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Economical Input Design for Identification of Multivariate Systems

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Abstract: In this article, we propose an optimization based formulation for design of optimal inputs for multivariate systems. A well designed identification experiment can generate good quality models while incurring significant costs. In a typical process plant, the nominal policy is to operate the plant at or near constraints to achieve economic benefits. In this work, we quantify the cost of the experiment carried out on such systems in terms of the deviation from the nominal operational policy. The objective is to minimize the cost incurred during the experiment without violating the operational constraints while guaranteeing model quality. The proposed economics based optimization formulation is non-convex and hence we present a two-step iterative algorithm. The inputs are realized as white noise sequence filtered through an M-tap multivariate FIR filter. The filter coefficients are obtained by the spectral factorization. A detailed simulation study is presented to illustrate the proposed approach.

Keywords: System identification, convex optimization, input design, multivariate system

1. INTRODUCTION

System identification has been a well studied research topic among the control community over the last several decades. An important component of any identification exercise is the selection of the perturbation signal. Hence, it is necessary to design an input which generates maximum information about the system dynamics using minimal resources. In general, we formulate the input design problem as an optimization problem where the objective is to minimize some scalar norm of the covariance of the estimated parameters subject to constraints on input, output, model quality etc. Based on the norm chosen, the design problem is classified as A, E,or D-optimal design problem [Goodwin and Payne 1977]. A dual form of the conventional input design problem is the least costly framework where the objective is to minimize the additional input and output deviations subject to constraints on the quality of the identified model [Bombois et al. 2004].

In process industries, we often encounter multi-input multi-output (MIMO) systems. Identification of such systems is challenging due to the interaction between the inputs and outputs [Conner and Seborg 2004]. A common method to identify MIMO systems is to perturb one input (using e.g., PRBS, step) at a time while the other inputs are kept constant. The identified SISO models are then combined to obtain a MIMO model. However, the test duration is high and the developed MIMO models perform poorly when used for Model Predictive Control (MPC), where multiple inputs change simultaneously [Morari and Lee 1999]. An alternative approach is simultaneous input excitation where more than one input is excited at a time which results in lowered experiment duration compared to sequential design. In a typical multivariate input design formulation, the cost is to minimize some norm of the information matrix subject to constraints on input and output which however, does not account for the cost of constraint violations [Kumar and Narasimhan 2015].

In practical situations, identification is carried out on running plants and any perturbation in the inputs will introduce additional variance in the inputs and outputs. In several process systems, it is often optimal to operate at or close to constraints. The additional variability in the inputs and outputs often leads to constraint violation. Hence, the operating point and the perturbation signal is appropriately chosen to minimize the risk of constraint violation by "backing-off" from constraints which has a significant impact on economics. Hence, it is more appropriate to quantify the cost of the experiment in terms of this "back-off" [Kumar et al. 2014]. This is in contrast to the traditional least costly framework, where the cost is quantified in terms of the additional variance introduced [Bombois et al. 2004].

The focus of this contribution is to design an appropriate multivariate identification experiment so that the deviation from optimality is minimized. A simultaneous input excitation approach is used for such identification . In this article, we extend the idea proposed for economic identification of SISO systems by Kumar et al. [2014]. The region in the input-output space where the process is expected to operate is called the expected dynamic operating region. We formally define and characterize the expected dynamic operating region as a function of the disturbances and additional perturbation introduced for the purpose of identification. The focus of this contribution is design of an appropriate multivariate identification experiment so that the departure from optimality is minimized. This is carried out subject to the overall goal of the experiment being satisfied and the dynamic operating region being feasible.

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This problem is formulated as a non-convex optimization problem, where the decision variables are the experiment conditions, viz., the operation point around which the perturbation has to be carried out, and the multi variable input spectrum. The multi variable input spectrum is finitely parameterized by requiring it to be realizable by an M-tap FIR filter. The non-convexity is addressed by a two-stage procedure where a convex problem is solved at each stage. This process is iterated until convergence. The input is realized as white noise filtered by an M-tap FIR filter. The filter coefficients are obtained by the spectral factorization.

2. PRELIMINARY ASPECT OF MULTIVARIATE IDENTIFICATION AND INPUT DESIGN

We consider identification of a discrete time, stable, linear time invariant MIMO system. The system is modeled as:

$$\mathcal{M} : y_k = G(q^{-1}, \theta)u_k + H(q^{-1}, \eta)e_k \tag{1}$$

where $G(q^{-1}, \theta) \in \mathbb{R}^{n_y \times n_u}$ and $H(q^{-1}, \theta) \in \mathbb{R}^{n_y \times n_y}$ are stable rational transfer functions, q^{-1} is the back-shift operator, $(i.e., q^{-1}u_k = u_{k-1})$, $y_k \in \mathbb{R}^{n_y}$, $u_k \in \mathbb{R}^{n_u}$ are outputs and inputs respectively. e_k is Gaussian white noise sequence with zero mean and variance $\Lambda = diag[\lambda_1, \lambda_2 \cdots \lambda_{n_y}]$. We assume that the identified model is flexible enough to capture dynamics of the true system $\mathcal{S}$, i.e., the true system is in the model set [Ljung 1999].

$$\mathcal{S} : y_k = G_0(q^{-1})u_k + H_0(q^{-1})e_k \tag{2}$$

Denote $\tilde{\theta} = \begin{bmatrix}\theta^T & \eta^T\end{bmatrix}^T$ and $\tilde{\theta}_0 = \begin{bmatrix}\theta_0^T & \eta_0^T\end{bmatrix}^T$ are the total estimated and true parameters vector.

The model parameters are identified within the prediction error framework in which the objective is to minimize one step ahead prediction error

$$\epsilon(k, \tilde{\theta}) = y_k - \hat{y}(k|k-1, \tilde{\theta}) \tag{3}$$

$$\hat{\tilde{\theta}} = \arg \min_{\tilde{\theta}} \frac{1}{N} \sum_{k=1}^{N} \epsilon(k, \tilde{\theta})^T \epsilon(k, \tilde{\theta}) \tag{4}$$

$\hat{y}(k|k-1, \theta)$ is the one step ahead prediction of y_k which is computed from the assumed model. Under some mild assumptions, the estimated parameter follows a normal distribution, when the number of sampled data tends to infinity [Ljung 1999].

$$\sqrt{N}(\hat{\tilde{\theta}} - \tilde{\theta}_0) \xrightarrow{N\to\infty} \mathcal{N}(0, P_{\tilde{\theta}}) \tag{5}$$

where $P_{\tilde{\theta}}$ is a covariance matrix which characterizes the uncertainty in the parameter estimates. For an asymptotic unbiased estimator such as prediction error method, $P_{\tilde{\theta}}$ can be expressed as

$$P_{\tilde{\theta}} = M_{\tilde{\theta}}^{-1} = \begin{bmatrix} M_d & 0 \\ 0 & M_\gamma \end{bmatrix}^{-1} \tag{6}$$

where M_θ is the Fisher information matrix which is further partitioned as input dependent matrix M_d and input independent matrix M_γ.The cross diagonal term of partitioned matrix is zero because in open loop noise are uncorrelated with the input signal. A method of finding information matrix for the multivariate process is presented in [Aguero et al. 2009].

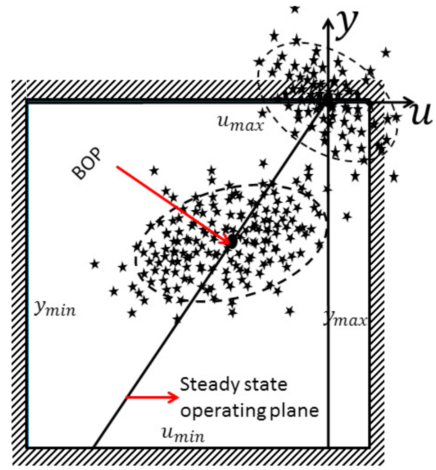

Fig. 1. Input excitation at the (a) nominal operating point showing infeasible dynamic operation, and (b) backed-off operating point showing feasible dynamic operation.

3. MOTIVATION AND PROBLEM FORMULATION

Nominal operation of a typical process system is usually obtained by solving a static, non-linear optimization problem by minimizing an economic cost function $J(\overline{u}_{ss}, \overline{y}_{ss})$, subject to the plant model and constraints. These constraints are usually in the form of input, output or resource constraints. Often the optimal operating point (OOP), $[u^{*T}, y^{*T}]^T$ is constrained, i.e., some constraints are active as shown in Figure1. The Expected Dynamic Operating Region (EDOR) is the region in input-output space where the process is expected to operate. Uncertainties in the form of disturbances can cause constraint violations. If an identification experiment is carried out at $[u^{*T}, y^{*T}]^T$, the nominally profitable operating point, the risk of constraint violation increases because of input and output perturbations. Hence, a conservative solution is to "back-off" or equivalently, operate the plant further inside the feasible region at the backed-off operating point (BOP) $[u_{ss}^T, y_{ss}^T]^T$ which will result in non-profitable operation [Nabil et al. 2012, 2013]. The expected dynamic operating region around the backed-off operating point is now a function of the disturbances and the perturbations introduced due to the identification experiment.

Hence, in such situations, a more appropriate cost associated with the identification experiment is the loss incurred by operating the plant at $[u_{ss}^T, y_{ss}^T]^T$ rather than at $[u^{*T}, y^{*T}]^T$. The focus of this contribution is to determine the operating conditions such that this loss is minimized. This is formulated as an optimization problem where the objective is to determine the experimint conditions that minimize this loss. These include the operating point around which the identification experiment is carried out and the input signal. The constraints imposed are that the dynamic region is feasible or equivalently, the constraints are not violated (up to a given confidence level) and the identified model is sufficiently accurate. The problem is formulated in the frequency domain and the input signal is characterized by the power spectrum.

We assume a linear form of the cost function $J(u_{ss}^T, y_{ss}^T)$ which is a good approximation around the constrained optimal point. The steady state behaviour of the plant is assumed to be linear with gain K. Defining $z = [u^T, y^T]^T$, we linearize the constraints to obtain $a_i^T z \leq b_i,\ i = 1, \dots, n_c$. The EDOR around z_{ss} is defined as follows [Kumar et al. 2014]:

$$\mathcal{E} = \{z | \mathbb{P}(z \in \mathcal{E}) \geq \zeta\}$$

where ζ is a user defined probability or confidence level. Define deviation variables as follows: $\tilde{y}_{ss} = y_{ss} - y^*$ and $\tilde{u}_{ss} = u_{ss} - u^*$, the optimization formulation can be expressed as

$$\begin{aligned} \min_{\Phi_u, \tilde{u}_{ss}, \tilde{y}_{ss}} \ & J_u^T \tilde{u}_{ss} + J_y^T \tilde{y}_{ss} \\ \text{s.t.} & \tilde{y}_{ss} = K \tilde{u}_{ss} \\ & M_d(\omega) \geq \eta I \\ & a_i^T z \leq b_i \ \forall z \in \mathcal{E} \end{aligned} \tag{7}$$

The proposed formulation is computationally intractable because the input spectrum is infinite dimensional and the constraint set $a_i^T z \leq b_i \ \forall z \in \mathcal{E}$ is also infinite dimensional. Therefore, we reformulate the problem to obtain a finite dimensional parameterization.

3.1 Parametrization of input spectrum

A common method of parameterizing the input spectrum is the following one [Jansson and Hjalmarsson 2005, Kumar and Narasimhan 2015]

$$\Phi_u(\omega) = \Psi(e^{j\omega}) + \Psi^*(e^{j}\omega) \succeq 0 \ \forall \ \omega \in [-\pi, \pi] \tag{8}$$

$$\Psi(e^{j\omega}) = \sum_{k=0}^{M-1} C_k e^{j\omega k} \succeq 0 \ \ \forall \ \omega \in [0, \pi] \tag{9}$$

where

$$C_k = \begin{bmatrix} c_1(k) & c_{12}(k) & \cdots & c_{1n_u}(k) \\ c_{12}^*(k) & c_2(k) & \cdots & c_{2n_u}(k) \\ \cdots & \cdots & \cdots & \cdots \\ c_{1N}^*(k) & c_{2N}^*(k) & \cdots & c_{n_u}(k) \end{bmatrix}$$

If we restrict our input signal to be white noise passing through an FIR filter of length M, than C_i can be interpreted as the correlation sequences ($R \in \mathbb{R}^{n_u \times n_u}$) of the input signal. In this case input spectrum (Φ_u) and its positive part (Ψ) can be written as:

$$\Phi_u(\omega) = R[0] + 2\textstyle\sum_{i=1}^{M-1} R[i] cos(i\omega) \succeq 0 \ \forall \ \omega \in [-\pi, \pi] \tag{10}$$

$$\Psi(e^{i\omega}) = \tfrac{R[0]}{2} + \textstyle\sum_{i=1}^{M-1} R[i] cos(i\omega) \succeq 0 \ \forall \ \omega \in [-\pi, \pi] \tag{11}$$

The advantage of parametrization of input spectrum as in (8) is that it is very easy to generate time-domain input sequences u_k. The semidefiniteness condition on the input spectrum is ensured by imposing an LMI given by the KYP Lemma [Jansson and Hjalmarsson 2005, Kumar and Narasimhan 2015].

$$\begin{bmatrix} Q_\phi - A_\phi^T Q_\phi A_\phi & C_\phi^T - A_\phi^T Q_\phi B_\phi \\ C_\phi - B_\phi^T Q_\phi A_\phi & D_\phi + D_\phi^T - B_\phi^T Q_\phi B_\phi \end{bmatrix} \succeq 0 \tag{12}$$

where

$$A_\phi = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ I_{n_u} & 0 & 0 & \cdots & 0 \\ 0 & I_{n_u} & 0 & \cdots & \vdots \\ \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & I_{n_u} & 0 \end{bmatrix} B_\phi = \begin{bmatrix} I_{n_u} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C_\phi = [R_1,\ R_2, \dots, R_{M-1}] \quad D_\phi = \frac{1}{2} R_0$$

3.2 Characterizing the expected dynamic operating region

Since the experiment time is large, the first and second moments of z can be conveniently expressed in terms of the frequency domain equivalents.

$$\mathbb{E} z = \tilde{z}_{ss} \tag{13}$$

$$\mathbb{E}(z - \tilde{z}_{ss})(z - \tilde{z}_{ss})^T = \Sigma_z(\omega) \tag{14}$$

$$\Sigma_z(\omega) = \begin{bmatrix} \Sigma_u(\omega) & \Sigma_{uy}(\omega) \\ \Sigma_{yu}(\omega) & \Sigma_y(\omega) \end{bmatrix} \tag{15}$$

where Σ_{yu} is the cross-correlation of input and output and obtained as follows:

$$\Sigma_{yu} = \Sigma_{uy} = \frac{1}{2\pi} \int_{-\pi}^{\pi} G(e^{i\omega}, \theta) \Phi_u(\omega) d\omega \tag{16}$$

Since the noise e_k is Gaussian and the input is obtained by filtering a Gaussian white noise signal through an M-tap FIR filter, the vector z is jointly Gaussian and is completely characterized by the first and second order moments. Hence, the EDOR is an ellipsoid and can be completely characterized in terms of the first and second order moments of z as follows:

$$\mathcal{E} = \{z | z = \tilde{z}_{ss} + \alpha \Sigma_z^{1/2} \tilde{z},\ ||\tilde{z}|| \leq 1\} \tag{17}$$

where α is indicative of the confidence level and can be obtained from the χ^2 distribution such that $\mathbb{P}(\chi^2(n_u + n_y) < \alpha^2 = \zeta)$. The infinite dimensional constraints $a_i^T z \leq b_i \forall z \in \mathcal{E}$ can be rewritten as follows [Boyd and Vandenberghe 2004]:

$$\alpha ||\Sigma_z^{1/2} a_i|| + a_i^T \tilde{z}_{ss} \leq b_i, \tag{18}$$

From the above discussions, we can write the finite dimensional formulation of the economically optimal input design problem as:

$$\begin{aligned} \min_{R, \tilde{u}_{ss}, \tilde{y}_{ss}} \ & J_u^T u_{ss} + J_y^T y_{ss} \\ \text{s.t.} & \tilde{y}_{ss} = K \tilde{u}_{ss} \\ & \Sigma_z(r) := \begin{bmatrix} \Sigma_u(R) & \Sigma_{uy}(R) \\ \Sigma_{yu}(R) & \Sigma_y(R) \end{bmatrix} \\ & M_d(R) \geq \eta I \\ & P(R) = \Sigma_z(R)^{1/2} \\ & \alpha ||P a_i|| + a_i^T z_{ss} \leq b_i \\ & \begin{bmatrix} Q_\phi - A_\phi^T Q_\phi A_\phi & C_\phi^T - A_\phi^T Q B_\phi \\ C_\phi - B_\phi^T Q_\phi A_\phi & D_\phi + D_\phi^T - B_\phi^T Q B_\phi \end{bmatrix} \succeq 0, \end{aligned} \tag{19}$$

where the last LMI follows from the KYP lemma and input parameterization described in Section 3.1.

4. SOLUTION METHODOLOGY

The input design problem proposed above is non-convex because of the constraint $P(R) = \Sigma_z(R)^{1/2}$.Therefore, we adapt the two-stage solution algorithm presented in [Nabil et al. 2014, 2012]. This is based on the geometric interpretation of the above formulation, viz., to determine the BOP (z_{ss}) and the associated ellipse Σ_z such that the ellipse lies within the feasible set and the cost $J_u^T u_{ss} + J_y^T y_{ss}$ is minimized and $M_d \geq \eta I$. The basic idea in the first stage is to find the smallest (in terms of trace) feasible ellipsoid Σ_z, that describes the feasible set of input signals for system identification. For this purpose, we impose the following constraints on the individual variances for obtaining the Σ_z that ensures feasibility in the second stage,

$$\sigma_{z,1}^2 < \frac{1}{4\alpha^2}\ (u_{max} - u_{min})^2 \tag{20}$$

where $\sigma_{z,1}^2$ is the variance of the input variable. It is important to note that, in a traditional input design problem, the upper limit is assumed to be provided by the designer (c_{in}, c_{op}) whereas, in the economical framework, it is defined in terms of the available feasible space given by the variable bounds (u_{min}, u_{max}) and (y_{min}, y_{max}). In the second stage, this covariance ellipsoid is used to determine the closest possible backed-off operating point (BOP) z_{ss} to the constrained optimal operating point OOP, $[u^{*T}, y^{*T}]^T$ such that the pre-determined expected dynamic operating region is within the feasible set. Equivalently, the inputs and outputs satisfy the constraints, modulo a confidence level. Information from the second stage (i.e., BOP) is used to create lower bounds on the variances by defining the parameter δ_{ij} describing the closeness to OOP. The parameter δ_{ij} is defined as

$$\delta_{ij} = \frac{distance\ of\ variable\ i\ from\ its\ closest\ bound}{distance\ of\ variable\ j\ from\ its\ closest\ bound} \tag{21}$$

To this end, we define the following constraints with respect to variance of the j^{th} variable, $\sigma_{z,j}^2$,

$$\sigma_{z,j}^2 > \frac{\delta_{ij}^2}{\alpha^2}\sigma_{z,i}^2 \tag{22}$$

where the iterative parameters δ_{ij}^2 are chosen such that the BOP selected in stage 2 is used to select the new minimum variance ellipsoid that forces the BOP close to OOP and recompute Σ_z in the first stage.

4.1 Stage 1

$$\begin{aligned}
\min_{R}\quad & tr(\Sigma_z) \\
\text{s.t.}\quad & M_d(R) \geq \eta I \\
& \begin{bmatrix} Q_\phi - A_\phi^T Q A_\phi & C_\phi^T - A_\phi^T Q B_\phi \\ C_\phi - B_\phi^T Q A_\phi & D_\phi + D_\phi^T - B_\phi^T Q B_\phi \end{bmatrix} \succeq 0 \\
& \Sigma_z(i,i) \leq \tfrac{1}{4\alpha^2}\ (d_{max}(i) - d_{min}(i))^2,\ i = 1, \cdots n_u + n_y \\
& \sigma_{z,2}^2 > \tfrac{\delta^2}{\alpha^2}\sigma_{z,1}^2
\end{aligned}$$

where $d_{max}, d_{min} \in \mathbb{R}^{P+N}$ are the maximum and minimum bound on corresponding inputs and outputs. Solution of Stage 1 results in a feasible covariance ellipsoid Σ_z. Let $P = \Sigma_z^{1/2}$. This is used to find the approximation to the backed-off operating point in stage 2 as follows.

4.2 Stage 2

$$\begin{aligned}
\min_{\tilde{u}_{ss},\tilde{y}_{ss}}\quad & {J_u}^T \tilde{u}_{ss} + {J_y}^T \tilde{y}_{ss} \\
\text{s.t.}\quad & \tilde{y}_{ss} = K\tilde{u}_{ss} \\
& \Sigma_z = \begin{bmatrix} R[0] & \Sigma_{uy}(R) \\ \Sigma_{yu}(R) & R^T\zeta_{op} + N_p \end{bmatrix} \succeq 0; \\
& \alpha||Pa_i|| + a_i^T \tilde{z}_{ss} \leq b_i
\end{aligned}$$

Solution of Stage 2 problem will result in a feasible backed off operating point. The parameter δ is updated based on the new BOP and used to resolve Stage 1. It is important to note that P is not a decision variable since Σ_z is known from first stage. Now it can be easily recognized that both the stages contains only convex constraints which could be easily solved using `CVX`, a package for specifying and solving convex programs ([Grant and Boyd 2011]). Initializing δ to zero and given two successive iterates, z_{ss}^{iter-1} and z_{ss}^{iter} this process is iterated until the convergence criteria $\|z_{ss}^{iter} - z_{ss}^{iter\text{-}1}\|_2 \leq \epsilon$ is satisfied where ϵ being the prescribed tolerance limit.

5. INPUT DESIGN

Given a set of R $\in \mathbb{R}^{n_u \times n_u}$, M-tap filter coefficients are calculated through spectral factorization using Fejér-Riesz spectral factorization [Dumitrescu 2007]:

Theorem 1 (Fejér-Riesz spectral factorization for multivariate polynomial) A matrix valued polynomial $\Phi_u(z) \in \mathbb{C}$ is nonnegative ($\Phi(\omega)$ is semidefinite) on the unit d-circle $\mathbb{T}^d$ if and only if Positive orthant polynomials (special class of causal polynomials)

$$H(z) = \sum_{i=0}^{(M-1)} h_i z^i \tag{23}$$

exists such that

$$\Phi_u(z) = \sum_{i=0}^{(M-1)} H(z)H^*(z^{-1}) \tag{24}$$

Several methods like functional analysis [Wilson 1972, Rozanov 1960], Newton Raphson type iterative algorithms [Yaglom 1960] etc., have been presented in literature to get solution of (24). Recently, a semidefinite programming approach is proposed while parameterizing the coefficients of the polynomial as a linear function of the elements of a positive semidefinite matrix Q (called Gram matrix). Such parameterization allows the description of a nonnegative trigonometric polynomial through a linear matrix inequality (LMI). Hence, SDP is applicable. The algorithm is as follows [Dumitrescu 2007]:

$$\begin{aligned}
\underset{Q}{\text{minimize}}\quad & tr(Q_{00}) \\
\text{s.t}\quad & TR(\Theta_i Q) = R_i \quad i = 0, 1, \cdots M-1 \\
& Q \succeq 0
\end{aligned} \tag{25}$$

Where Q_{00} is the upper $n_u \times n_u$ left block of Gram matrix. TR(.) is the block trace operator defined in [Dumitrescu 2007]. Θ is the block Toeplitz matrix, with unit matrices of size $n_u \times n_u$ on its i-th block diagonal and zeros elsewhere. The filter coefficient is obtained by the factorization (e.g. Cholesky factorization) of optimal Q^*. The factorization of Q matrix is not always trivial due to the tolerance limit of the solver. For such situation an equivalent matrix (Q_{eqv}) can be obtained as follow:

$$\begin{aligned} &\underset{Q_{eqv}}{\text{minimize}} && \|(Q_{eqv} - Q)\|_{frobenius} \\ &\text{s.t} && Q_{eqv} \succeq 0 \end{aligned}$$

$$Q_{eqv} = \begin{bmatrix} h_0^H \\ . \\ . \\ . \\ h_{M-1}^H \end{bmatrix} [\, h_0 \ . \ . \ h_{M-1} \,] \tag{26}$$

Where $h_k \in \mathbb{R}^{n_u \times n_u}$ is the desired filter coefficient.

6. CASE STUDY

To illustrate our proposed method, we have considered a Van de Vusse reactor as an example. It is basically a continuous stirred tank reactor (CSTR) where three endothermic chemical reactions $A \to B \to C$ and 2A$\to D$ takes place [Rothfuss et al. 1996]. The model equations are

$$\dot{c_A} = r_A(c_A, T) + (c_{in} - c_A)\, u_1 \tag{27}$$

$$\dot{c_B} = r_B(c_A, c_B, T) - c_B u_1 \tag{28}$$

$$\dot{T} = h\,(c_{A,}, c_B, T) + \alpha\,(u_2 - T) + (T_{in} - T)\, u_1 \tag{29}$$

The rate of reaction is given by following equation:

$$r_A = -k_1\,(T)\, c_A - k_2\,(T)\, c_A^2 \; r_B = k_1\,(c_A - c_B) \tag{30}$$

$$h = -\delta \left(k_1\,(T) \;\; (c_A \Delta H_{AB} + c_B \Delta H_{BC}) + \tilde{h} \right) \tag{31}$$

$$\tilde{h} = k_2\,(T)\, c_A^2 \Delta H_{AD} \quad k_i\,(T) = k_{i0} exp \frac{-E_i}{T + T_0} \tag{32}$$

Where c_A, c_B are the concentration (mole/liter) and T is the temperature of cooling jacket in degree Celsius. The inputs u_1 and u_2 are the normalized flow rate through the reactor in 1/h. c_{in} (mole/liter) and T_{in} (°C) are considered as disturbances. For simplicity, we assume that the disturbance are normally distributed and are uncorrelated with each other. All states are measurable and considered as outputs of the system. The outputs and inputs are constrained as :

$$\begin{aligned} &c_A \in [0, 4.5] \text{ mol/l} \quad c_B \in [0, 4] \text{ mol/l} \quad T \in [70, 200]\ ^\circ\text{C} \\ &u_1 \in [3, 200]\ \frac{1}{h} \quad u_2 \in [60, 170]\ ^\circ\text{C} \end{aligned} \tag{33}$$

The system parameters are considered as

$$\begin{aligned} &\alpha = 30.828 \;\; \delta = 0.3522 \;\; m^3 K\; kj^{-1} \; E_1 = 9758.3 \\ &E_2 = 8560 \quad k_{10} = 1.28 \times 10^{12}\, \frac{1}{h} \\ &\Delta H_{ab} = 4.2\; mol^{-1} \;\; \Delta H_{bc} = -11\; mol^{-1} \\ &T_0 = 273.15\ ^\circ\text{C} \;\; c_{in} = 10\, \frac{mol}{l} \quad T_{in} = 114.9^\circ\text{C} \\ &\gamma = 0.1\; Kkj^{-1} \; k_{20} = 9.04 \times 10^9\; m^3 (mol.h)^{-1} \\ &\Delta H_{ad} = -41.85\; mol^{-1} \end{aligned}$$

Here, the economic objective is to maximize the production rate of c_B

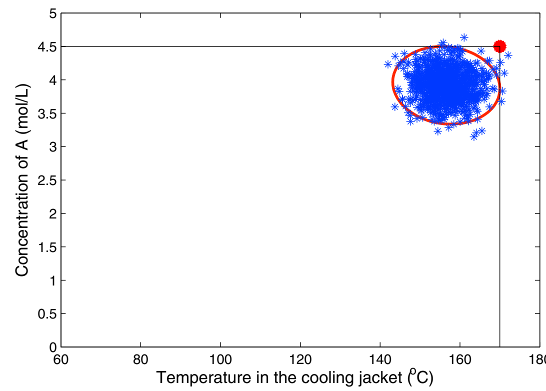

Fig. 2. Profitable and feasible dynamic operating region between flow rate u_2 and c_A

$$J\,(c_B, u_1) = \beta c_B u_1 \quad \beta = 86.7\; h^{-1} \tag{34}$$

The underlaying system is linearized around the optimal steady state point $y^* = [4.50, 1.99, 148.71]^T$ and $u^* = [146.01, 170]^T$. At the optimal operating points the production rate of c_B is 2.5192×10^4 mole/hr. Here, we can observe that y_1 and u_2 are active at their respective upper bounds. A state space form of linear system and corresponding discrete time transfer functions are obtained by considering 7 sec sampling time, which is not presented here in the interest of brevity. This obtained linear model is used to generate inputs and outputs data for the purpose of identification.

Since, the objective function and constraints depends upon the system parameters an initial estimates of parameters are obtained by perturbing system with a PRBS signals. We have considered M = 10 to shape the input spectrum, a higher value of M can also be selected but it is computationally intensive. The lower bound on the information matrix M_d is considered as I. The convex optimization problems arising in the two stage procedure described above are solved using `CVX` [Grant and Boyd 2011]. The backed-off operating point is $z_{ss} = [3.918, 2.047, 148.844, 105.802, 156.58]^T$. The production rate of product B at the backed-off operating point is $1.8776 \times 10^4 mole/hr$. This is the maximal achievable production rate that ensures dynamic feasible operation during excitation of input signals for the purpose of system identification. The backed-off operating point and the feasible dynamic region are shown in Figures 2,3,4 and 5 respectively. From the figures, it is seen that for the most part, the constraints are obeyed.

7. CONCLUSION

We have proposed an economic input design problem for multi input multi output systems. The objective is to minimize the important cost, viz., the deviation from optimality subject to constraints on feasibility and quality of the identified model. Since, the optimization formulation is non-convex, a two stage solution is adopted where a convex problem is solved at each stage. The proposed ideas are demonstrated using simulations of a CSTR.

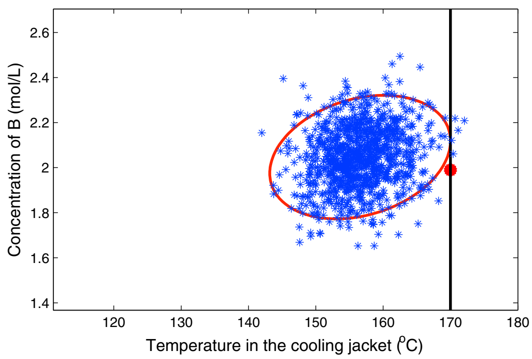

Fig. 3. Profitable and feasible dynamic operating region between flow rate u_2 and c_B

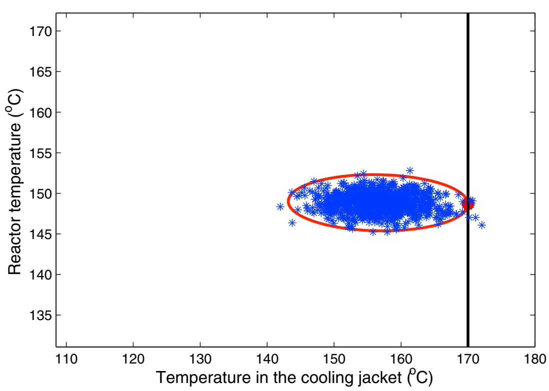

Fig. 4. Profitable and feasible dynamic operating region between flow rate u_2 and T

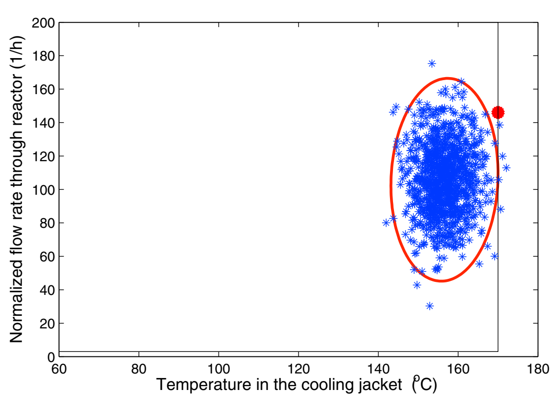

Fig. 5. Profitable and feasible dynamic operating region between u_2 and u_1

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