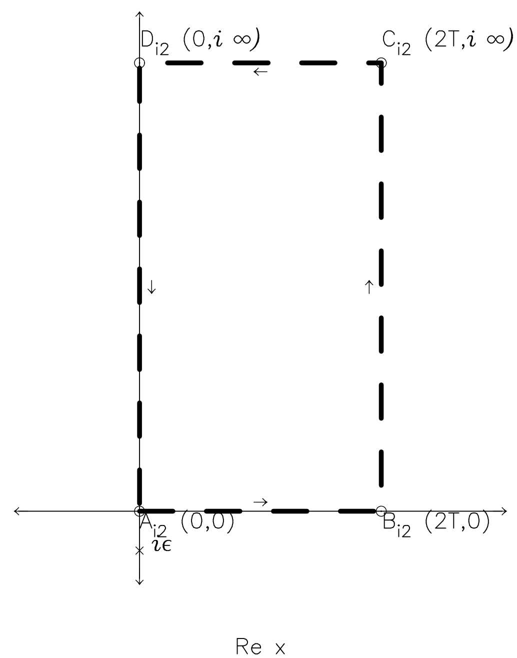

Figure 2, Contour for $\mathcal{F}_{ine2A}$

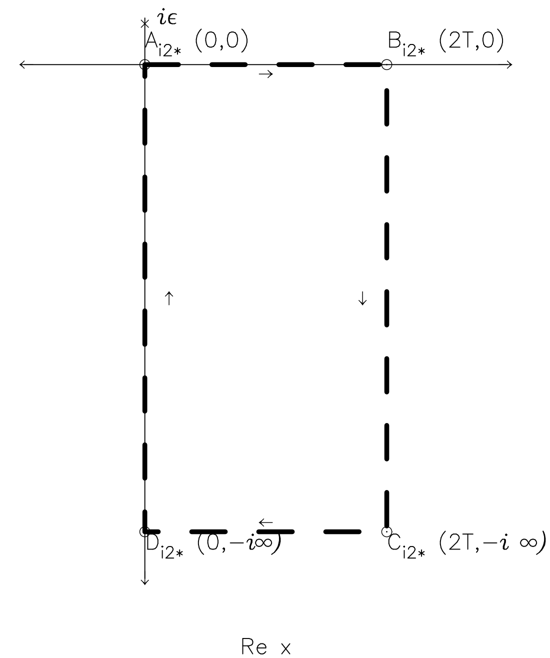

Figure 3, Contour for $\mathcal{F}_{ine2B}$

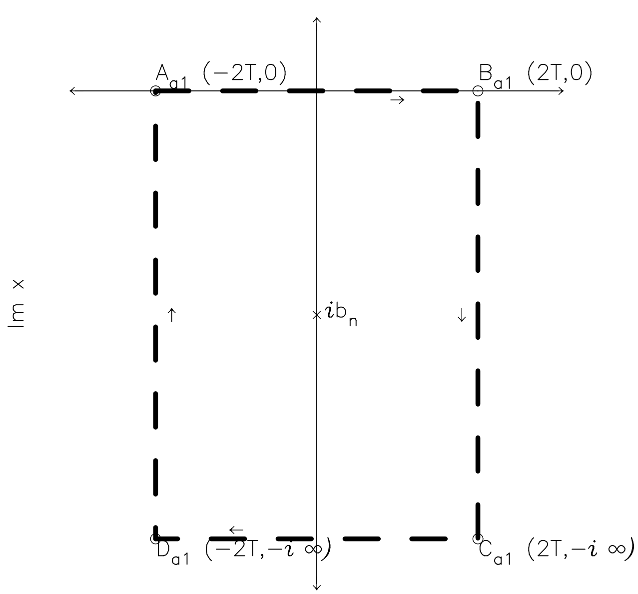

Figure 4, Contour for $\mathcal{F}_{acc1n}$

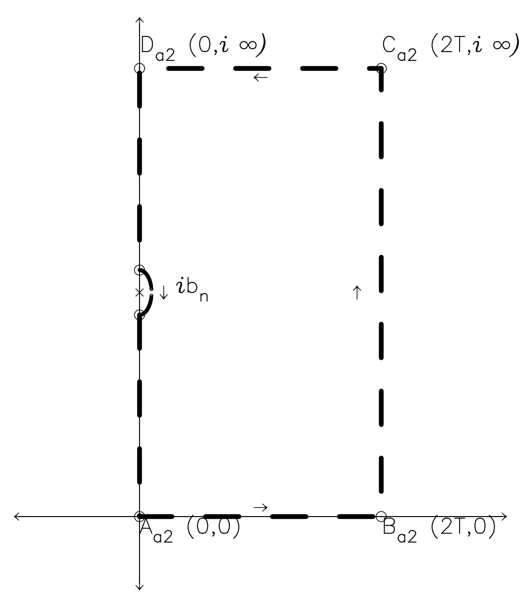

Figure 5, Contour for $\mathcal{F}_{acc2nA}$

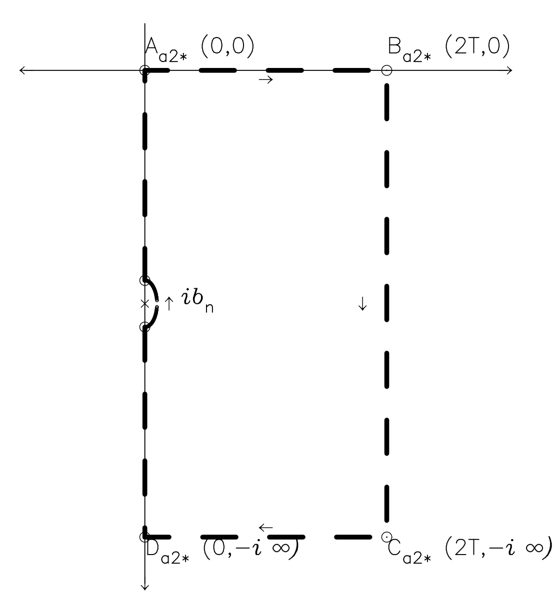

Figure 6, Contour for $\mathcal{F}_{acc2nB}$

Contours on the complex x−plane