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# Equi-prime intuitionistic fuzzy ideals of nearrings

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**Abstract.** The aim of this paper is to introduce the notions of equi-prime intuitionistic fuzzy ideal of a near-ring. We characterize these intuitionistic fuzzy ideals using level subsets and intuitionistic fuzzy points.

## 1. Introduction

Zadeh [18] in 1965 presented fuzzy sets after which a few specialists investigated on the generalization of the thought of fuzzy sets and its application to numerous scientific branches.

Abou-Zaid [1], presented the thought of a fuzzy subnear-ring and contemplated fuzzy ideals(FI) of a near ring( $\mathcal{R}_N$ ).

This idea is additionally talked about by numerous scientists, among them Biswas, Davvaz, [3, 8] have accomplished some intriguing work. The possibility of intuitionistic fuzzy (IF) sets was presented by Atanassov [2] as a generalization of the idea of fuzzy sets. In [3], Biswas applied the idea of IF sets to the theory of groups and examined IF subgroups of a groups. The idea of an IF R-subgroups of a  $\mathcal{R}_N$  is given by Jun, Yon and Cho in [6]. Zhan Jianming and Ma Xueling [19], examined the different properties of intuitionistic fuzzy ideals(IFIs) of  $\mathcal{R}_N$ s.

Booth [5] demonstrated that each equiprime(EP)  $\mathcal{R}_N$  is zerosymmetric, That is, on the off chance that 0 is an EP ideal of  $\mathcal{R}_N$ , at that point  $a0 = 0$  for each  $a \in \mathcal{R}_N$ .

In that paper, they demonstrated that for a common ( thresholds  $\zeta = 0$  and  $\eta = 1$ ) EP-FI  $\mu$  of  $\mathcal{R}_N$ ,  $\mu(a0) = \mu(0)$  for each  $a \in \mathcal{R}_N$ . Also it was indicated that this outcome need not hold for a generalized EP-IFI  $\mu$  of  $\mathcal{R}_N$ . However the method of reasoning in referring to the above outcomes is to take note of that significantly after the procedure of fuzzy generalizatin, at whatever point required we can copy effectively the huge writing of crisp algebra by selecting suitable thresholds.

Davvaz [7] utilized this thought and further generalized the idea of  $(\epsilon, \epsilon \vee q)$  FI to a FI with threshold  $\zeta$  and  $\eta$ . When  $\zeta = 0$  and  $\eta = 1$  we get the ordinary FI given by Abou-Zaid [1] and when  $\zeta = 0$  and  $\eta = 0.5$  we get the  $(\epsilon, \epsilon \vee q)$ -FI characterized by Davvaz [7], Bhakat and Das [4] for rings. Clearly, the essential advantage of the idea of threshold is the decision for threshold which offers ascend to the fuzzy character in the models. This inspires us to utilize the idea of threshold and study the ideas of EP-IFI of a  $\mathcal{R}_N$ .



## 2. Preliminaries

For the definition and preliminaries of  $\mathcal{R}_N$ , fuzzy ideals and IFI of  $\mathcal{R}_N$  etc. in fuzzy case and IF case see [7,8,9,10,11,12,13,15,16].

## 3. Equiprime intuitionistic fuzzy ideals

### 3.1. Definition

An IFI  $A = (\mu_A, \lambda_A)$  of  $\mathcal{R}_N$  is called EP-IFI if  $\forall u, v, b \in \mathcal{R}_N$ ,

$$(i) \zeta \vee \mu_A(b) \vee \mu_A(u - v) \geq \eta \wedge \inf_{r \in N} \mu_A(bru - brv).$$

$$(ii) (1 - \zeta) \wedge \lambda_A(b) \wedge \lambda_A(u - v) \leq (1 - \eta) \vee \sup_{r \in N} \lambda_A(bru - brv).$$

### 3.2. Example

Consider  $Z_{12} = \{\bar{0}, \bar{1}, \bar{2}, \dots, \bar{11}\}$ . Let  $u, v \in (0.2, 0.8)$  with  $u \neq v$ . Define  $\mu_A : Z_{12} \rightarrow [0, 1]$  and  $\lambda_A : z_{12} \rightarrow [0, 1]$  by

$$\mu_A(w) = \begin{cases} 0.9 & \text{if } w = \bar{6} \\ 0.8 & \text{if } w = \bar{0} \\ u & \text{if } w \in \{\bar{3}, \bar{9}\} \\ v & \text{if } w \in \{\bar{2}, \bar{4}, \bar{8}, \bar{10}\} \\ 0.2 & \text{elsewhere} \end{cases}$$

$$\lambda_A(w) = \begin{cases} 0.2 & \text{if } w = \bar{0} \\ 0.1 & \text{if } w = \{\bar{6}\} \\ 1 - u & \text{if } w \in \{\bar{3}, \bar{9}\} \\ 1 - v & \text{if } w \in \{\bar{2}, \bar{4}, \bar{8}, \bar{10}\} \\ 0.7 & \text{if } w = \{\bar{1}, \bar{5}, \bar{7}, \bar{11}\}. \end{cases}$$

Take entry  $\zeta = u \wedge v$  and  $\eta = u \vee v$ . Then  $A = \{\mu_A, \lambda_A\}$  is an EF-IFI  $Z_{12}$ . Note that  $A_{(0.9,0.1)} = \{\bar{6}\}$  is not an ideal of  $Z_{12}$ .

### 3.3. Lemma

Let  $A = (\mu_A, \lambda_A)$  be an EP-IFI  $\mathcal{R}_N$ .

(i) For  $u, v, b \in \mathcal{R}_N$ , if  $\mu_A(bru - brv) \geq \eta$  ( $\mu_A(bru) \geq \eta$ )  $\forall r \in \mathcal{R}_N, \Rightarrow \mu_A(b) \geq \eta$  or  $\mu_A(u - v) \geq \eta$  (resp.  $\mu_A(u) \geq \eta$ ).  
if  $\lambda_A(bru - brv) \leq (1 - \eta)$  (resp.  $\lambda_A(bru) \leq 1 - \eta$ )  $\forall r \in \mathcal{R}_N, \Rightarrow \lambda_A(b) \leq (1 - \eta)$  or  $\lambda_A(u - v) \leq (1 - \eta)$  (resp.  $\lambda_A(u) \leq (1 - \eta)$ )

(ii) Let  $A = (\mu_A, \lambda_A)$  be an ordinary EP-IFI. For  $u, v, b \in \mathcal{R}_N$ , if  $\mu_A(bru - brv) = \mu_A(0) \forall r \in \mathcal{R}_N, \Rightarrow \mu_A(b) = \mu_A(0)$  or  $\mu_A(u) = \mu_A(v)$  and if  $\lambda_A(bru - brv) = \lambda_A(0)$  for all  $r \in \mathcal{R}_N$ , then  $\lambda_A(b) = \lambda_A(0)$  or  $\lambda_A(u) = \lambda_A(v)$ .

(iii) For  $u, v \in \mathcal{R}_N, \mu_A(u0) \geq \eta$  and  $\zeta \vee \mu_A(u - v0) \geq \mu_A(u) \wedge \eta$ . If  $A$  is an ordinary EP-IFI, then  $\mu_A(u0) = \mu_A(0)$  and  $\mu_A(u - v0) = \mu_A(u)$  and if  $\lambda_A(u0) \leq (1 - \eta)$  and  $(1 - \zeta) \wedge \lambda_A(u - v0) \leq \lambda_A(a) \vee (1 - \eta)$ . If  $A$  is an ordinary EP-IFI, then  $\lambda_A(u0) = \lambda_A(0)$  and  $\lambda_A(u - v0) = \lambda_A(u)$ .

(iv) If there have  $u, v \in \mathcal{R}_N \ni u\mathcal{R}_N = v$  then  $\mu_A(v) \geq \eta$  and if  $\lambda_A(v) \leq (1 - \eta)$ .

Proof: We prove (iii) (as (i) and (ii) are evident), contemplate

$$\begin{aligned} \zeta \vee \mu_A(u0) \vee \mu_A(u0 - 0) &\geq \eta \wedge \inf_{r \in \mathcal{R}_N} \mu_A((u0)r(u0) - (u0)r0) = \eta \wedge \mu_A(0) = \eta. \\ (1 - \zeta) \wedge \lambda_A(u0) \wedge \lambda_A(u0 - 0) \\ &\leq (1 - \eta) \vee \sup_{r \in \mathcal{R}_N} \lambda_A((u0)r(u0) - (u0)r0) = (1 - \eta) \vee \lambda_A(0) = 1 - \eta. \end{aligned}$$

This gives  $\mu_A(u0) \geq \eta$ . and  $\lambda_A(u0) \leq (1 - \eta)$ .

Now  $\zeta \vee \mu_A(u - v0) \geq \eta \wedge \mu_A(u) \wedge \mu_A(v0)$  and  $(1 - \zeta) \vee \lambda_A(a - b0) \leq (1 - \eta) \wedge \lambda_A(u) \wedge \lambda_A(v0) = \mu_A(u) \wedge (\eta \wedge \mu_A(v0)) = \mu_A(u) \wedge \eta$ . and  $= \lambda_A(u) \wedge ((1 - \eta) \wedge \lambda_A(v0)) = \lambda_A(u) \wedge (1 - \eta)$ .

This implies  $\zeta \vee \mu_A(u - v0) \geq \mu_A(u) \wedge \eta$ . and  $(1 - \zeta) \vee \lambda_A(u - v0) \leq \lambda_A(u) \wedge (1 - \eta)$ .

Now  $\zeta = 0$  and  $\eta = 1$ . In the other way we assume there has  $u \in \mathcal{R}_N \ni \mu_A(u0) \neq \mu_A(0)$ . Note that  $\mu_A(0) = \mu_A((u0)r(u0) - (u0)r0), \forall r \in \mathcal{R}_N$ . Using (ii), we get  $\mu_A(u0) = \mu_A(0)$ , a conflict.  $\mu_A(u - v0) = \mu_A(u)$  is clear. In the same manner  $\lambda_A(u0) = \lambda_A(u)$  and  $\lambda_A(u - v0) = \lambda_A(u)$ . For (iv), if there have  $u, v \in \mathcal{R}_N \ni u\mathcal{R}_N = v$ . Now, (iii) provide  $\eta \leq \mu_A(u0) = \mu_A(v)$  and  $(1 - \eta) \geq \lambda_A(u0) = \lambda_A(v)$ . By an illustration we exhibit  $\mu_A(u0) = \mu_A(0)$  and  $\lambda_A(0)$  need not hold in general for an EP-IFI  $\mu_A$ .

### 3.4. Theorem

Let  $A = (\mu_A, \lambda_A)$  be an IF sets  $\mathcal{R}_N$ . Then we have two equivalent conditions:

(I)  $A = (\mu_A, \lambda_A)$  is an IFI  $\mathcal{R}_N$ .

(II)  $\forall a, b \in (0, 1]$ , satisfying  $a + b \leq 1$ ,  $U[A : (a, b)]_{\vee q}$  is an ideal of  $\mathcal{R}_N$ .

Proof : Let  $a, b \in (0, 1]$ .

(i) Let  $u, v \in U[A : (a, b)]_{\vee q}$ . We profess  $u - v \in U[A : (a, b)]_{\vee q}$ .

We have  $\zeta \vee \mu_A(u) \geq a$  or  $\zeta \vee \mu_A(u) + a > 2\eta$  and  $\zeta \vee \mu_A(v) \geq a$  or  $\zeta \vee \mu_A(v) + a \geq 2\eta$  and  $(1 - \zeta) \wedge \lambda_A(u) \leq b$  or  $(1 - \zeta) \wedge \lambda_A(u) + a > 2(1 - \eta)$  and  $(1 - \eta) \wedge \lambda_A(v) \leq b$  or  $(1 - \zeta) \wedge \lambda_A(v) + b < 2(1 - \eta)$ .

As  $A$  is an IFI of  $\mathcal{R}_N$ ,  $\zeta \vee \mu_A(u - v) = \zeta \vee \zeta \vee \mu_A(u - v)$

$$\begin{aligned} &\geq \zeta \vee (\eta \wedge \mu_A(u) \wedge \mu_A(v)) \\ &= (\zeta \vee \eta) \wedge (\zeta \vee \mu_A(u)) \wedge (\zeta \vee \mu_A(v)) \\ &= \eta \wedge (\zeta \vee \mu_A(u)) \wedge (\zeta \vee \mu_A(v)). \text{ and} \\ (1 - \zeta) \wedge \lambda_A(u - v) &= (1 - \zeta) \wedge (1 - \zeta) \wedge \lambda_A(u - v) \\ &\leq (1 - \zeta) \wedge ((1 - \eta) \vee \lambda_A(u) \vee \lambda_A(v)) \\ &= ((1 - \zeta) \wedge (1 - \eta)) \vee ((1 - \zeta) \wedge \lambda_A(u)) \vee ((1 - \zeta) \wedge \lambda_A(y)) \\ &= (1 - \eta) \vee ((1 - \zeta) \wedge \lambda_A(u)) \vee ((1 - \zeta) \wedge \lambda_A(v)). \end{aligned}$$

Case 1 (i): If  $\zeta \vee \mu_A(u) \geq a$  and  $\zeta \vee \mu(v) \geq a$ . Consequently  $\zeta \vee \mu(u - v) \geq \eta \wedge a \wedge a = \eta \wedge a$ . If  $\eta \wedge a = a$  so  $u - v \in U[A : (a, b)]_{\vee q}$ . This verification holds if  $a = \eta$ . Hence take for granted  $a \neq \eta$ . If  $\eta \wedge a = \eta$  so  $\zeta \vee \mu(u - v) \geq \eta$  and  $a > \eta$ . Then we obtain  $\zeta \vee \mu(u - v) + a \geq \eta + a > \eta + \eta = 2\eta$ .

Case 1 (ii): Presume  $(1 - \zeta) \wedge \lambda_A(u) \leq b$  and  $(1 - \zeta) \wedge \lambda(v) \leq b$ . As a result  $(1 - \zeta) \vee \lambda(u - v) \leq (1 - \eta) \vee b \vee b = (1 - \eta) \vee b$ . If  $1 - \eta \vee b = b$  then  $u - v \in U[A : (a, b)]_{\vee q}$ . This testament holds if  $b = (1 - \eta)$ . Thus premise  $t \neq 1 - \eta$ . If  $1 - \eta \vee b = 1 - \eta$  then  $1 - \zeta \wedge \lambda(u - v) \leq 1 - \eta$  and  $b < 1 - \eta$ . Then we obtain  $1 - \zeta \wedge \lambda_A(u - v) + b \leq (1 - \eta) + b < (1 - \eta) + (1 - \eta) = 2(1 - \eta)$ . Hence  $u - v \in U[A : (a, b)]_{\vee q}$ .

Case 2 (i): If  $\zeta \vee \mu_A(u) \geq b$  and  $\zeta \vee \mu_A(v) + b \geq 2\eta. \Rightarrow \zeta \vee \mu_A(u - v) \geq \eta \wedge a \wedge (2\eta - a)$ . If  $\eta \wedge a \wedge (2\eta - a) = a$  so  $u - v \in \mu_{a \vee q}$ . This declaration holds if  $a = \eta$ . Hence say  $a \neq \eta$ . If  $\eta \wedge a \wedge (2\eta - a) = \eta$  then  $\zeta \vee \mu_A(u - v) \geq \eta$  and  $a > \eta$ . Then we get  $\zeta \vee \mu_A(u - v) + s \geq \eta + a > \eta + \eta = 2\eta$ . Hence  $u - v \in U[A : (u, v)]_{\vee q}$ . If  $\eta \wedge a \wedge (2\eta - a) = (2\eta - a)$  then  $a > (2\eta - a)$  and  $\zeta \vee \mu_A(u - v) > (2\eta - a)$ . Now  $\zeta \vee \mu_A(u - v) + a > (2\eta - a) + a = 2\eta$ .

Case 2 (ii) : Say  $(1 - \zeta) \wedge \lambda_A(a) \leq b$  and  $(1 - \zeta) \wedge \lambda_A(v) + b \leq 2(1 - \eta)$ . So  $(1 - \zeta) \wedge \lambda_A(u - v) \leq 1 - \eta \vee b \vee (2(1 - \eta) - b)$ . If  $(1 - \eta) \vee b \vee (2(1 - \eta) - b) = b$  then  $u - v \in U$ . This declaration holds if  $b \neq 1 - \eta$ . Hence hypothecate  $b = 1 - \eta$ . If  $1 - \eta \vee b \vee (2(1 - \eta) - b) = 1 - \eta$  then  $(1 - \zeta) \wedge \mu_A(u - v) \leq 1 - \eta$  and  $b < 1 - \eta$ . Then we acquire  $1 - \zeta \wedge \mu_A(u - v) + b \leq 1 - \eta + b < 1 - \eta + 1 - \eta = 2(1 - \eta)$ . Thereupon  $u - v \in \mu_{b \wedge q}$ . If  $1 - \eta \vee b \vee (2(1 - \eta) - b) = (2(1 - \eta) - a)$  then  $b < (2(1 - \eta) - b)$  and  $(1 - \zeta) \wedge \lambda_A(u - v) > (2(1 - \eta) - b)$ . Now  $(1 - \zeta) \wedge \lambda_A(u - v) + b < (2(1 - \eta) - b) + b = 2(1 - \eta)$ . Accordingly  $u - v \in U[A : (a, b)]_{\vee q}$ .

Case 3(i) : If  $\zeta \vee \mu_A(u) + a > 2\eta$  and  $\zeta \vee \mu_A(v) \geq a$ . We omit the testament as it is same to case 2.

Case 3 (ii) : Take  $(1 - \zeta) \wedge \lambda(u) + b < 2(1 - \eta)$  and  $(1 - \zeta) \wedge \lambda_A(v) \leq b$ . We omit the declaration as it is analogous to case 2.

Case 4 (i): If  $\zeta \vee \mu_A(u) + a > 2\eta$  and  $\zeta \vee \mu_A(v) + a > 2\eta$ . Consequently  $\zeta \vee \mu_A(u - v) \geq \eta \wedge (2\eta - a) \wedge (2\eta - a) = \eta \wedge (2\eta - a)$ . If  $\eta \wedge (2\eta - a) = \eta \Rightarrow \zeta \vee \mu_A(u - v) \geq \eta$  and  $(2\eta - a) \geq \eta$ . i.e,  $\zeta \vee \mu_A(u - v) \geq \eta$  and  $\eta \geq a$ . This gives  $\zeta \vee \mu_A(u - v) \geq a$ . Hence  $u - v \in U[A : (a, b)]_{\vee q}$ . This tetament holds if  $a \neq \eta$ . Hence presume  $a \neq \eta$ . If  $\eta \wedge (2\eta - a) = (2\eta - a)$  then  $\zeta \vee \mu_A(u - v) > (2\eta - a)$ . Now  $\zeta \vee \mu_A(a - b) + a > (2\eta - a) + a = 2\eta$ . The following case is analogous to this.

Case 4 (ii) : Say  $(1 - \zeta) \wedge \lambda_A(u) + a < 2(1 - \eta)$  and  $(1 - \zeta) \wedge \lambda_A(v) + a < 2(1 - \eta)$ . Then  $(1 - \zeta) \wedge \lambda_A(u - v) \leq (1 - \eta) \vee (2(1 - \eta) - a) \vee (2(1 - \eta) - a) = (1 - \eta) \vee (2(1 - \eta) - a)$ . If  $(1 - \eta) \vee (2(1 - \eta) - a) = (1 - \eta)$  so  $(1 - \zeta) \wedge \lambda(u - v) \leq (1 - \eta)$  and  $(2(1 - \eta) - a) \leq (1 - \eta)$ . i.e,  $(1 - \zeta) \wedge \lambda_A(u - v) \leq (1 - \eta)$  and  $(1 - \eta) \leq a$ . This confer  $(1 - \zeta) \wedge \lambda_A(u - v) \leq a$ . Hence  $u - v \in \lambda_{a \vee q}$ . Reult holds if  $a = 1 - \eta$ . Thereupon assume  $a < 1 - \eta$ . If  $(1 - \eta) \vee (2(1 - \eta) - a) = (2(1 - \eta) - a)$  then  $(1 - \zeta) \wedge \lambda_A(u - v) < (2(1 - \eta) - a)$ .

Now  $(1 - \zeta) \wedge \lambda_A(u - v) + a < (2(1 - \eta) - a) + a = 2(1 - \eta)$ . Henceforth  $u - v \in U[A : (a, b)]_{\vee q}$ . The following case is the carbon copy this.

(ii) If  $u \in \mu_{a \vee q}, v \in \mathcal{R}_N \Rightarrow v + u - v \in U[A : (a, b)]_{\vee q}$ . and

If  $u \in \lambda_{a \wedge q}, v \in \mathcal{R}_N \Rightarrow v + u - v \in U[A : (a, b)]_{\vee q}$ .

(iii) If  $u \in \mu_{a \vee q}, y \in \mathcal{R}_N$  so  $uv \in U[A : (a, b)]_{\vee q}$ . and

If  $u \in \lambda_{a \wedge q}, v \in \mathcal{R}_N \Rightarrow uv \in U[A : (a, b)]_{\vee q}$ .

(iv) If  $i \in \mu_{a \vee q}, u, v \in \mathcal{R}_N \Rightarrow u(v + i) - uv \in U[A : (a, b)]_{\vee q}$ . and

If  $i \in \lambda_{a \wedge q}, u, v \in \mathcal{R}_N \Rightarrow u(v + i) - uv \in U[A : (a, b)]_{\vee q}$ .

Applying (i)-(iv), the level subset  $U[A : (a, b)]_{\vee q}$  is an ideal of  $\mathcal{R}_N$  and

(II)  $\Rightarrow$ (I); We will prove (i)  $\zeta \vee \mu_A(u - v) \geq \eta \wedge \mu_A(u) \wedge \mu_A(v)$  and  $(1 - \zeta) \wedge \lambda_A(u - v) \leq (1 - \eta) \wedge \lambda_A(u) \vee \lambda_A(v) \forall u, v \in \mathcal{R}_N$ .

If possible say that there has  $u, v \in \mathcal{R}_N \ni \zeta \vee \mu_A(u - v) < \eta \wedge \mu_A(u) \wedge \mu_A(v)$  and  $(1 - \zeta) \wedge \lambda_A(u - v) > (1 - \eta) \vee \lambda_A(u) \vee \lambda_A(v)$ . Select  $a, b \in (\zeta, \eta) \ni \zeta \vee \mu_A(u - v) < a < \eta \wedge \mu_A(u) \wedge \mu_A(v)$ , and  $(1 - \zeta) \wedge \lambda_A(u - v) > b > (1 - \eta) \vee \lambda_A(u) \vee \lambda_A(v)$ .

Note that  $\zeta \vee \mu_A(u - v) < a$  and  $\zeta \vee \mu_A(u - v) + a < a + a < 2\eta$ , and  $(1 - \zeta) \wedge \lambda_A(u - v) > b$  and  $(1 - \zeta) \wedge \lambda_A(u - v) + b > b + b > 2(1 - \eta)$ . This provides  $u - v \notin U[A : (a, b)]_{\vee q}$ . As  $t < \eta \wedge \mu_A(u) \wedge \mu_A(v)$ , we have  $\mu_A(u) > a$  and  $\mu_A(v) > a, \Rightarrow \zeta \vee \mu_A(u - v) > a$ , and  $\zeta \vee \mu_A(v) > a$ , and  $1 - \zeta \wedge \lambda_A(u) < b$ , and  $1 - \zeta \wedge \lambda_A(v) < b$ .

Thus we acquire  $u, v \in U[A : (a, b)]_{\vee q}$  but  $u - v \notin U[A : (a, b)]_{\vee q}$ , and  $U[A : (a, b)]_{\vee q}$ . This is absurd to the fact that  $U[A : (a, b)]_{\vee q}$  is an ideal of  $\mathcal{R}_N$ . Evenly for  $u, v, i \in \mathcal{R}_N$ , we can demonstrate the following:

(ii)  $\zeta \vee \mu_A(v + u - v) \geq \eta \wedge \mu_A(u)$  and  $1 - \zeta \wedge \lambda_A(v + u - v) \leq 1 - \eta \vee \lambda_A(u)$

(iii)  $\zeta \vee \mu_A(uv) \geq \eta \wedge \mu_A(u)$ , and  $1 - \zeta \wedge \lambda_A(uv) \leq 1 - \eta \vee \lambda_A(u)$ ,

(iv)  $\zeta \vee \mu_A(u(v+i) - uv) \geq \eta \wedge \mu_A(i)$ , and  $1 - \zeta \wedge \lambda_A(u(v+i) - uv) \leq 1 - \eta \vee \lambda_A(i)$ .

Applying (i)-(iv),  $A$  is an IFI of  $\mathcal{R}_N$ .

### 3.5. Theorem

Let  $A = (\mu_A, \lambda_A)$  be an IFI of  $\mathcal{R}_N$ . Then  $A$  is an EP-IFI of  $\mathcal{R}_N$  if and only if  $\forall a, b \in (\zeta, \eta]$ , with  $a + b \leq 1$ , the level subset  $U[A; (a, b)]$  is an EP ideal of  $\mathcal{R}_N$ .

Proof :  $A$  is an IFI of  $\mathcal{R}_N$  if and only if  $\forall a, b \in (\zeta, \eta]$ , the level subset  $U[A : (a, b)]$  is an ideal of  $\mathcal{R}_N$ . Let  $A$  be an EP-IFI of  $\mathcal{R}_N$ . Consider  $a, b \in (\zeta, \eta], u, v, n \in \mathcal{R}_N, \exists nru - nrsv \in U[A : (a, b)] \forall r \in \mathcal{R}_N$ . This means  $\mu_A(nru - nrsv) \geq a$  and  $\lambda_A(nru - nrsv) \leq b \forall r \in \mathcal{R}_N$ . Henceforth

$$\inf_{r \in \mathcal{R}_N} \mu_A(nru - nrsv) \geq a, \text{ and } \sup_{r \in \mathcal{R}_N} \lambda_A(nru - nrsv) \leq b.$$

As  $A$  is an EP-IFI of  $\mathcal{R}_N$ , we acquire

$$\zeta \vee \mu_A(n) \vee \mu_A(u - v) \geq \eta \wedge \inf_{r \in \mathcal{R}_N} \mu_A(nru - nrsv) \geq \eta \wedge a = a \text{ and } (1 - \zeta) \wedge \lambda_A(a) \wedge \lambda_A(u - v) \leq (1 - \eta) \vee \sup_{r \in \mathcal{R}_N} \lambda_A(nru - nrsv) \leq (1 - \eta) \vee b = b.$$

Consequently  $n \in U[A : (a, b)]$  or  $u - v \in U[A : (a, b)]$  and  $n \in U[A : (a, b)]$  or  $u - v \in U[A : (a, b)]$ .

Accordingly  $U[A : (a, b)]$  is an EP-IFI of  $\mathcal{R}_N$ .

Conversely, let there have  $n, u, v \in \mathcal{R}_N \ni$

$$\zeta \vee \mu_A(n) \vee \mu_A(u - v) < \eta \wedge \inf_{r \in \mathcal{R}_N} \mu_A(nru - nrsv) \text{ and } (1 - \zeta) \wedge \lambda_A(n) \wedge \lambda_A(u - v) > (1 - \eta) \vee \sup_{r \in \mathcal{R}_N} \lambda_A(nru - nrsv).$$

$$\text{Select } a \text{ and } b \zeta \vee \mu_A(n) \vee \mu_A(u - v) < a < \eta \wedge \inf_{r \in \mathcal{R}_N} \mu_A(nru - nrsv) \text{ and } (1 - \zeta) \wedge \lambda_A(n) \wedge \lambda(u - v) > b > (1 - \eta) \vee \sup_{r \in \mathcal{R}_N} \lambda_A(nru - nrsv).$$

This shows  $\mu_A(n) < a, \mu_A(u - v) < a$  and  $\inf_{r \in \mathcal{R}_N} \mu_A(nru - nrsv) > a$ .

$$\lambda_A(n) > b, \lambda_A(u - v) > b \text{ and } \sup_{r \in \mathcal{R}_N} \lambda(nru - nrsv) < b.$$

Again it gives  $n \notin U[A : (a, b)], u - v \notin U[A : (a, b)]$  and  $nru - nrsv \in U[A : (a, b)], \forall r \in \mathcal{R}_N$ . This is a mismatch to the supposition that  $U[A : (a, b)]$  is an EP-IFI of  $\mathcal{R}_N \forall a, b \in (\zeta, \eta]$  with  $a + b \leq 1$ .

The other results are analogous to this.

### 3.6. Theorem

Let  $A$  be an IFS of  $\mathcal{R}_N$ . Then the following are alike:

(I)  $A = (\mu_A, \lambda_A)$  is an EP-IFI of  $\mathcal{R}_N$ .

(II)  $\forall a \in (0, 1], b \in [0, 1) U[A : (a, b)]_{\vee q}$  is an EP-IFI of  $\mathcal{R}_N$ .

Proof : (I)  $\Rightarrow$  (II) Let  $a \in (0, 1]$ . By Theorem 3.5,  $U[A : (a, b)]_{\vee q}$  is an ideal of  $\mathcal{R}_N$ . Let  $nru - nrsv \in U[A : (a, b)]_{\vee q}$  for all  $r$  in  $\mathcal{R}_N$ . We assert  $n \in U[A : (a, b)]_{\vee q}$  or  $u - v \in U[A : (a, b)]_{\vee q}$ . As  $A = (\mu_A, \lambda_A)$  is an EP-IFI of  $\mathcal{R}_N$ , we have

$$\begin{aligned} \zeta \vee \mu_A(n) \vee \mu_A(u - v) &\geq \eta \wedge \inf_{r \in \mathcal{R}_N} \mu_A(nru - nrsv) \\ \Rightarrow \zeta \vee \zeta \vee \mu_A(n) \vee \mu_A(u - v) &\geq \eta \wedge \left( \zeta \vee \inf_{r \in \mathcal{R}_N} \mu_A(nru - nrsv) \right) \\ \Rightarrow \zeta \vee \mu_A(n) \vee \mu_A(u - v) &\geq \eta \wedge a \wedge (2\eta - a). \end{aligned}$$

$$\begin{aligned}
 (1 - \zeta) \wedge \lambda_A(n) \wedge \lambda_A(u - v) &\leq (1 - \eta) \vee \sup_{r \in \mathcal{R}_N} \lambda_A(nru - nrv) \\
 \Rightarrow (1 - \zeta) \wedge (1 - \zeta) \wedge \lambda_A(n) \wedge \lambda_A(u - v) &\leq (1 - \eta) \vee \left( (1 - \zeta) \wedge \sup_{r \in \mathcal{R}_N} \lambda_A(nru - nrv) \right) \\
 \Rightarrow (1 - \zeta) \wedge \lambda_A(n) \wedge \lambda_A(u - v) &\leq (1 - \eta) \wedge a \vee (2(1 - \eta) - a).
 \end{aligned}$$

Case 1 (i) : If  $\eta \wedge a \wedge (2\eta - a) = a$ . Then  $\zeta \vee \mu_A(n) \geq a$  or  $\zeta \vee \mu_A(u - v) \geq a$ . Wherefore  $n \in U[A : (a, b)]_{\vee q}$  or  $u - v \in U[A : (a, b)]_{\vee q}$ .

The proof exhibited in case 1 holds for  $a = \eta$ . Accordingly say  $a \neq \eta$  in the forthcoming cases.

Case 1 (ii) : If  $(1 - \eta) \vee a \vee (2(1 - \eta) - a) = a$ . So  $(1 - \zeta) \wedge \lambda_A(n) \leq a$  or  $(1 - \zeta) \wedge \lambda_A(u - v) \leq a$ . therefore  $n \in U[A : (a, b)]_{\wedge q}$ , or  $u - v \in U[A : (a, b)]_{\wedge q}$ .

The proof offered in case 1 holds for  $a = (1 - \eta)$ . Hence say  $a < (1 - \eta)$  in the following cases.

Case 2 (i) : Imagine  $\eta \wedge a \wedge (2\eta - a) = \eta$ . So  $a > \eta$  and  $\zeta \vee \mu_A(n) \vee \mu_A(u - v) \geq \eta$ .  $\Rightarrow \zeta \vee \mu_A(a) \geq \eta$  or  $\zeta \vee \mu_A(u - v) \geq \eta$ . If  $\zeta \vee \mu_A(n) \geq \eta$  then  $\zeta \vee \mu_A(n) + a \geq \eta + a > \eta + \eta = 2\eta$ . Consequently  $n \in U[A : (a, b)]_{\vee q}$ , if  $\zeta \vee \mu_A(u - v) \geq \eta$  then  $\zeta \vee \mu_A(u - v) + a \geq \eta + a > \eta + \eta = 2\eta$ . Hence  $u - v \in U[A : (a, b)]_{\vee q}$ .

Case 2 (ii): Let us say  $(1 - \eta) \vee a \vee (2(1 - \eta) - a) = (1 - \eta)$ . Then  $a < (1 - \eta)$  and  $(1 - \zeta) \wedge \lambda_A(n) \wedge \lambda_A(u - v) \leq (1 - \eta)$ .  $\Rightarrow (1 - \zeta) \wedge \lambda_A(n) \leq (1 - \eta)$  or  $(1 - \zeta) \wedge \lambda_A(u - v) \leq (1 - \eta)$ . If  $(1 - \zeta) \wedge \lambda_A(n) \leq (1 - \eta)$  then  $(1 - \zeta) \wedge \lambda_A(n) + a \leq (1 - \eta) + a < (1 - \eta) + (1 - \eta) = 2(1 - \eta)$ . Hence  $n \in U[A : (a, b)]_{\vee q}$ . If  $(1 - \zeta) \wedge \lambda_A(u - v) \leq (1 - \eta)$  then  $(1 - \zeta) \wedge \lambda_A(u - v) + a \leq (1 - \eta) + a < (1 - \eta) + (1 - \eta) = 2(1 - \eta)$ . Hence  $u - v \in U[A : (a, b)]_{\vee q}$ .

Case 3 : If  $\eta \wedge a \wedge (2\eta - a) = (2\eta - a)$ . Then  $(2\eta - a) < \eta$  and  $\zeta \vee \mu_A(n) \vee \mu_A(u - v) \geq (2\eta - a)$  and

$(1 - \eta) \vee a \vee [2(1 - \eta) - a] = 2[(1 - \eta) - a]$ . Then

$[2(1 - \eta) - n] > (1 - \eta)$  and  $(1 - \zeta) \wedge \mu_A(a) \wedge \mu_A(u - v) \leq [2(1 - \eta) - a]$ .

As infimum and supremum are vital in the definition of an EP-IFI, consider the cases

$\zeta \vee \mu_A(n) \vee \mu_A(u - v) = (2\eta - a)$  and  $(1 - \zeta) \wedge \lambda_A(n) \wedge \lambda_A(u - v) = [2(1 - \eta) - a]$ . The following subcases are must.

(i) Take  $\zeta \vee \mu_A(n) \vee \mu_A(u - v) > 2\eta - a$ .

Then  $\zeta \vee \mu_A > (2\eta - a)$  or  $\zeta \vee \mu_A(u - v) > (2\eta - a)$ .

$\Rightarrow \zeta \vee \mu_A(n) + a > (2\eta - a) + a = 2\eta$  or  $\zeta \vee \mu_A(u - v) + a > (2\eta - a) + a = 2\eta$ .

And consider  $(1 - \zeta) \wedge \lambda_A(n) \wedge \lambda_A(u - v) < [2(1 - \eta) - a]$ .

So  $(1 - \zeta) \wedge \lambda_A < [2(1 - \eta) - a]$  or  $(1 - \zeta) \wedge \lambda_A(u - v) < [2(1 - \eta) - a]$ .

$\Rightarrow (1 - \zeta) \wedge \lambda_A(n) + a < [2(1 - \eta) - a] + a = [2(1 - \eta)]$  or  $(1 - \zeta) \wedge \lambda_A(u - v) + t < [2(1 - \eta) - a] + a = [2(1 - \eta)]$ .

As a consequence  $n \in U[A : a, b]$  or  $u - v \in U[A : a, b]$ .

(ii) Now take  $\zeta \vee \mu_A(n) \vee \mu_A(u - v) > (2\eta - a)$ .  $\Rightarrow \zeta \vee \mu_A(n) = (2\eta - a)$  or  $\zeta \vee \mu_A(u - v) = (2\eta - a)$ . If  $\eta < a$  then our presupposition  $\eta \wedge a \wedge (2\eta - a) = (2\eta - a)$  gives  $\eta \wedge (2\eta - a) = 2\eta - a$ .  $\Rightarrow \eta \geq 2\eta - a$ . This gives the absurd result  $a \geq \eta$ .

Hence  $\eta > a$ , and  $\zeta \vee \mu_A(n) = (2\eta - a) > a$  or  $\zeta \vee \mu_A(u - v) = (2\eta - a) > a$ .

And so  $(1 - \zeta) \wedge \lambda_A(n) \wedge \lambda_A(u - v) = [2(1 - \eta) - a]$ .

This means  $(1 - \zeta) \wedge \lambda_A(n) = [2(1 - \eta) - a]$  or  $(1 - \zeta) \wedge \lambda_A(u - v) = [2(1 - \eta) - a]$ .

If  $1 - \eta > a$  then our presumption

$(1 - \eta) \vee a \vee [2(1 - \eta) - a] = [2(1 - \eta) - a]$

gives  $(1 - \eta) \vee [2(1 - \eta) - a] = [2(1 - \eta) - a]$

Implied  $(1 - \eta) \leq [2(1 - \eta) - a]$ .

Provides contradiction  $a \leq 1 - \eta$

Hence  $1 - \eta < a$ . So we have

$1 - \zeta \wedge \lambda_A(n) = [2(1 - \eta) - a] < a$  or  $(1 - \zeta) \wedge \lambda_A(u - v) = [2(1 - \eta) - a] < a$   
Henceforth  $n \in U[A : a, b]$  or  $u - v \in U[A : a, b]$ .

Eventually  $U[A : a, b]$  is an EP ideal of  $\mathcal{R}_N$ .

(II)  $\Rightarrow$  (I):  $A$  is an IFI of  $\mathcal{R}_N$ . If possible say that there have  $n, u, v \in \mathcal{R}_N \ni$   
 $\zeta \vee \mu_A(n) \vee \mu_A(u - v) < \eta \wedge \inf_{r \in \mathcal{R}_N} \mu_A(nru - nrv)$ . and  
 $(1 - \zeta) \wedge \lambda_A(n) \wedge \lambda_A(u - v) > (1 - \eta) \vee \sup_{r \in \mathcal{R}_N} \lambda_A(nru - nrv)$ .

Select  $a, b \in (\zeta, \eta) \ni$

$\zeta \vee \mu_A(n) \vee \mu_A(u - v) < a < \eta \wedge \inf_{r \in \mathcal{R}_N} \mu_A(nru - nrv)$ .

$\Rightarrow \zeta \vee \mu_A(n) < a, \zeta \vee \mu_A(u - v) < a,$

$\zeta \vee \mu_A(n) + a < a + a = 2a < 2\eta$  and

$\zeta \vee \mu_A(u - v) + a < a + a = 2a < 2\eta.$

And,  $(1 - \zeta) \wedge \lambda_A(n) \wedge \lambda_A(u - v) > b > (1 - \eta) \vee \sup_{r \in \mathcal{R}_N} \lambda_A(nru - nrv)$ .

$\Rightarrow (1 - \zeta) \wedge \lambda_A(n) > b, (1 - \zeta) \wedge \lambda_A(u - v) > b,$

$(1 - \zeta) \wedge \lambda_A(n) + b > b + b = 2b > 2(1 - \eta)$  and

$(1 - \zeta) \wedge \lambda_A(u - v) + b > b + b = 2b > 2(1 - \eta).$

So we get  $n \notin U[A : (a, b)], u - v \notin U[A : (a, b)].$

Also,  $a < \eta \wedge \inf_{r \in \mathcal{R}_N} \mu_A(nru - nrv)$

$\Rightarrow \mu_A(nru - nrv) > a$  for all  $r \in \mathcal{R}_N$

$\Rightarrow \zeta \vee \mu_A(nru - nrv) > a \forall r \in \mathcal{R}_N$

and,  $b > (1 - \eta) \vee \sup_{r \in \mathcal{R}_N} \lambda_A(nru - nrv)$

$\Rightarrow \lambda_A(nru - nrv) < b \forall r \in \mathcal{R}_N$

$\Rightarrow (1 - \zeta) \wedge \lambda_A(nru - nrv) < b \forall r \in \mathcal{R}_N$

$\Rightarrow nr u - nr v \in U[A : (a, b)] \forall r \in \mathcal{R}_N.$

This is a violation to the supposition that  $U[A : (a, b)]$  is an EP-IFI of  $\mathcal{R}_N$  for every  $a, b \in (0, 1]$  with  $a + b \leq 1$ . This accomplishes the proof.

#### 4. Conclusion

In this article, EP-IFI of a nearring has been expounded.

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