The effect of rare regions on a disordered itinerant quantum antiferromagnet with cubic anisotropy

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We study the quantum phase transition of an itinerant antiferromagnet with cubic anisotropy in the presence of quenched disorder, paying particular attention to the locally ordered spatial regions that form in the Griffiths region. We derive an effective action where these rare regions are described in terms of static annealed disorder. A one loop renormalization group analysis of the effective action shows that for order parameter dimensions $p < 4$ the rare regions destroy the conventional critical behavior. For order parameter dimensions $p > 4$ the critical behavior is not influenced by the rare regions, it is described by the conventional dirty cubic fixed point. We also discuss the influence of the rare regions on the fluctuation-driven first-order transition in this system.

I. INTRODUCTION

Quenched disorder can have very drastic influences on the critical behavior of a system undergoing a continuous phase transition. According to the Harris criterion[1] the critical behavior of a clean system is unaltered by disorder, if the correlation length critical exponent ν obeys the inequality $\nu > 2/d$, where d is the spatial dimensionality of the system. In the opposite case, $\nu < 2/d$, the clean critical behavior is unstable, and the disorder either leads to a new, different universality class, or to an unconventional critical point, or even to the destruction of the phase transition.

Another, less well understood consequence of quenched disorder is the formation of rare locally ordered regions in the disordered phase. For a transition occuring at a finite temperature, this can be explained in the following way. In general, disorder leads to the suppression of the critical temperature from its clean value T_c^0 to T_c. In the temperature region between T_c^0 and T_c the system does not show long-range order. However, there will be arbitrarily large regions which are devoid of impurities and thus order locally. The probability of finding such regions usually decreases exponentially with their size, they represent non-perturbative degrees of freedom. These locally ordered regions are known as rare regions, and the order parameter fluctuations induced by them as local moments or instantons. Griffiths[2] showed that the rare regions lead to a non-analytic free energy everywhere in the temperature region between T_c^0 and T_c, now called the Griffiths region or Griffiths phase. In generic classical systems this is a very weak effect, and the non-analyticity in the free energy is only an essential one. However, the Griffiths singularities become stronger if the disorder is spatially correlated. McCoy and Wu[3] studied a two-dimensional Ising model where the disorder is perfectly correlated in one spatial direction and uncorrelated in the other. In this model the rare regions lead to the divergence of the susceptibility at some temperature T_χ within the Griffiths region.

A very interesting question is what is the influence of the rare regions on the critical behavior of a system. Dotsenko et al.[4] studied this question for a weakly disordered classical ferromagnet. They found that the conventional theory of critical behavior[5] in this system is unstable with respect to replica symmetry breaking. They also showed that the rare regions actually induce replica symmetry breaking perturbations and thus destabilize the conventional critical fixed point. While so far no final conclusion about the fate of the transition in the weakly disordered ferromagnet could be reached, the occurrence of replica symmetry breaking raises the possibility of an unconventional transition with activated scaling, as is believed to occur in the random field Ising model[6].

For quantum phase transitions[7] which occur at zero temperature as a function of some non-thermal control parameter, one expects an even stronger influence of the rare regions than for classical transitions. The reason is that a quantum model with uncorrelated quenched disorder is effectively equivalent to a classical model with the disorder being perfectly correlated in one dimension (the imaginary time dimension). Fisher[8] investigated the critical behavior of a one-dimensional quantum Ising spin chain in a transverse field which is equivalent to the classical McCoy-Wu model. He found that due to the rare regions the critical behavior is of the activated form. This has been confirmed by numerical simulations[9] which also suggest[10] that this sort of behavior may not be restricted to one-dimensional systems.

In two recent papers[11] we have considered the effect of rare regions on quantum phase transitions of itinerant electrons in $d > 1$. We have developed a systematic approach, representing the local moments by inhomogeneous saddle point solutions of the field theory. The interaction between the local moments and the fluctuations leads to a new term in the effective action which is of the form of annealed static disorder. In the case of the quantum antiferromagnetic transition this new term results in the destruction of the conventional critical fixed point if the number p of order parameter components is smaller than 4. No new fixed point could be identified, the system displays runaway flow to large disorder strength. On the other hand, for the quantum ferromagnetic transition the rare regions do not affect the critical behavior since a self-induced long-range interaction suppresses all fluctuations including those produced by the local moments.

In this paper we apply the approach developed in Ref. 11 to a model of an itinerant antiferromagnet with an additional interaction term with cubic symmetry. This model is equivalent to a weakly disordered classical ferromagnet with cubic anisotropy in which the disorder is perfectly correlated in some of the spatial dimensions but uncorrelated in the remaining dimensions. The conventional theory for this model (without taking rare regions into account) has been developed by Yamazaki, Holz, Ochiai and Fukuda[12].

The purpose for this work is threefold. We want investigate (i) whether the conventional critical fixed point is stable under the influence of the rare regions. If it is unstable we want to find out (ii) whether a new stable fixed fixed point exists which describes a rare region driven transition. Finally we want to study (iii) the influence of the rare regions on the fluctuation-driven first-order transition occurring in our system. The layout of the paper is as follows. In Sec. II we derive the effective field theory by taking into account the disorder induced rare regions. In Sec. III, we carry out the renormalization group analysis. Finally, Sec. IV is left for a summary of our results.

II. AN EFFECTIVE ACTION FOR DISORDERED ANTIFERROMAGNETS WITH CUBIC ANISOTROPY

A. The model

In 1976 Hertz[13] derived an order parameter field theory for the description of the antiferromagnetic quantum phase transition of itinerant electrons. Later this model was generalized to the dirty case by making the distance from the critical point a random function of position[11,14]. Here we consider an extension of this order parameter field theory by incorporating an additional ϕ^4 term which possesses a (hyper-)cubic symmetry.

In terms of the p-component order parameter field $\boldsymbol{\phi}$ (with components ϕ_i) the total action can be written as

$$S[\boldsymbol{\phi}] = S_{\rm G}[\boldsymbol{\phi}] + S_{\rm int}[\boldsymbol{\phi}] + S_{\rm cubic}[\boldsymbol{\phi}] \quad , \qquad (2.1a)$$

with the Gaussian part, the interaction part and the cubic anisotropic part given by

$$S_{\rm G}[\boldsymbol{\phi}] = \frac{1}{2}\int dx\, dy \;\sum_i \phi_i(x)\,\Gamma(x-y)\,\phi_i(y) \quad , \qquad (2.1b)$$

$$S_{\rm int}[\boldsymbol{\phi}] = u\int dx\, \sum_{i,j} \phi_i(x)\,\phi_i(x)\,\phi_j(x)\,\phi_j(x) \quad , \qquad (2.1c)$$

$$S_{\rm cubic}[\boldsymbol{\phi}] = \lambda \int dx\, \sum_i {\phi_i}^4(x) \quad . \qquad (2.1d)$$

Here we use a 4-vector notation to combine the real space coordinate $\mathbf{x}$ and imaginary time τ, $x=(\mathbf{x},\tau)$, $\int dx = \int d\mathbf{x} \int_0^{1/T} d\tau$. The bare two point function,

$$\Gamma(\mathbf{x}-\mathbf{y},\tau-\tau') = \Gamma_0(\mathbf{x}-\mathbf{y},\tau-\tau') + \delta(\mathbf{x}-\mathbf{y})\,\delta(\tau-\tau')\,\delta t(\mathbf{x}) \quad , \qquad (2.2)$$

consists of the deterministic part derived by Hertz[13] whose Fourier transform reads

$$\Gamma_0(\mathbf{q},\omega_n) = t_0 + \mathbf{q}^2 + |\omega_n| \quad , \qquad (2.3)$$

and a disorder part in the form of a "random mass" term. Here $\mathbf{q}$ is the wave vector, ω_n is a bosonic Matsubara frequency and $\delta t(\mathbf{x})$ is a random function of position and is endowed with the following statistical properties:

$$\langle \delta t(\mathbf{x})\rangle = 0 \quad , \qquad (2.4a)$$

$$\langle \delta t(\mathbf{x})\,\delta t(\mathbf{y})\rangle = \Delta\,\delta(\mathbf{x}-\mathbf{y}) \quad . \qquad (2.4b)$$

B. Inhomogeneous saddle points and annealed disorder

In the conventional approach to critical behavior in systems with quenched disorder[5] the disorder average is carried out at the beginning of the calculation by means of the replica trick[15]. A subsequent perturbative analysis of the resulting, spatially homogeneous effective theory misses the rare regions we are interested in since they are non-perturbative degrees of freedom.

We therefore follow the approach developed in Ref. 11, and work with a particular realization of the disorder rather than integrating it out. Let us consider spatially inhomogeneous, but time-independent saddle point solutions of the action (2.1) (time-dependent saddle-point solutions – if any – will always have a higher free energy since the disorder is static). Depending on the sign of the cubic interaction term the structure of the saddle points in the p-dimensional order parameter space will be different. When $\lambda > 0$ the free energy is minimized by saddle point solutions that lie on the diagonals of a p-dimensional hypercube, while when $\lambda < 0$ the free energy is minimized by solutions that lie on the axis of the hypercube. In either case the modulus $\phi_{\rm sp}$ of these minimizing saddle point solutions fulfills the equation

$$\left(t_0 + \delta t(\mathbf{x}) - \partial_{\mathbf{x}}^{\mathbf{2}}\right) \, |\boldsymbol{\phi}_{\rm sp}(\mathbf{x})| + 4\, u_{\rm eff}\, |\boldsymbol{\phi}_{\rm sp}(\mathbf{x})|^3 = 0 \quad , \tag{2.5a}$$

$$u_{\rm eff} = \begin{cases} u + \frac{\lambda}{p} & \text{for } \lambda > 0 \\ u + \lambda & \text{for } \lambda < 0 \end{cases} \quad . \tag{2.5b}$$

Although $\phi_{\rm sp}(\mathbf{x}) = 0$ is always a solution, there will be spatially inhomogeneous solutions if $\delta t(\mathbf{x})$ has sufficiently deep and wide troughs[11]. Let us now consider the Griffiths region, i.e. the region where the average distance t_0 from the critical point is positive but where there are isolated islands which support a non-zero $\phi_{\rm sp}$. If we have N such islands which are sufficiently apart from each other the global saddle point solutions may be written as

$$\boldsymbol{\phi}_{\rm sp}^{\{\sigma_I\}}(\mathbf{x}) \equiv \Phi^{\{\sigma_I\}}(\mathbf{x}) = \sum_{I=1}^{N} \psi_I(\mathbf{x})\, \sigma_I \tag{2.6}$$

where $\psi_I(\mathbf{x})$ is a solution of (2.5) on the island I and σ_I is a unit vector in spin space (on one of the axis for $\lambda < 0$ or on one of the diagonals for $\lambda > 0$).

Since the direction of the order parameter on each of the N islands can be chosen independently, (2.6) describes an exponentially large number of degenerate saddle points, $(2p)^N$ for $\lambda < 0$ and $(2^p)^N$ for $\lambda > 0$. To be precise, the saddle points are not exactly degenerate due to the residual interaction of the (exponentially small) tails of the order parameter between the islands. The complicated structure of the free energy landscape connected with the existence of an exponentially large number of almost degenerate saddle points will finally turn out to be responsible for the failure of the conventional approach.

We now consider fluctuations around the saddle points (2.6). Since the saddle points are separated by large free energy barriers an expansion around one of them will not give a good representation of the partition function of the entire system. Instead we will restrict ourselves to small fluctuations and simply add the contributions coming from *all* of the saddle points. Thus the partition function for a particular realization $\delta t(\mathbf{x})$ of the disorder can be written as

$$Z[\delta t(\mathbf{x})] \approx \sum_{\{\sigma_I\}} \int_< D[\varphi(x)]\; e^{-S[\Phi^{\{\sigma_I\}}(\mathbf{x}) + \boldsymbol{\varphi}(x), \delta t(\mathbf{x})]} \quad . \tag{2.7}$$

Here $\int_<$ indicates that the integration is restricted to small fluctuations $\boldsymbol{\varphi}$ only.

We now carry out the sum over the saddle point configurations. The residual interaction between the islands will lead to slight deviations of the saddle point function from the ideal one given in (2.6). This is taken into account by replacing the sum over the saddle points by an integral over a probability distribution

$$P[\Phi] \sim e^{-\frac{1}{T}\int dx \mathcal{L}^{\rm sp}(\Phi)} \quad . \tag{2.8}$$

The temperature factor in the exponent reflects the fact that the saddle points are classical (static) degrees of freedom[17]. Expanding in powers of the fluctuations, we obtain the following effective action for the fluctuations $\boldsymbol{\varphi}$ (still for a particular disorder realization)

$$\begin{aligned} S_{\rm eff} - S^{\rm SP} &= S_{\rm G}[\boldsymbol{\varphi}] + S_{\rm int}[\boldsymbol{\varphi}] + S_{\rm cubic}[\boldsymbol{\varphi}] \\ &\quad + T\bar{w} \int dx dy\, C(x,y) \sum_{i,j} \varphi_i^2(x) \varphi_j^2(y) \\ &\quad + \text{higher order terms} \,. \end{aligned} \tag{2.9}$$

The correlation function $C(x, y)$ measures, up to a constant factor determined by the precise form of $\mathcal{L}$, whether $\mathbf{x}$ and $\mathbf{y}$ belong to the same island, and $\bar{w} = [(2+4/p)u + 6\lambda/p]$ is a positive constant. The $\bar{w}$ term is produced by the interaction of the fluctuations with the rare regions. It is our approximation of the effect of these non-perturbative degrees of freedom. Terms of higher than fourth order in $\boldsymbol{\varphi}$ also arise, but they are renormalization group irrelevant at both the Gaussian and the nontrivial fixed points of the conventional theory (see below).

Having identified the effects of the rare regions we now use the replica trick[15] to perform the quenched disorder average over $\delta t(\mathbf{x})$ which implies an average over position and size of the rare regions. The resulting effective action reads

$$\begin{aligned} & S_{\rm \;eff}[\boldsymbol{\varphi}^\alpha(x)] = \\ & = \frac{1}{2} \sum_\alpha \sum_i \int dx\, dy\, \Gamma_0(x-y)\, \boldsymbol{\varphi}_i^\alpha(x) \boldsymbol{\varphi}_i^\alpha(y) \\ & + u \sum_\alpha \sum_{i,j} \int d\mathbf{x}\, d\tau\, \left(\boldsymbol{\varphi}_i^\alpha(\mathbf{x},\tau)\right)^2 \left(\boldsymbol{\varphi}_j^\alpha(\mathbf{x},\tau)\right)^2 \\ & + \lambda \sum_\alpha \sum_i \int d\mathbf{x}\, d\tau\, \left(\boldsymbol{\varphi}_i^\alpha(\mathbf{x},\tau)\right)^4 \\ & - \Delta \sum_{\alpha,\beta} \sum_{i,j} \int d\mathbf{x}\, d\tau d\tau'\, \left(\boldsymbol{\varphi}_i^\alpha(\mathbf{x},\tau)\right)^2 \left(\boldsymbol{\varphi}_j^\beta(\mathbf{x},\tau')\right)^2 \\ & - T\bar{w} \sum_{\alpha,\beta} \sum_{i,j} \int d\mathbf{x}\, d\tau d\tau'\, \left(\boldsymbol{\varphi}_i^\alpha(\mathbf{x},\tau)\right)^2 \left(\boldsymbol{\varphi}_j^\alpha(\mathbf{x},\tau')\right)^2 \end{aligned} \tag{2.10}$$

Here the first four terms are identical to the result of the conventional treatment. The 5th term has the form of static, annealed disorder and represents the interaction of the fluctuations with the rare regions in the Griffiths phase. For more details of this derivation see Ref. 11.

III. RENORMALIZATION GROUP ANALYSIS

A. Flow equations

We first consider the effective action (2.10) at tree level. As usual, let us define the scale dimension of a length L to be $[L] = -1$, and that of imaginary time τ to be $[\tau] = -z$ with z being the dynamical critical exponent. We first analyze the Gaussian fixed point. From the Gaussian part of the action (2.10) we see that ω_n scales as q^2, implying that $z = 2$. The scale dimension of the field is $[\boldsymbol{\varphi}] = d/2$. Power counting for the interaction and disorder terms of the action gives the scale dimensions of u, λ, Δ and $\bar{w}$ as $[u] = [\lambda] = [\bar{w}] = 2 - d$ and $[\Delta] = 4 - d$. Here we have used the fact that in Matsubara formalism the temperature scales like a frequency, $[T] = z$. Consequently, u, λ and $\bar{w}$ are irrelevant for $d > 2$, while Δ is irrelevant only for $d > 4$. This implies that in the physical dimension $d = 3$ the Gaussian fixed point is unstable, and we must carry out a loop expansion of the effective action (2.10) close to $d = 4$. All terms of higher order in $\boldsymbol{\varphi}$ that arise in addition to those given in (2.10) have negative scale dimensions at and close to $d = 4$. Thus, they are irrelevant by power counting with respect to both the Gaussian and the conventional non-trivial fixed points.

As in the conventional theory[12,14,16] we carry out the perturbation theory in $d = 4 - \epsilon$ spatial dimensions and ϵ_τ time dimensions. In this way the perturbation expansion becomes a double expansion in terms of ϵ and ϵ_τ. The renormalization group flow equations are obtained by performing a frequency momentum shell RG procedure.[13] To one-loop order, we obtain the following flow equations,

$$\frac{du}{dl} = \tilde{\epsilon}u - 4(p+8)u^2 + 48u\Delta - 24u\lambda, \tag{3.1a}$$

$$\frac{d\lambda}{dl} = \tilde{\epsilon}\lambda - 36\lambda^2 + 48\lambda\Delta - 48u\lambda, \tag{3.1b}$$

$$\frac{d\Delta}{dl} = \epsilon\Delta + 32\Delta^2 - 8(p+2)u\Delta + 8p\Delta\bar{w} - 24\Delta\lambda, \tag{3.1c}$$

$$\frac{d\bar{w}}{dl} = \tilde{\epsilon}\bar{w} + 4p\bar{w}^2 - 8(p+2)u\bar{w} + 48\Delta\bar{w} - 24\lambda\bar{w}. \tag{3.1d}$$

Here we have defined $\tilde{\epsilon} = \epsilon - 2\epsilon_\tau$. Of course, also the distance t from the critical point will be renormalized. However, we only consider the flow on the critical surface $t = 0$ since we are interested in the stability of the critical fixed points. Note that the coefficient of the rare region term $\bar{w}$ only couples to Δ. The flow of u and λ is only indirectly influenced by the rare regions (via a modification of the flow of Δ). This will be important later on.

B. Fixed points and their stability

The flow equations (3.1) possess sixteen fixed points. Their fixed point values are given in Table I, the eigenvalues of the corresponding linearized renormalization group transformations are listed in Table II. For eight of the sixteen fixed points (Nos. 1–8 in Table I) the fixed point value of the rare region term is $\bar{w}^* = 0$. These fixed points have already been studied in Ref. 12 using the conventional approach. In the following, we concentrate on the case $\epsilon > 0$ and $\tilde{\epsilon} = \epsilon - 2\epsilon_\tau < 0$ relevant for the itinerant quantum antiferromagnet.

We first consider the dirty Heisenberg fixed point (No. 6) and the dirty cubic fixed point (No. 8). These are the stable fixed points of the conventional theory for the cases of $p < 4$ and $p > 4$, respectively. Analyzing the stability matrix for the dirty Heisenberg fixed point shows that it is unstable since the eigenvalue e_4 is positive for $p < 4$. In contrast, the dirty cubic fixed point remains stable for $p > 4$ since all eigenvalues of the stability matrix are negative. Thus we conclude that the rare regions destroy the conventional dirty Heisenberg critical behavior for $p < 4$ while they do not influence the conventional dirty cubic critical behavior for $p > 4$.

We now turn to the new fixed points with $\bar{w}^* \neq 0$ (Nos. 9 – 16 in Table I). Fixed points 9, 11, 13 and 15 are unphysical because their fixed point values $\bar{w}^*$ are negative. Since the bare $\bar{w}$ is positive and according to eq. (3.1d) the flow cannot cross the $(\bar{w} = 0)$-plane these fixed points can never be reached. Depending on the number p of order parameter components the remaining fixed points (Nos. 10, 12, 14, and 16) are either also unphysical, or they are unstable. Consequently, for $p < 4$ and to one-loop order there is no stable fixed point. Renormalization group trajectories which in the conventional theory would go to the dirty Heisenberg fixed point show runaway flow to large disorder strength. This runaway flow could either indicate a unconventional phase transition, e.g. an infinite disorder critical point as in the one-dimensional random Ising model[8] or a percolative rather than a homogeneous transition or even a destruction of the phase transition. Within the present approach we cannot be decide between these alternatives.

The influence of the rare regions on the stability of the fixed points in our model is similar to that in the isotropic case[11]. For $p < 4$ the conventional fixed point is destroyed in both models. For $p > 4$ the conventional fixed point is stable. In our model this is the dirty cubic fixed point while in the isotropic case this stable fixed point is the dirty Heisenberg fixed point.

C. The fluctuation-driven first-order transition

In addition to the continuous phase transitions associated with the critical points discussed above there is also the possibility for a first-order transition in the model

No.	FP values			
	u^*	λ^*	Δ^*	$\bar{w}^*$
1	0	0	0	0
2	$\tilde{\epsilon}/4(p+8)$	0	0	0
3	0	$\tilde{\epsilon}/36$	0	0
4	$\tilde{\epsilon}/12p$	$\tilde{\epsilon}(p-4)/36p$	0	0
5	0	0	$-\epsilon/32$	0
6	$(3\epsilon-2\tilde{\epsilon})/16(p-1)$	0	$[(p+8)\epsilon-2(p+2)\tilde{\epsilon}]/64(p-1)$	0
7	0	$\mathcal{O}(\epsilon^{1/2})$	$\mathcal{O}(\epsilon^{1/2})$	0
8	$(3\epsilon-2\tilde{\epsilon})/24(p-2)$	$[(3\epsilon-2\tilde{\epsilon})(p-4)]/72(p-2)$	$[3p\epsilon-4(p-1)\tilde{\epsilon}]/96(p-2)$	0
9	0	0	0	$-\tilde{\epsilon}/4p$
10	$\tilde{\epsilon}/4(p+8)$	0	0	$[(p-4)\tilde{\epsilon}]/4p(p+8)$
11	0	$\tilde{\epsilon}/36$	0	$-\tilde{\epsilon}/12p$
12	$\tilde{\epsilon}/12p$	$[(p-4)\tilde{\epsilon}]/36p$	0	$(p-4)\tilde{\epsilon}/12p^2$
13	0	0	$(\epsilon-2\tilde{\epsilon})/64$	$(2\tilde{\epsilon}-3\epsilon)/16p$
14	$(3\epsilon-2\tilde{\epsilon})/8(10-p)$	0	$[(p+8)\epsilon-12\tilde{\epsilon}]/32(10-p)$	$[(3\epsilon-2\tilde{\epsilon})(p-4)]/8p(10-p)$
15	0	$(3\epsilon-2\tilde{\epsilon})/72$	$(9\epsilon-12\tilde{\epsilon})/288$	$-3(3\epsilon-2\tilde{\epsilon})/72p$
16	$(3\epsilon-2\tilde{\epsilon})/48$	$(3\epsilon-2\tilde{\epsilon})(p-4)/144$	$(3p\epsilon-2(p+2)\tilde{\epsilon})/192$	$(3\epsilon-2\tilde{\epsilon})(p-4)/48p$

TABLE I. Fixed points of the flow equations, p is the number of order parameter components.

considered here. Let us first discuss the mechanism for a clean system and discuss the effects of disorder and rare regions later.

According to a mean-field stability analysis of the effective action (2.10) with $\Delta = \bar{w} = 0$ the inequalities $u + \lambda > 0$ (for $u > 0$) and $u + \lambda/p > 0$ (for $u < 0$) have to be fulfilled for the theory to be stable. Now consider a bare theory with $u < 0, \lambda > 0$ or $u > 0, \lambda < 0$ but still fulfilling the above stability conditions. In these cases the flow equations (3.1) can lead the renormalization group trajectories to the mean-field unstable region. This indicates a fluctuation-driven first-order transition[18,19]. It was later shown[20,21] that the fluctuation-driven first-order in this model survives the presence of quenched disorder, at least within the conventional theory. Let us now consider the influence of the rare regions. As already mentioned, the rare region coefficient $\bar{w}$ does not couple into the flow equations for u and λ but only into the flow equation for Δ. Thus a renormalization group trajectory going to the mean-field unstable region within the conventional theory will generically also do so in the presence of rare regions, the only modification being a different disorder value at the stability boundary.

Therefore, we conclude that the fluctuation-driven first-order transition also occurs when taking the rare regions into account. However, since the rare regions modify the flow of the disorder strength Δ, the boundaries of the first-order region may change.

IV. SUMMARY AND CONCLUSIONS

We have investigated the influence of rare regions on the quantum phase transition of a disordered itinerant antiferromagnet with cubic anisotropy. The local magnetic moments forming on the rare regions in the Griffiths phase generate a new term in the order parameter field theory which has the form of static annealed disorder. We have found that for order parameter dimension $p > 4$ this new term does not change the critical behavior, which is characterized by the dirty cubic fixed point. In contrast, for $p < 4$ the rare region term renders the conventional critical fixed point unstable. The renormalization group trajectories show runaway flow to large disorder. Within our approach which is essentially perturbative, even though it includes some non-perturbative degrees of freedom (the local moments) we cannot determine the ultimate fate of the transition. It could be an unconventional phase transition, e.g. an infinite disorder critical point or a percolative rather than a homogeneous transition or even the destruction of the phase transition. We have also found that the fluctuation-driven first-order transition occurring in this model remains qualitatively unchanged by the rare regions, while the precise position of the first-order region in parameter space will change.

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No.	eigenvalues			
	e_1	e_2	e_3	e_4
1	$\tilde{\epsilon}$	$\tilde{\epsilon}$	ϵ	$\tilde{\epsilon}$
2	$-\tilde{\epsilon}$	$(p-4)\tilde{\epsilon}/(p+8)$	$\epsilon-2(p+2)\tilde{\epsilon}/(p+8)$	$-(p-4)\tilde{\epsilon}/(p+8)$
3	$\tilde{\epsilon}/3$	$-\tilde{\epsilon}$	$\epsilon-2\tilde{\epsilon}/3$	$\tilde{\epsilon}/3$
4	$-\tilde{\epsilon}$	$-\tilde{\epsilon}(p-4)/3p$	$\epsilon-4\tilde{\epsilon}(p-1)/3p$	$-\tilde{\epsilon}(p-4)/3p$
5	eigenvalues not calculated since FP is unphysical			
6	$\frac{-A+\sqrt{A^2-B}}{p-1}$	$\frac{-A-\sqrt{A^2-B}}{p-1}$	$(p-4)(3\epsilon-2\tilde{\epsilon})/4(p-1)$	$-(p-4)(3\epsilon-2\tilde{\epsilon})/4(p-1)$
7	$\mathcal{O}(\epsilon^{1/2})$	$\mathcal{O}(\epsilon^{1/2})$	$\mathcal{O}(\epsilon^{1/2})$	$\mathcal{O}(\epsilon^{1/2})$
8	$\frac{-E+\sqrt{E^2-F}}{12(p-2)}$	$\frac{-E-\sqrt{E^2-F}}{12(p-2)}$	$-(3\epsilon-2\tilde{\epsilon})(p-4)/6(p-2)$	$-(3\epsilon-2\tilde{\epsilon})(p-4)/6(p-2)$
9	eigenvalues not calculated since FP is unphysical			
10	$-\tilde{\epsilon}$	$(p-4)\tilde{\epsilon}/(p+8)$	$\epsilon-12\tilde{\epsilon}/(p+8)$	$(p-4)\tilde{\epsilon}/(p+8)$
11	eigenvalues not calculated since FP is unphysical			
12	$-\tilde{\epsilon}$	$-\tilde{\epsilon}(p-4)/3p$	$\epsilon-2\tilde{\epsilon}(p+2)/3p$	$\tilde{\epsilon}(p-4)/3p$
13	eigenvalues not calculated since FP is unphysical			
14	$\frac{-C+\sqrt{C^2-D}}{4\,(10-p)}$	$\frac{-C-\sqrt{C^2-D}}{4\,(10-p)}$	$(p-4)(3\epsilon-2\tilde{\epsilon})/2(10-p)$	$(p-4)(3\epsilon-2\tilde{\epsilon})/2(10-p)$
15	eigenvalues not calculated since FP is unphysical			
16	$\frac{-G+\sqrt{G^2-H}}{24}$	$\frac{-G-\sqrt{G^2-H}}{24}$	$(3\epsilon-2\tilde{\epsilon})(p-4)/12$	$-(3\epsilon-2\tilde{\epsilon})(p-4)/12$

TABLE II. Eigenvalues of the corresponding linearized RG transformation. p is the number of order parameter components. A, B, C, and D are defined as $A=(p+8)\epsilon-2(p-4)\tilde{\epsilon}$, $B=16(p-1)\,(3\epsilon-2\tilde{\epsilon})\,[(p+8)\epsilon-2(p+2)\tilde{\epsilon}]$, $C=(p+8)\epsilon-2(p-4)\tilde{\epsilon}$, $D=8(10-p)\,(3\epsilon-2\tilde{\epsilon})\,[8\epsilon-12\tilde{\epsilon}+p\epsilon]$. Analogously, $E=3p\epsilon+2(p-4)\tilde{\epsilon}$, $F=24(p-2)\,(3\epsilon-2\tilde{\epsilon})\,[4\tilde{\epsilon}+3p\epsilon-4p\tilde{\epsilon}]$, $G=8\tilde{\epsilon}+3p\epsilon-2p\tilde{\epsilon}$, $H=48(3\epsilon-2\tilde{\epsilon})[-4\tilde{\epsilon}+3p\epsilon-2p\tilde{\epsilon}]$.

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