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Duality and scale invariant magnetic fields from bouncing universes

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Recently, we numerically showed that, for a nonminimal coupling that is a simple power of the scale factor, scale invariant magnetic fields arise in a class of bouncing universes. In this work, we *analytically* evaluate the spectrum of magnetic and electric fields generated in a subclass of such models. We illustrate that, for cosmological scales which have wave numbers much smaller than the wave number associated with the bounce, the shape of the spectrum is preserved across the bounce. Using the analytic solutions obtained, we also illustrate that the problem of backreaction is severe at the bounce. Finally, we show that the power spectrum of the magnetic field remains invariant under a two-parameter family of transformations of the nonminimal coupling function.

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I. INTRODUCTION

Magnetic fields are ubiquitous in the Universe. Coherent magnetic fields have been observed over a wide variety of scales, ranging from astrophysical systems such as stars and galaxies, to cosmological systems such as the large scale structures (LSS) (in this context, see Refs. [1,2]; for some recent reviews, see Refs. [3–6]). More recently, magnetic fields have been observed even in the intergalactic medium [7,8]. While the strength of magnetic fields observed in galaxies and clusters of galaxies are typically about a few microgauss, in the intergalactic medium, the lower bounds on their strengths have been inferred to be of the order of 10^{-17} G at 1 Mpc from the Fermi/LAT and HESS observations of TeV blazars (see Refs. [7–10]; also see Refs. [11–16]). This should be contrasted with the upper bound of a few nanogauss which has been arrived at from the cosmic microwave background (CMB) observations (for current constraints from the Planck and POLARBEAR data, see Refs. [17,18], and references therein). Similar upper limits have also been obtained independently using the rotation measures from the National Radio Astronomy Observatory Very Large Array Sky Survey [19]. These bounds are broadly in agreement with the limits arrived at from the LSS data, either alone or when combined with the CMB data (in this context, see Refs. [20–22]; for improved limits from LSS and reionization, see Refs. [23–27]). Though astrophysical processes such as the dynamo mechanism can, in principle, boost the amplitude of magnetic fields in galaxies, a seed field is nevertheless required for such mechanisms to work. Therefore, a primordial origin for the magnetic fields seems inevitable to explain their prevalence, particularly on the largest scales.

Inflation is currently considered the most promising paradigm to describe the origin of perturbations in the early Universe. Hence, it seems natural to consider the generation of magnetic fields in the inflationary scenario. It is well known that the conformal invariance of the electromagnetic field has to be broken in order to generate magnetic fields of observable strengths in the early Universe [28,29]. In fact, the issue has been studied extensively. There exist many inflationary models which lead to nearly scale invariant magnetic fields of appropriate strength and correlation scales to match with the observations [30–47]. However, most models of inflationary magnetogenesis typically suffer from the so-called backreaction and strong coupling problems (see, for instance, Refs. [32,48,49]).

Under such circumstances, it seems worthwhile to examine the generation of magnetic fields in alternative scenarios of the early Universe. A reasonably popular alternative are bouncing models, wherein the Universe undergoes a period of contraction until the scale factor attains a minimum value, after which it begins to expand (see, for instance, Refs. [50–65], and the following reviews [66–69]). Such bouncing scenarios provide an alternative to inflation to overcome the horizon problem. These models allow well-motivated, Minkowski-like initial conditions to be imposed on the perturbations at early times during the contracting phase. The generation of magnetic fields in such scenarios has been explored only to a limited extent [70–72].

In a recent work [72], we numerically showed that, for nonminimal couplings that are a simple power of the scale factor, scale invariant magnetic fields can be generated in certain bouncing scenarios. In this work, we investigate the

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problem analytically in a subclass of these models wherein the nonminimal coupling is a positive power of the scale factor. We consider a specific form for the scale factor, leading to a nonsingular bounce, which reduces to a power law form far from the bounce. We find that, in such situations, we can obtain analytical solutions for modes of the electromagnetic vector potential that are much smaller than the natural scale associated with the bounce. We divide the time before the bounce into two domains, one that corresponds to very early times and another closer to the bounce. We analytically evaluate the electromagnetic modes during these domains and arrive at the corresponding power spectra for the electric and magnetic fields. It can be easily shown that scale invariant magnetic fields can be generated before the bounce for specific values of the parameters involved. We evolve these modes across the bounce and calculate the power spectra in the final and third domain, i.e. in the early stages of the expanding phase *after* the bounce. We show that the shapes of the spectra are preserved for scales of cosmological interest as the modes evolve across the bounce.

At this stage, it is important that we comment on the theoretical and observational status of bouncing models. Theoretically, the main issue that plagues these models is the rapid growth of perturbations as one approaches the bounce. Evidently, this raises questions about the validity of linear perturbation theory around the bounce [67,68]. As far as scalar perturbations are concerned, this is typically circumvented by working in a specific gauge where the amplitude of the perturbations remains small (in this context, see, for instance, Ref. [58]). Another concern that had been pointed out early in the literature is the rapid growth of vector perturbations in a contracting universe [73]. But this issue does not arise if one assumes that there are no vector sources. As far as the observational constraints are concerned, one finds that nearly scale invariant scalar and tensor power spectra can indeed be generated in bouncing scenarios [58,74]. However, one of the primary problems that confronts bouncing models seems to be the fact that the tensor-to-scalar ratio r generated in these models may prove to be much larger than the present upper bound of $r \lesssim 0.1$ from Planck [75]. For instance, in certain matter bounce scenarios, the tensor-to-scalar ratio r has been found to be as large as $\mathcal{O}(10)$ [58,67,69,76,77], which is considerably beyond the constraints arrived at from the CMB observations. Nonetheless, these exist other models—such as the matter bounce curvaton scenario [78] and other versions of the matter bounce scenario [79,80]—which lead to values of r that seem to be consistent with the observations.

This paper is organized as follows. In the following section, we shall describe a few essential aspects of the electromagnetic field that is coupled nonminimally to a scalar field. We shall discuss the equation of motion governing the electromagnetic potential, the quantization of the potential in terms of the normal modes in an evolving universe, and the power spectra describing the electric and magnetic fields. We shall also introduce the forms of the scale factor and the coupling function that we consider. In Sec. III, we shall divide the bounce into three domains and evaluate the modes analytically in each of these domains. We shall evaluate the power spectra prior to the bounce as well as soon after the bounce and illustrate that the shape of the power spectra is preserved across the bounce. In Sec. IV, we shall study the issue of backreaction using the analytic solutions for the modes. In Sec. V, we shall illustrate that the power spectrum of the magnetic field is form invariant under a two-parameter family of transformations of the coupling function. Finally, we shall conclude with a brief discussion in Sec. VI.

We shall work with natural units such that $\hbar = c = 1$ and set the Planck mass to be $M_{\rm Pl} = (8\pi G)^{-1/2}$. We shall adopt the metric signature of $(-,+,+,+)$. Greek indices shall denote the spacetime coordinates, whereas Latin indices shall represent the spatial coordinates, except for k which shall be reserved for denoting the wave number. Last, an overprime shall denote differentiation with respect to the conformal time coordinate.

II. THE BOUNCE, NONMINIMAL ACTION, EQUATIONS OF MOTION, AND POWER SPECTRA

Recall that, if A^μ is the electromagnetic vector potential, then the corresponding field tensor $F_{\mu\nu}$ is given in terms of A^μ by the relation

$$F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu} = A_{\nu,\mu} - A_{\mu,\nu}. \tag{1}$$

We shall consider the case wherein the electromagnetic field is coupled nonminimally to a scalar field ϕ through a function $J(\phi)$ and is described by the action (see, for instance, Refs. [31,33,36])

$$S[\phi, A^\mu] = -\frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}J^2(\phi)F_{\mu\nu}F^{\mu\nu}. \tag{2}$$

Evidently, it is the coupling function J which is responsible for breaking the conformal invariance of the action. The scalar field ϕ, for example, can be the primary source that is driving the evolution of the bouncing model. The variation of the above action leads to the following equation of motion of the electromagnetic field:

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}J^2(\phi)F^{\mu\nu}] = 0. \tag{3}$$

We shall consider the background to be the spatially flat, Friedmann-Lemaître-Robertson-Walker (FLRW) metric that is described by the line element

$$\mathrm{d}s^2 = a^2(\eta)(-\mathrm{d}\eta^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j), \tag{4}$$

where $a(\eta)$ is the scale factor and η denotes the conformal time coordinate. In order to study the evolution of the vector potential, we shall choose to work in the Coulomb gauge wherein $A_0 = 0$ and $\partial_i A^i = 0$. On quantization, the vector potential $\hat{A}_i$ can be Fourier decomposed as follows [33,36,48]:

$$\hat{A}_i(\eta, \boldsymbol{x}) = \sqrt{4\pi}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}} \times \sum_{\lambda=1}^{2} \tilde{\epsilon}_{\lambda i}(\boldsymbol{k})[\hat{b}_{\boldsymbol{k}}^{\lambda}\bar{A}_k(\eta)\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}_{\boldsymbol{k}}^{\lambda\dagger}\bar{A}_k^{*}(\eta)\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}}], \tag{5}$$

where the Fourier modes $\bar{A}_k$ satisfy the differential equation [cf. Eq. (3)]

$$\bar{A}_k'' + 2\frac{J'}{J}\bar{A}_k' + k^2\bar{A}_k = 0. \tag{6}$$

The quantities $\tilde{\epsilon}_{\lambda i}$ represent polarization vectors, and the summation corresponds to the two orthonormal transverse polarizations. The operators $\hat{b}_{\boldsymbol{k}}^{\lambda}$ and $\hat{b}_{\boldsymbol{k}}^{\lambda\dagger}$ are the annihilation and creation operators, respectively, satisfying the following standard commutation relations:

$$[\hat{b}_{\boldsymbol{k}}^{\lambda}, \hat{b}_{\boldsymbol{k}'}^{\lambda'}] = [\hat{b}_{\boldsymbol{k}}^{\lambda\dagger}, \hat{b}_{\boldsymbol{k}'}^{\lambda'\dagger}] = 0, \qquad [\hat{b}_{\boldsymbol{k}}^{\lambda}, \hat{b}_{\boldsymbol{k}'}^{\lambda'\dagger}] = \delta_{\lambda\lambda'}\delta^{(3)}(\boldsymbol{k} - \boldsymbol{k}'). \tag{7}$$

Let us now define a new variable $\mathcal{A}_k = J\bar{A}_k$, which, as we shall see, proves to be convenient to deal with. In terms of the new variable, Eq. (6) for $\bar{A}_k$ simplifies to

$$\mathcal{A}_k'' + \left(k^2 - \frac{J''}{J}\right)\mathcal{A}_k = 0. \tag{8}$$

Let $\hat{\rho}_{\mathrm{E}}$ and $\hat{\rho}_{\mathrm{B}}$ denote the operators corresponding to the energy densities associated with the electric and magnetic fields, respectively. Upon using the decomposition (5) of the vector potential, the expectation values of the energy densities $\hat{\rho}_{\mathrm{E}}$ and $\hat{\rho}_{\mathrm{B}}$ can be evaluated in the vacuum state, say, $|0\rangle$, that is annihilated by the operator $\hat{b}_{\boldsymbol{k}}^{\lambda}$. It can be shown that the spectral energy densities of the magnetic and electric fields can be expressed in terms of the modes $\bar{A}_k$ and $\mathcal{A}_k$, their derivatives $\bar{A}_k'$ and $\mathcal{A}_k'$, and the coupling function J as follows [33,36]:

$$\mathcal{P}_{\mathrm{B}}(k) = \frac{\mathrm{d}\langle 0|\hat{\rho}_{\mathrm{B}}|0\rangle}{\mathrm{d}\ln k} = \frac{J^2(\eta)}{2\pi^2}\frac{k^5}{a^4(\eta)}|\bar{A}_k(\eta)|^2 = \frac{1}{2\pi^2}\frac{k^5}{a^4(\eta)}|\mathcal{A}_k(\eta)|^2, \tag{9a}$$

$$\mathcal{P}_{\mathrm{E}}(k) = \frac{\mathrm{d}\langle 0|\hat{\rho}_{\mathrm{E}}|0\rangle}{\mathrm{d}\ln k} = \frac{J^2(\eta)}{2\pi^2}\frac{k^3}{a^4(\eta)}|\bar{A}_k{}'(\eta)|^2 = \frac{1}{2\pi^2}\frac{k^3}{a^4(\eta)}\left|\mathcal{A}_k'(\eta) - \frac{J'(\eta)}{J(\eta)}\mathcal{A}_k(\eta)\right|^2. \tag{9b}$$

The spectral energy densities $\mathcal{P}_{\mathrm{B}}(k)$ and $\mathcal{P}_{\mathrm{E}}(k)$ are often referred to as the power spectra for the generated magnetic and electric fields, respectively. A flat or scale invariant magnetic field spectrum corresponds to a constant, i.e. k-independent, $\mathcal{P}_{\mathrm{B}}(k)$.

We shall model the nonsingular bounce by assuming that the scale factor $a(\eta)$ behaves as follows [72,81]:

$$a(\eta) = a_0\left(1 + \frac{\eta^2}{\eta_0^2}\right)^q = a_0(1 + k_0^2\eta^2)^q, \tag{10}$$

where a_0 is the value of the scale factor at the bounce (i.e. when $\eta = 0$), $\eta_0 = 1/k_0$ denotes the time scale of the duration of the bounce, and $q > 0$. Note that, when $q = 1$, during very early times wherein $\eta \ll -\eta_0$, the scale factor behaves as in a matter-dominated universe (i.e. $a \propto \eta^2$). Therefore, the $q = 1$ case is often referred to as the matter bounce scenario. We shall assume that the scale k_0 associated with the bounce is of the order of the Planck scale M_{Pl}. We should mention here that, for certain values of the parameters involved, the above scale factor leads to tensor power spectra that are consistent with the CMB observations (see, for instance, Refs. [81,82]). However, we should hasten to add that determining the corresponding scalar power spectra requires a detailed modeling of the bounce.

The scale factor (10) above can be achieved, for instance, if we consider that the Universe is composed of two noninteracting fluids with constant equation of state parameters. Let the energy densities and pressure of the two fluids be denoted by ρ_i and p_i, respectively, with $i = (1, 2)$. Also, let the equations of state for the two fluids be given by $p_i = w_i\rho_i$, where w_i is a constant. Since the two fluids are noninteracting, the equation governing the conservation of energy associated with the fluids can be integrated to yield that $\rho_i = M_i/a^{r_i}$, where M_i is a constant. As is well known, the index r_i is related to the equation of state parameter w_i by the relation $r_i = 3(1 + w_i)$. It can be easily shown that the equation of state parameters w_1 and w_2 are related to the quantity q through the relations $w_1 = (1-q)/(3q)$ and $w_2 = (2-q)/(3q)$. Furthermore, one can show that $M_1 = 12k_0^2M_{\mathrm{Pl}}^2a_0^{1/q}$ and $M_2 = -M_1a_0^{1/q}$. It is important to note that, while M_1 is positive, M_2 is negative. In other words, the energy density of the second fluid is always negative. This seems inevitable, as the total energy density has to vanish at the bounce. For the specific case of $q = 1$, which is the matter bounce scenario, the first fluid corresponds to matter. The second fluid behaves in a manner similar to radiation as far as its time evolution is concerned, but it has a negative energy density. For our discussion, we shall assume that the evolution of the universe is achieved with the aid of suitable scalar field(s) which effectively mimic the behavior of the fluids (in this context, see, for example, Ref. [83]).

Given a scale factor, in order to arrive at the behavior of the electromagnetic modes in a FLRW universe, we shall also require the form of the nonminimal coupling function J. We shall assume that the coupling function can be conveniently expressed in terms of the scale factor as follows:

$$J(\eta) = J_0 a^n(\eta). \tag{11}$$

It can be easily argued that the resulting power spectra are independent of the constant J_0 (in this context, see Ref. [72]). As we shall discuss in the following section, in this work, for the problem to be tractable completely analytically, we shall restrict ourselves to cases wherein n is positive.

III. ANALYTICAL EVALUATION OF THE MODES AND THE POWER SPECTRA

To arrive at analytic solutions to the electromagnetic modes, let us divide the period prior to the bounce into two domains, one far away from the bounce and another closer to the bounce. Let these two domains correspond to $-\infty < \eta < -\alpha\eta_0$ and $-\alpha\eta_0 < \eta < 0$, where α is a relatively large number, say, of the order of 10^5 or so.

During the first domain, the scale factor (10) reduces to the following power law form: $a(\eta) \propto \eta^{2q}$. In such a case, the nonminimal coupling function J also simplifies to a power law form and it behaves as $J(\eta) \propto \eta^{\gamma}$, where we have set $\gamma = 2nq$. Under these conditions, we have $J''/J \simeq \gamma(\gamma-1)/\eta^2$. This behavior is exactly what is encountered for a similar coupling function in power law inflation. Because of this reason, it is straightforward to show that the solutions to the modes of the electromagnetic vector potential $\bar{A}_k$ in the first domain can be expressed in terms of the Bessel functions $J_\nu(x)$ [33,36,48,71,72]. One finds that the solutions can be expressed in terms of the quantity $\mathcal{A}_k$ as follows:

$$\mathcal{A}_k(\eta) = \sqrt{-k\eta}[C_1(k)J_{\gamma-1/2}(-k\eta) + C_2(k)J_{-\gamma+1/2}(-k\eta)], \tag{12}$$

where the coefficients $C_1(k)$ and $C_2(k)$ are to be fixed by the initial conditions. On imposing the Bunch-Davies initial conditions at early times during the contracting phase, i.e. as $k\eta \to -\infty$, one obtains that

$$C_1(k) = \sqrt{\frac{\pi}{4k}}\frac{\mathrm{e}^{-i\pi\gamma/2}}{\cos(\pi\gamma)}, \qquad C_2(k) = \sqrt{\frac{\pi}{4k}}\frac{\mathrm{e}^{i\pi(\gamma+1)/2}}{\cos(\pi\gamma)}. \tag{13}$$

At this stage, it is also useful to note that

$$\mathcal{A}_k'(\eta) - \frac{J'}{J}\mathcal{A}_k(\eta) = k\sqrt{-k\eta}[C_1(k)J_{\gamma+1/2}(-k\eta) - C_2(k)J_{-\gamma-1/2}(-k\eta)], \tag{14}$$

an expression we shall require to evaluate the power spectrum of the electric field.

Let us now evaluate the power spectra of the magnetic and electric fields as one approaches the bounce, i.e. in the limit $k|\eta| \ll 1$. It should be mentioned that, in order for the solutions we have obtained above to be applicable, we need to remain in the first domain (i.e. $-\infty < \eta < -\alpha\eta_0$) even as we consider this limit. This condition implies that we have to restrict ourselves to modes such that $k \ll k_0/\alpha$. The power spectra of the magnetic and electric fields can be arrived at from the above expressions for $\mathcal{A}_k$ and $\mathcal{A}_k' - (J'/J)\mathcal{A}_k$ and the asymptotic forms of the Bessel functions. As is to be expected, the resulting spectra have the same form as one encounters in power law inflation. One finds that the spectrum of the magnetic field can be written as [33,36,48,72]

$$\mathcal{P}_{\mathrm{B}}(k) = \frac{\mathcal{F}(m)}{2\pi^2}\left(\frac{H}{2q}\right)^4(-k\eta)^{4+2m}, \tag{15}$$

where $H \simeq (2q/a_0\eta)(\eta_0/\eta)^{2q}$, while $m = \gamma$ for $\gamma \le 1/2$ and $m = 1-\gamma$ for $\gamma \ge 1/2$. Moreover, the quantity $\mathcal{F}(m)$ is given by

$$\mathcal{F}(m) = \frac{\pi}{2^{2m+1}\Gamma^2(m+1/2)\cos^2(\pi m)}. \tag{16}$$

Clearly, the case $m = -2$ leads to a scale invariant spectrum for the magnetic field, which corresponds to either $\gamma = 3$ or $\gamma = -2$. The associated spectrum for the electric field can be evaluated to be

$$\mathcal{P}_{\mathrm{E}}(k) = \frac{\mathcal{G}(m)}{2\pi^2}\left(\frac{H}{2q}\right)^4(-k\eta)^{4+2m}, \tag{17}$$

where $m = 1+\gamma$ if $\gamma \le -1/2$ and $m = -\gamma$ for $\gamma \ge -1/2$, while $\mathcal{G}(m)$ is given by

$$\mathcal{G}(m) = \frac{\pi}{2^{2m+3}\Gamma^2(m+3/2)\cos^2(\pi m)}. \tag{18}$$

It should be noted that, when $\gamma = 3$ and $\gamma = -2$, the power spectrum of the electric field behaves as k^{-2} and k^2, respectively. These results imply that, in the bouncing scenario, one can expect these cases to lead to scale invariant spectra (corresponding to wave numbers such that $k \ll k_0/\alpha$) for the magnetic field before the bounce. Using the analytic expressions (12) and (14), in Fig. 1, we have plotted the spectra of the magnetic and electric fields

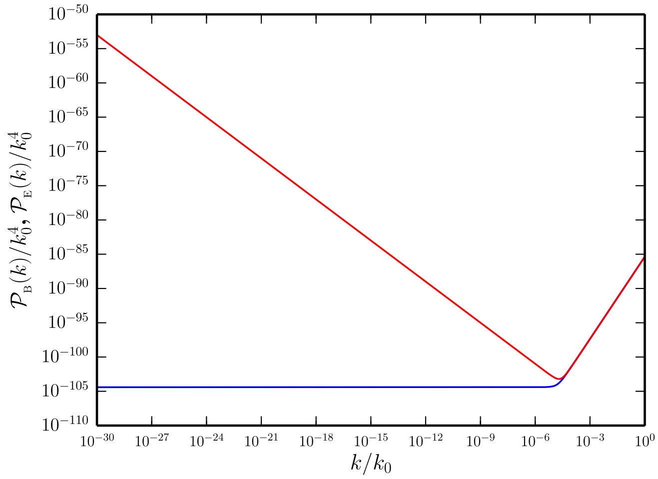

FIG. 1. The power spectra of the magnetic (in blue) and electric (in red) fields, evaluated before the bounce at $\eta = -\alpha\eta_0$ using the analytical expressions (12) and (14), have been plotted as a function of k/k_0 for $\gamma = 3$, $q = 1$, $a_0 = 8.73 \times 10^{10}$, and $\alpha = 10^5$. Note that the dimensionless quantities $\mathcal{P}_{\rm B}(k)/k_0^4$ and $\mathcal{P}_{\rm E}(k)/k_0^4$ that we have plotted depend only on the combination k/k_0. Also, we should mention that our analytical approximations are valid only for scales such that $k \ll k_0/\alpha$. Over this domain, while the spectrum of the magnetic field is strictly scale invariant, the spectrum of the electric field behaves as k^{-2}, as is suggested by the spectra (15) and (17) arrived at from the asymptotic forms of the Bessel functions. Needless to add, the question of interest is whether these power spectra will retain their shape after the bounce.

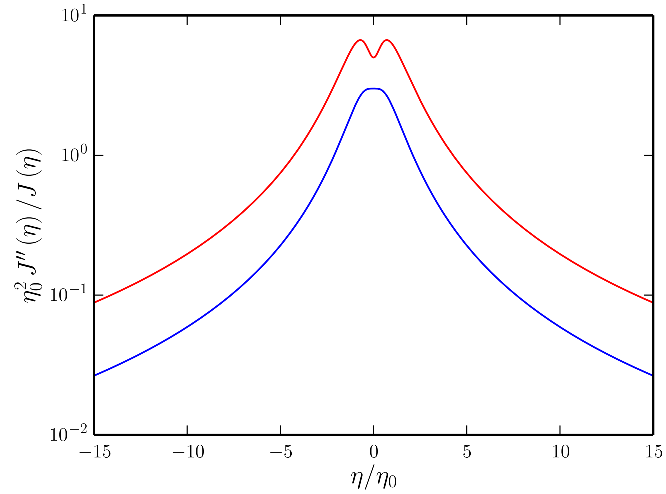

FIG. 2. The behavior of $\eta_0^2 J''/J$, which depends only on η/η_0, has been plotted for $\gamma = 3$ (in blue) and $\gamma = 5$ (in red). The figure has been plotted over a very narrow range of η/η_0 in order to illustrate the presence of a single maximum for $\gamma = 3$ and two maxima and one minimum for $\gamma = 5$.

evaluated at $\eta = -\alpha\eta_0$ as a function of k/k_0, for $\gamma = 3$ and a set of values for the parameters q, a_0 and α. In the domain $k \ll k_0/\alpha$, where our approximations are valid, it is evident from the figure that, while the spectrum of the magnetic field is scale invariant, the spectrum of the electric field behaves as k^{-2}. These are exactly the asymptotic forms (15) and (17) that we have arrived at above. The question that naturally arises is whether these spectra will retain their form as they traverse across the bounce.

Our analysis until now applies to both positive and negative values of n. However, as we mentioned, we shall hereafter restrict our analysis to the cases wherein $n > 0$. We shall illustrate that, in such cases, one can arrive at an analytic expression for the electromagnetic modes even during the bounce for wave numbers such that $k \ll k_0$. When $n > 0$, J grows away from the bounce, and, hence, it seems natural to expect that J''/J will exhibit its maximum near the bounce. Actually, J''/J has a single maximum at the bounce for indices n and q such that $\gamma \leq 3$. One finds that, for other values of γ, there arise two maxima and a minimum close to the bounce. The minimum occurs exactly at the bounce, and its value proves to be γk_0^2. These behavior are clear from Fig. 2, wherein we have plotted the quantity J''/J for two different values of γ. Therefore, when $n > 0$, for scales of cosmological interest such that $k \ll k_0$, $k^2 \ll J''/J$ around the bounce. Hence, near the bounce, we can neglect the k^2 term in (6) [to be precise, we can ignore the k^2 term in Eq. (8)] so that we have

$$\bar{A}_k'' + 2\frac{J'}{J}\bar{A}_k' \simeq 0. \tag{19}$$

This equation can be immediately integrated to yield

$$\bar{A}_k'(\eta) \simeq \bar{A}_k'(\eta_*)\frac{J^2(\eta_*)}{J^2(\eta)}, \tag{20}$$

where η_* is a time when $k^2 \ll J''/J$ before the bounce. The above equation can be further integrated to arrive at

$$\begin{aligned}\bar{A}_k(\eta) &\simeq \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)\int_{\eta_*}^{\eta} {\rm d}\tilde{\eta}\,\frac{J^2(\eta_*)}{J^2(\tilde{\eta})}\\ &= \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)a^{2n}(\eta_*)\int_{\eta_*}^{\eta}\frac{{\rm d}\tilde{\eta}}{a^{2n}(\tilde{\eta})},\end{aligned} \tag{21}$$

where we have set the constant of integration to be $\bar{A}_k(\eta_*)$. If we substitute the expression (10) for the scale factor, we find that the integral can be carried out for an arbitrary γ to obtain that

$$\begin{aligned}\bar{A}_k(\eta) &\simeq \bar{A}_k(\eta_*) + \bar{A}_k'(\eta_*)\frac{a^{2n}(\eta_*)}{a_0^{2n}}\\ &\quad\times\left[\eta\,{}_2F_1\left(\frac{1}{2},\gamma;\frac{3}{2};-\frac{\eta^2}{\eta_0^2}\right) - \eta_*\,{}_2F_1\left(\frac{1}{2},\gamma;\frac{3}{2};-\frac{\eta_*^2}{\eta_0^2}\right)\right],\end{aligned} \tag{22}$$

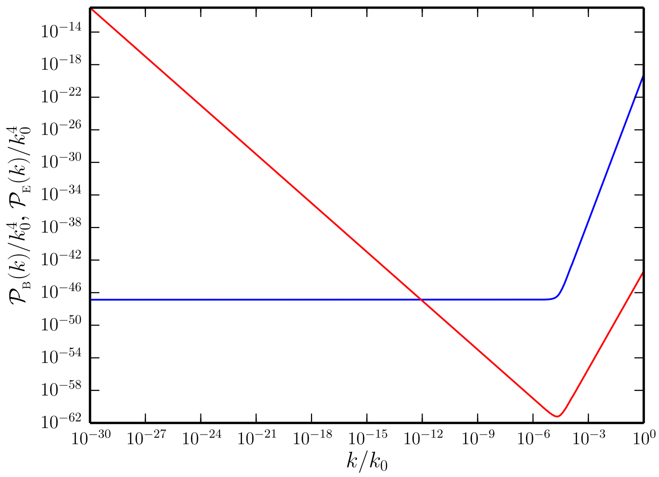

FIG. 3. The power spectra of the electric (in red) and magnetic (in blue) fields, evaluated at $\eta = \beta\eta_0$, with $\beta = 10^2$, have been plotted for the same set of values of the parameters as in Fig. 1. Note that the shape of the spectra generated before the bounce is retained for scales such that $k \ll k_0/\alpha$ even after the bounce. We should mention that the values of the parameters we have worked with lead to magnetic fields of observed strengths today corresponding to a few femtogauss. It should also be added that the electric field dominating the strength of the magnetic field is considered to be undesirable (see, for instance, Ref. [36]). This seems inevitable for positive n that we are considering here, but it can be, for example, circumvented by choosing n to be negative (in this context, see Ref. [72]).

where ${}_2F_1(a,b;c;z)$ denotes the hypergeometric function [84]. We can now choose $\eta_* = -\alpha\eta_0$ to arrive at the behavior of $\bar{A}_k(\eta)$ and $\bar{A}_k'(\eta)$ in the second domain. In such a case, we can make use of the solution (12) in the first domain to determine the values of $\bar{A}_k(\eta_*)$ and $\bar{A}_k'(\eta_*)$.

In fact, the solutions we have obtained above can be expected to be valid even after the bounce until the condition $k^2 \ll J''/J$ is violated. While the bounce is symmetric, the solution $\bar{A}_k(\eta_*)$ and its time derivative $\bar{A}_k'(\eta_*)$ are not symmetric [72]. Numerical analysis suggests that the analytical solutions will cease to be valid well before the condition $k^2 = J''/J$ is satisfied after the bounce. For this reason, we evaluate the spectra after the bounce at $\eta = \beta\eta_0$ with β chosen to be about 10^2. This choice of β can be said to roughly correspond to the time of reheating after the more conventional inflationary scenario [72]. We can now evaluate the spectra *after* the bounce at $\eta = \beta\eta_0$ using the analytic expressions for $\bar{A}_k$ and $\bar{A}_k'$ we have obtained above [cf. Eqs. (21) and (20)]. Recall that, while the power spectrum of the electric field depends on $\bar{A}_k'$, the power spectrum of the magnetic field depends on $\bar{A}_k$. Since $\bar{A}_k'$ after the bounce is related to the corresponding $\bar{A}_k'$ at the end of the first domain only by a time-dependent factor [cf. Eq. (20)], it is obvious that the shape of the electric field will not be affected by the bounce. In contrast, the quantity $\bar{A}_k$ after the bounce depends on a combination of $\bar{A}_k$ and $\bar{A}_k'$ evaluated at the end of the first domain [cf. Eq. (21)]. So, it is not immediately evident that the shape of magnetic field will be preserved across the bounce. In Fig. 3, we have plotted these spectra after the bounce. Upon comparing Figs. 1 and 3, it is clear that, while the amplitudes of the spectra change, the shapes of the spectra before and after the bounce are identical.

IV. THE ISSUE OF BACKREACTION

Note that, since the electromagnetic field is a test field, the energy density associated with it must always remain much smaller than the energy density that drives the background evolution. However, in certain cases, it is found that the energy density associated with the electromagnetic field can grow and dominate the background energy density [35,37]. This issue is regularly encountered in the context of inflation [33]. Such a situation is not viable, and the energy density associated with the background must be dominant at all times. It is therefore imperative that we examine the issue of backreaction in bouncing models. In what follows, with the analytical results at hand, we shall evaluate the energy density in the generated electromagnetic field and investigate the issue of backreaction in the bouncing scenario of our interest.

Using the Friedmann equation, the background energy density, say, $\rho_{\rm bg}$, can immediately be written as

$$\rho_{\rm bg} = 3M_{\rm Pl}^2 H^2. \tag{23}$$

Upon using the expression (10) for the scale factor, we obtain that

$$\rho_{\rm bg} = \frac{12 M_{\rm Pl}^2 q^2 \eta^2}{a_0^2 \eta_0^4 (1 + k_0^2\eta^2)^{2(q+1)}}. \tag{24}$$

The energy density in a particular mode k of the electromagnetic field is given by

$$\rho_{\rm EB}^k = \mathcal{P}_{\rm B}(k) + \mathcal{P}_{\rm E}(k). \tag{25}$$

For the effects of backreaction to be negligible, the condition $\rho_{\rm bg} > \rho_{\rm EB}^k$ must be satisfied by all modes of cosmological interest at all times. However, we find that this condition is violated in this scenario, particularly around the bounce. To illustrate this issue, we have plotted the ratio of the background energy density and the electromagnetic energy density, viz. $\rho_{\rm r} = \rho_{\rm bg}/\rho_{\rm EB}^k$. We should mention that we have evaluated the quantity $\rho_{\rm EB}^k$ from the analytic solutions we have obtained in the last section. To cover a wide range in time, one often uses the e-fold as a time variable in the context of inflation. In symmetric bouncing models, it proves to be convenient to use a new time variable $\mathcal{N}$ called the e-$\mathcal{N}$-fold which is related to the scale factor as follows [72]:

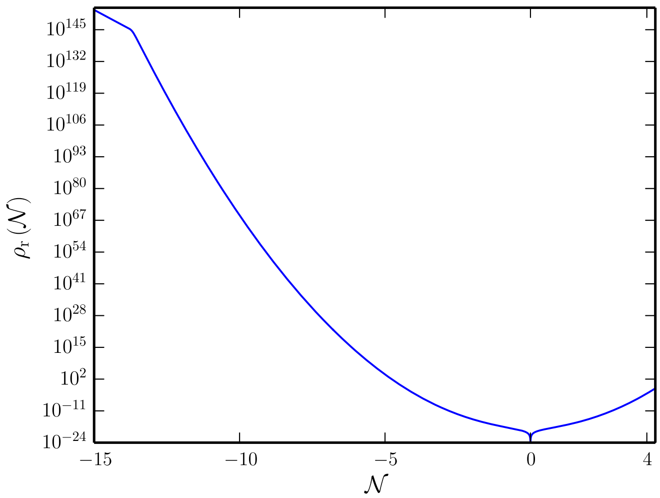

FIG. 4. The evolution of the ratio of the background energy density to the sum of the energy densities in the electric and magnetic fields for a given mode ($k = 10^{-20}k_0$) has been plotted against e-$\mathcal{N}$-folds. We should mention that we have assumed the same set of values for the various parameters as in Figs. 1 and 3. Evidently, the ratio has to remain large in order to avoid the backreaction problem. However, we find that the energy in the generated electromagnetic field rises sharply as one approaches the bounce, indicating that the problem of backreaction is the most severe at the bounce.

$$a(\mathcal{N}) = a_0 \exp(\mathcal{N}^2/2). \tag{26}$$

In Fig. 4, we have plotted the quantity ρ_r as a function of e-$\mathcal{N}$-folds. Since the Hubble parameter vanishes at the bounce, the background energy density also vanishes. Hence, any nonzero amount of electromagnetic energy density at the bounce would lead to a violation of the condition $\rho_r > 1$. We find that the condition is actually violated even as one approaches the bounce, indicating that the problem is indeed a severe one.

We had mentioned earlier that no vector perturbations arise in the absence of vector sources. Note that the evolution of metric vector perturbations depends on the behavior of the scale factor [73]. In contrast, the evolution of the electromagnetic modes is determined by the form of the coupling function J. Evidently, we do not have any vector sources in the scenario of our interest here, and, in fact, the electromagnetic modes we have considered have a quantum origin. For the form of the coupling function we have assumed here [J given by Eq. (11), with positive n], the amplitude of the generated modes indeed grows rapidly during the contracting phase close to the bounce [72]. Therefore, in this case, the issue of backreaction can be considered as a manifestation of the strong growth of the vector modes that are expected to occur as one approaches the bounce [73].

V. DUALITY INVARIANCE

The primordial scalar and tensor perturbations are governed by the so-called Mukhanov-Sasaki equations (see, for instance, Refs. [85,86]). In these cases, it can be shown that the corresponding power spectra will remain invariant under a two-parameter family of transformations of the homogeneous background quantity that determines the evolution of the perturbations (viz. the scale factor a in the case of tensor perturbations and a quantity often denoted as z in the case of scalar perturbations) [87]. The new forms of the background quantities obtained as a transformation of the original quantities are called the dual functions. For instance, the conventional slow roll solutions can lead to dual functions which may be away from the slow roll limit but still produce the same power spectra. In this section, we shall extend these duality arguments to the generation of magnetic fields.

The argument is in fact relatively simple. Equation (8) that governs the dynamics of quantity $\mathcal{A}_k$ has the same form as the Mukhanov-Sasaki equations that describe the scalar and tensor perturbations. Note that the quantity $\mathcal{A}_k$ is determined by J''/J. Evidently, the solution $\mathcal{A}_k$ to the differential equation can be expected to behave in the same fashion and, hence, lead to the same power spectrum for the magnetic field if we can construct another coupling function that leads to the same J''/J. Given a coupling function J, its dual function, say, $\tilde{J}$, which leads to the same $\tilde{J}''/\tilde{J}$, is found to be

$$J(\eta) \to \tilde{J}(\eta) = CJ(\eta)\int_{\eta_*}^{\eta} \frac{\mathrm{d}\bar{\eta}}{J^2(\bar{\eta})}, \tag{27}$$

where C and η_* are constants. These constants can be suitably chosen to arrive at a physically reasonable form for $\tilde{J}$.

Let us now construct the dual form of the coupling function (11) that we had considered. The corresponding dual solution is described by the integral

$$\tilde{J}(\eta) = \frac{C}{J_0} a^n(\eta) \int_{\eta_*}^{\eta} \frac{\mathrm{d}\bar{\eta}}{a^{2n}(\bar{\eta})}. \tag{28}$$

Let us first consider the behavior at very early times when the scale factor (10) reduces to the simple power law form. Recall that, in such a situation, the coupling function J behaves as $J(\eta) \propto \eta^{\gamma}$. In such a case, the dual function $\tilde{J}$ can be easily evaluated to be

$$\tilde{J}(\eta) = \frac{C\eta^{-\gamma+1}}{-2\gamma+1}\left(1 - \frac{\eta^{2\gamma-1}}{\eta_*^{2\gamma-1}}\right). \tag{29}$$

We are specifically interested in the cases where $\gamma = 3$ and $\gamma = -2$, as these lead to scale invariant spectra for the magnetic field. When $\gamma = 3$, we have

$$\tilde{J}(\eta) = -\frac{C}{5\eta^2}\left(1 - \frac{\eta^5}{\eta_*^5}\right), \tag{30}$$

and, if we set $\eta_* \to -\infty$, we obtain that $\tilde{J}(\eta) \propto 1/\eta^2$. Also, when $\gamma = -2$, we have

$$\tilde{J}(\eta) = \frac{C\eta^3}{5}\left(1 - \frac{\eta_*^5}{\eta^5}\right), \tag{31}$$

and, if we can choose η_* to be some large, but finite positive value, then at very early times, i.e. as $\eta \to -\infty$, we find that $\tilde{J}(\eta) \propto \eta^3$. Therefore, clearly, the coupling functions corresponding to $\gamma = 3$ and $\gamma = -2$ are dual to each other. Given that one of these two cases leads to a scale invariant spectrum for the magnetic field before the bounce, their dual nature suggests that the other, too, will lead to the same spectrum, exactly as we have seen.

Let us now construct the dual form of the coupling function using the complete scale factor (10), which we had used to model the bounce. On substituting the expression for the scale factor in Eq. (27), we find that we can write the dual coupling function $\tilde{J}(\eta)$ in terms of the hypergeometric function as follows:

$$\tilde{J}(\eta) = \frac{C}{J_0 a_0^n}\left(1 + \frac{\eta^2}{\eta_0^2}\right)^{\gamma/2}\left[\eta\, {}_2F_1\left(\frac{1}{2}, \gamma; \frac{3}{2}; -\frac{\eta^2}{\eta_0^2}\right) - \eta_*\, {}_2F_1\left(\frac{1}{2}, \gamma; \frac{3}{2}; -\frac{\eta_*^2}{\eta_0^2}\right)\right]. \tag{32}$$

This expression, though it is exact and is applicable to arbitrary γ, does not reveal the behavior of the coupling function easily. However, we find that for the cases corresponding to $\gamma = 3$ and $\gamma = -2$, $\tilde{J}(\eta)$ can be written in terms of simple functions. When $\gamma = 3$ (say, $n = 3/2$ and $q = 1$), the dual form of the coupling function can be expressed as

$$\tilde{J}(\eta) = \frac{C\eta_0}{8 J_0 a_0^{3/2}}\left(1 + \frac{\eta^2}{\eta_0^2}\right)^{3/2}\left[\frac{5(\eta/\eta_0) + 3(\eta/\eta_0)^3}{(1 + \eta^2/\eta_0^2)^2} - \frac{5(\eta_*/\eta_0) + 3(\eta_*/\eta_0)^3}{(1 + \eta_*^2/\eta_0^2)^2} + 3\tan^{-1}\left(\frac{\eta}{\eta_0}\right) - 3\tan^{-1}\left(\frac{\eta_*}{\eta_0}\right)\right]. \tag{33}$$

Note that the power spectrum for the electric field depends on the quantity J'/J [see Eq. (9b)]. Clearly, we shall require a well-behaved $\tilde{J}'/\tilde{J}$ to ensure that the electric field evolves smoothly. For this reason, it seems desirable to demand that the dual function $\tilde{J}$ does not vanish over the domain of interest. We find that, if we set $\eta_* \to -\infty$, then with a suitable choice of the constant C we can ensure that the above $\tilde{J}$ remains positive at all times. We also find that at early times, i.e. as $\eta \to -\infty$, the above $\tilde{J}$ reduces to $\tilde{J}(\eta) \propto 1/\eta^2$, as required. Let us now turn to the case $\gamma = -2$ (say, $n = -1$ and $q = 1$). In this case, the dual form of this coupling function is given by

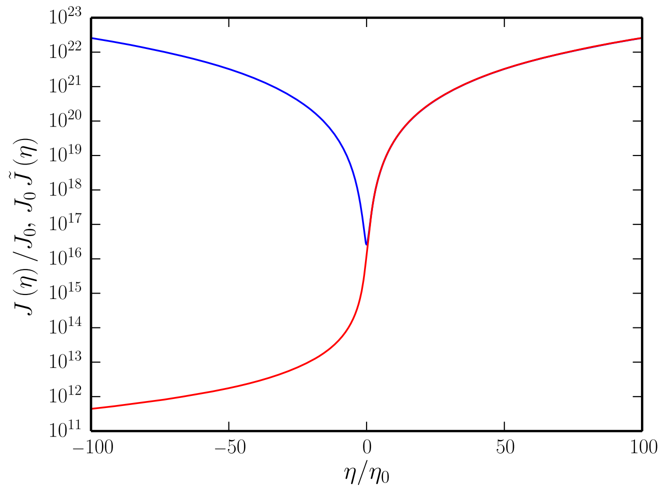

FIG. 5. The coupling function J (in blue) and its dual $\tilde{J}$ (in red) have been plotted as a function of η/η_0 for $\gamma = 3$ and $\eta_* \to -\infty$ [cf. Eq. (33)]. Also, we have chosen the constant C to be $C/k_0 = 5.7 \times 10^{32}$ so that the dual function $\tilde{J}$ matches the original coupling function J after the bounce.

$$\tilde{J}(\eta) = \frac{C a_0 \eta_0}{J_0}\left(1 + \frac{\eta^2}{\eta_0^2}\right)^{-1}\left[\frac{\eta}{\eta_0} - \frac{\eta_*}{\eta_0} + \frac{2}{3}\left(\frac{\eta^3}{\eta_0^3} - \frac{\eta_*^3}{\eta_0^3}\right) + \frac{1}{5}\left(\frac{\eta^5}{\eta_0^5} - \frac{\eta_*^5}{\eta_0^5}\right)\right]. \tag{34}$$

We find that, in such a case, if we choose η_* to be a suitably large positive value (say, $\eta > \beta\eta_0$), then we can ensure that $\tilde{J}(\eta)$ remains positive over the domain that we are interested in. Also, we should point out that, at early times, i.e. as $\eta \to -\infty$, the $\tilde{J}(\eta)$ above reduces $\tilde{J}(\eta) \propto \eta^3$, as required.

In Fig. 5, we have plotted the coupling function J and its dual $\tilde{J}$ for the case $\gamma = 3$, with a suitable choice of the parameters. Recall that our original choice for the coupling function J was symmetric about the bounce. While the dual function $\tilde{J}$ behaves in a similar fashion as J after the bounce (for a suitable choice of the constant C), we find that the dual function behaves very differently before the bounce. In fact, $\tilde{J}$ is asymmetric about the bounce. For the case of $\gamma = -2$, as we had discussed, η_* has to be chosen to be a large positive value in order to ensure that $\tilde{J}$ does not vanish, which seems to pose difficulties for the evolution of the electric field.

VI. DISCUSSION

In the present work, we have *analytically* studied the generation of primordial electromagnetic fields in a class of nonsingular and symmetric bouncing scenarios. We have assumed that the electromagnetic field is coupled nonminimally to a background scalar field which is expected to drive the bounce. Considering specific forms of the scale factor and the coupling function, we have arrived at analytical expressions for the power spectra for the electric and magnetic fields. We find that a scale invariant spectrum for the magnetic field arises before the bounce for certain values of the parameters involved, while the corresponding electric field spectrum has a certain power law scale dependence. Interestingly, we have shown that, as the modes evolve across the bounce, these shapes of the power spectra are preserved. However, a severe backreaction due to the generated electromagnetic fields seems unavoidable close to the bounce. This issue needs to be circumvented if the scenario has to be viable. We have further illustrated the existence of a two-parameter family of transformations of the original coupling function under which the spectrum of the magnetic field remains invariant. The dual transformation leads to asymmetric forms for the coupling function, and it seems to be a worthwhile exercise to explore these new forms. We are currently investigating the generation of magnetic fields in symmetric bounces with asymmetric coupling functions.

We need to emphasize a few points at this stage of our discussion. One may be concerned by the fact that the presence of radiation prior to the bounce can modify the equations of motion of the electromagnetic field which would, in turn, affect the process of magnetogenesis. We had described earlier as to how the class of bouncing models that we have considered can be driven with the aid of two fluids. We are envisaging a situation wherein such a behavior is actually achieved with the help of scalar fields. If, in addition to the scalar fields, radiation is also present before the bounce, its energy density can dominate close to the bounce, modifying the evolution of the background in the vicinity of the bounce and altering the form of the scale factor. Therefore, in our discussion, we have assumed that there is no radiation present before the bounce. We believe that, after the bounce, the scalar fields driving the bounce can decay into radiation via some mechanism (as it occurs immediately after inflation) and lead to the standard radiation-dominated epoch. However, we should add that the phenomenon of reheating in bouncing scenarios and its effects on the process of magnetogenesis is not yet well understood.

Before concluding, we would like to make a few further clarifying remarks about some of the issues concerning the bouncing models which we discussed in the introduction section. As we had described, while many bouncing models seem to lead to a large tensor-to-scalar ratio [58,67,69,76,77], it also seems possible to construct models which result in scalar and tensor power spectra that are consistent with the CMB observations [78–80]. Our aim in this work was twofold. The first was to show analytically that scale invariant magnetic fields of observable strengths can indeed be generated in bouncing scenarios. The second aim was to illustrate that, just as in the case of the scalar and the tensor perturbations, the power spectrum of the magnetic fields is invariant under a certain duality transformation. Needless to say, it is important to study the generation of magnetic fields in those specific bouncing models which satisfy the observational constraints at the level of scalar and tensor power spectra. As we pointed out earlier, for certain values of the parameters involved, the form of the scale factor that we have considered here [viz. Eq. (10)] leads to tensor power spectra that are consistent with the observations [81,82]. Evaluating the corresponding scalar power spectra requires a detailed modeling of the source that drives the bounce. We have been able to construct scalar field models that lead to such scale factors. However, we find that these scenarios require numerical efforts to arrive at the scalar power spectrum. We are currently developing codes to study these situations and compare them with the CMB data.

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[1] D. Grasso and H. R. Rubinstein, Magnetic fields in the early universe, Phys. Rep. **348**, 163 (2001).

[2] L. M. Widrow, Origin of galactic and extragalactic magnetic fields, Rev. Mod. Phys. **74**, 775 (2002).

[3] A. Kandus, K. E. Kunze, and C. G. Tsagas, Primordial magnetogenesis, Phys. Rep. **505**, 1 (2011).

[4] L. M. Widrow, D. Ryu, D. R. G. Schleicher, K. Subramanian, C. G. Tsagas, and R. A. Treumann, The first magnetic fields, Space Sci. Rev. **166**, 37 (2012).

[5] R. Durrer and A. Neronov, Cosmological magnetic fields: Their generation, evolution and observation, Astron. Astrophys. Rev. **21**, 62 (2013).

[6] K. Subramanian, The origin, evolution and signatures of primordial magnetic fields, Rep. Prog. Phys. **79**, 076901 (2016).

[7] A. Neronov and I. Vovk, Evidence for strong extragalactic magnetic fields from Fermi observations of TeV blazars, Science **328**, 73 (2010).

[8] F. Tavecchio *et al.*, The intergalactic magnetic field constrained by Fermi/LAT observations of the TeV blazar 1ES 0229+200, Mon. Not. R. Astron. Soc. **406**, L70 (2010).

[9] C. D. Dermer, M. Cavadini, S. Razzaque, J. D. Finke, and B. Lott, Time delay of cascade radiation for TeV blazars and the measurement of the intergalactic magnetic field, Astrophys. J. **733**, L21 (2011).

[10] I. Vovk, A. M. Taylor, D. Semikoz, and A. Neronov, Fermi/LAT observations of 1ES 0229+200: Implications for extragalactic magnetic fields and background light, Astrophys. J. **747**, L14 (2012).

[11] F. Tavecchio, G. Ghisellini, G. Bonnoli, and L. Foschini, Extreme TeV blazars and the intergalactic magnetic field, Mon. Not. R. Astron. Soc. **414**, 3566 (2011).

[12] K. Dolag, M. Kachelriess, S. Ostapchenko, and R. Tomas, Lower limit on the strength and filling factor of extragalactic magnetic fields, Astrophys. J. **727**, L4 (2011).

[13] A. M. Taylor, I. Vovk, and A. Neronov, Extragalactic magnetic fields constraints from simultaneous GeV-TeV observations of blazars, Astron. Astrophys. **529**, A144 (2011).

[14] K. Takahashi, M. Mori, K. Ichiki, and S. Inoue, Lower bounds on intergalactic magnetic fields from simultaneously observed GeV-TeV light curves of the blazar Mrk 501, Astrophys. J. **744**, L7 (2012).

[15] H. Huan, T. Weisgarber, T. Arlen, and S. P. Wakely, A new model for gamma-ray cascades in extragalactic magnetic fields, Astrophys. J. **735**, L28 (2011).

[16] J. D. Finke, L. C. Reyes, M. Georganopoulos, K. Reynolds, M. Ajello, S. J. Fegan, and K. McCann, Constraints on the intergalactic magnetic field with gamma-ray observations of blazars, Astrophys. J. **814**, 20 (2015).

[17] P. Ade *et al.* (Planck Collaboration), Planck 2015 results. XIX. Constraints on primordial magnetic fields, arXiv: 1502.01594.

[18] P. A. R. Ade *et al.* (POLARBEAR Collaboration), POLARBEAR constraints on cosmic birefringence and primordial magnetic fields, Phys. Rev. D **92**, 123509 (2015).

[19] M. S. Pshirkov, P. G. Tinyakov, and F. R. Urban, New Limits on Extragalactic Magnetic Fields from Rotation Measures, Phys. Rev. Lett. **116**, 191302 (2016).

[20] D. G. Yamazaki, K. Ichiki, T. Kajino, and G. J. Mathews, New constraints on the primordial magnetic field, Phys. Rev. D **81**, 023008 (2010).

[21] T. Kahniashvili, A. G. Tevzadze, S. K. Sethi, K. Pandey, and B. Ratra, Primordial magnetic field limits from cosmological data, Phys. Rev. D **82**, 083005 (2010).

[22] C. Caprini, Limits for primordial magnetic fields, *Proc. Sci.*, TEXAS20102010 (2010) 222 [arXiv:1103.4060].

[23] J. R. Shaw and A. Lewis, Constraining primordial magnetism, Phys. Rev. D **86**, 043510 (2012).

[24] D. R. G. Schleicher and F. Miniati, Primordial magnetic field constraints from the end of reionization, Mon. Not. R. Astron. Soc. **418**, L143 (2011).

[25] C. Fedeli and L. Moscardini, Constraining primordial magnetic fields with future cosmic shear surveys, J. Cosmol. Astropart. Phys. 11 (2012) 055.

[26] T. Kahniashvili, Y. Maravin, A. Natarajan, N. Battaglia, and A. G. Tevzadze, Constraining primordial magnetic fields through large scale structure, Astrophys. J. **770**, 47 (2013).

[27] K. L. Pandey, T. R. Choudhury, S. K. Sethi, and A. Ferrara, Reionization constraints on primordial magnetic fields, Mon. Not. R. Astron. Soc. **451**, 1692 (2015).

[28] M. S. Turner and L. M. Widrow, Inflation produced, large scale magnetic fields, Phys. Rev. D **37**, 2743 (1988).

[29] B. Ratra, Cosmological "seed" magnetic field from inflation, Astrophys. J. **391**, L1 (1992).

[30] K. Bamba and J. Yokoyama, Large scale magnetic fields from inflation in dilaton electromagnetism, Phys. Rev. D **69**, 043507 (2004).

[31] K. Bamba and M. Sasaki, Large-scale magnetic fields in the inflationary universe, J. Cosmol. Astropart. Phys. 02 (2007) 030.

[32] V. Demozzi, V. Mukhanov, and H. Rubinstein, Magnetic fields from inflation?, J. Cosmol. Astropart. Phys. 08 (2009) 025.

[33] J. Martin and J. Yokoyama, Generation of large-scale magnetic fields in single-field inflation, J. Cosmol. Astropart. Phys. 01 (2008) 025.

[34] L. Campanelli, Helical magnetic fields from inflation, Int. J. Mod. Phys. D **18**, 1395 (2009).

[35] S. Kanno, J. Soda, and M.-a. Watanabe, Cosmological magnetic fields from inflation and backreaction, J. Cosmol. Astropart. Phys. 12 (2009) 009.

[36] K. Subramanian, Magnetic fields in the early universe, Astron. Nachr. **331**, 110 (2010).

[37] F. R. Urban, On inflating magnetic fields, and the backreactions thereof, J. Cosmol. Astropart. Phys. 12 (2011) 012.

[38] R. Durrer, L. Hollenstein, and R. K. Jain, Can slow roll inflation induce relevant helical magnetic fields?, J. Cosmol. Astropart. Phys. 03 (2011) 037.

[39] C. T. Byrnes, L. Hollenstein, R. K. Jain, and F. R. Urban, Resonant magnetic fields from inflation, J. Cosmol. Astropart. Phys. 03 (2012) 009.

[40] R. K. Jain, R. Durrer, and L. Hollenstein, Generation of helical magnetic fields from inflation, J. Phys. Conf. Ser. **484**, 012062 (2014).

[41] T. Kahniashvili, A. Brandenburg, L. Campanelli, B. Ratra, and A. G. Tevzadze, Evolution of inflation-generated magnetic field through phase transitions, Phys. Rev. D **86**, 103005 (2012).

[42] S.-L. Cheng, W. Lee, and K.-W. Ng, Inflationary dilaton-axion magnetogenesis, Chin. J. Phys. **53**, 110105 (2015).

[43] K. Bamba, Generation of large-scale magnetic fields, non-Gaussianity, and primordial gravitational waves in inflationary cosmology, Phys. Rev. D **91**, 043509 (2015).

[44] T. Fujita, R. Namba, Y. Tada, N. Takeda, and H. Tashiro, Consistent generation of magnetic fields in axion inflation models, J. Cosmol. Astropart. Phys. 05 (2015) 054.

[45] L. Campanelli, Lorentz-violating inflationary magnetogenesis, Eur. Phys. J. C **75**, 278 (2015).

[46] T. Fujita and R. Namba, Pre-reheating magnetogenesis in the kinetic coupling model, Phys. Rev. D **94**, 043523 (2016).

[47] C. G. Tsagas, Causality, initial conditions, and inflationary magnetogenesis, Phys. Rev. D **93**, 103529 (2016).

[48] R. J. Ferreira, R. K. Jain, and M. S. Sloth, Inflationary magnetogenesis without the strong coupling problem, J. Cosmol. Astropart. Phys. 10 (2013) 004.

[49] R. J. Ferreira, R. K. Jain, and M. S. Sloth, Inflationary magnetogenesis without the strong coupling problem II: Constraints from CMB anisotropies and B-modes, J. Cosmol. Astropart. Phys. 06 (2014) 053.

[50] F. Finelli and R. Brandenberger, On the generation of a scale invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase, Phys. Rev. D **65**, 103522 (2002).

[51] J. Martin, P. Peter, N. Pinto-Neto, and D. J. Schwarz, Passing through the bounce in the ekpyrotic models, Phys. Rev. D **65**, 123513 (2002).

[52] S. Tsujikawa, R. Brandenberger, and F. Finelli, On the construction of nonsingular pre—big bang and ekpyrotic cosmologies and the resulting density perturbations, Phys. Rev. D **66**, 083513 (2002).

[53] P. Peter and N. Pinto-Neto, Primordial perturbations in a non singular bouncing universe model, Phys. Rev. D **66**, 063509 (2002).

[54] J. Martin and P. Peter, Parametric amplification of metric fluctuations through a bouncing phase, Phys. Rev. D **68**, 103517 (2003).

[55] J. Martin and P. Peter, On the Causality Argument in Bouncing Cosmologies, Phys. Rev. Lett. **92**, 061301 (2004).

[56] P. Peter, N. Pinto-Neto, and D. A. Gonzalez, Adiabatic and entropy perturbations propagation in a bouncing universe, J. Cosmol. Astropart. Phys. 12 (2003) 003.

[57] J. Martin and P. Peter, On the properties of the transition matrix in bouncing cosmologies, Phys. Rev. D **69**, 107301 (2004).

[58] L. E. Allen and D. Wands, Cosmological perturbations through a simple bounce, Phys. Rev. D **70**, 063515 (2004).

[59] T. Battefeld, S. P. Patil, and R. Brandenberger, Non-singular perturbations in a bouncing brane model, Phys. Rev. D **70**, 066006 (2004).

[60] P. Creminelli, A. Nicolis, and M. Zaldarriaga, Perturbations in bouncing cosmologies: Dynamical attractor versus scale invariance, Phys. Rev. D **71**, 063505 (2005).

[61] G. Geshnizjani, T. J. Battefeld, and G. Geshnizjani, A note on perturbations during a regular bounce, Phys. Rev. D **73**, 048501 (2006).

[62] L. R. Abramo and P. Peter, K-Bounce, J. Cosmol. Astropart. Phys. 09 (2007) 001.

[63] P. Peter and N. Pinto-Neto, Cosmology without inflation, Phys. Rev. D **78**, 063506 (2008).

[64] F. T. Falciano, M. Lilley, and P. Peter, A classical bounce: Constraints and consequences, Phys. Rev. D **77**, 083513 (2008).

[65] A. Cardoso and D. Wands, Generalised perturbation equations in bouncing cosmologies, Phys. Rev. D **77**, 123538 (2008).

[66] M. Novello and S. E. P. Bergliaffa, Bouncing cosmologies, Phys. Rep. **463**, 127 (2008).

[67] D. Battefeld and P. Peter, A critical review of classical bouncing cosmologies, Phys. Rep. **571**, 1 (2015).

[68] R. Brandenberger and P. Peter, Bouncing cosmologies: Progress and problems, arXiv:1603.05834.

[69] Y.-F. Cai, Exploring bouncing cosmologies with cosmological surveys, Sci. China Phys. Mech. Astron. **57**, 1414 (2014).

[70] J. Salim, N. Souza, S. E. P. Bergliaffa, and T. Prokopec, Creation of cosmological magnetic fields in a bouncing cosmology, J. Cosmol. Astropart. Phys. 04 (2007) 011.

[71] F. A. Membiela, Primordial magnetic fields from a non-singular bouncing cosmology, Nucl. Phys. **B885**, 196 (2014).

[72] L. Sriramkumar, K. Atmjeet, and R. K. Jain, Generation of scale invariant magnetic fields in bouncing universes, J. Cosmol. Astropart. Phys. 09 (2015) 010.

[73] T. J. Battefeld and R. Brandenberger, Vector perturbations in a contracting universe, Phys. Rev. D **70**, 121302 (2004).

[74] F. Finelli, P. Peter, and N. Pinto-Neto, Spectra of primordial fluctuations in two-perfect-fluid regular bounces, Phys. Rev. D **77**, 103508 (2008).

[75] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2015 results. XX. Constraints on inflation, arXiv:1502.02114.

[76] Y.-F. Cai, J. Quintin, E. N. Saridakis, and E. Wilson-Ewing, Nonsingular bouncing cosmologies in light of BICEP2, J. Cosmol. Astropart. Phys. 07 (2014) 033.

[77] Y.-F. Cai, T. Qiu, R. Brandenberger, and X. Zhang, A nonsingular cosmology with a scale-invariant spectrum of cosmological perturbations from Lee-Wick theory, Phys. Rev. D **80**, 023511 (2009).

[78] Y.-F. Cai, R. Brandenberger, and X. Zhang, The matter bounce curvaton scenario, J. Cosmol. Astropart. Phys. 03 (2011) 003.

[79] Y.-F. Cai, D. A. Easson, and R. Brandenberger, Towards a nonsingular bouncing cosmology, J. Cosmol. Astropart. Phys. 08 (2012) 020.

[80] Y.-F. Cai, E. McDonough, F. Duplessis, and R. H. Brandenberger, Two field matter bounce cosmology, J. Cosmol. Astropart. Phys. 10 (2013) 024.

[81] D. Chowdhury, V. Sreenath, and L. Sriramkumar, The tensor bi-spectrum in a matter bounce, J. Cosmol. Astropart. Phys. 11 (2015) 002.

[82] D. J. Stargen and V. Sreenath, Quantum-to-classical transition and imprints of wavefunction collapse in bouncing universes, arXiv:1605.07311.

[83] S. Unnikrishnan and L. Sriramkumar, A note on perfect scalar fields, Phys. Rev. D **81**, 103511 (2010).

[84] Wolfram Research, Inc., Mathematica, Version 8.0. Champaign, IL, 2010, https://www.wolfram.com/mathematica/.

[85] L. Sriramkumar, An introduction to inflation and cosmological perturbation theory, arXiv:0904.4584.

[86] J. Martin, The observational status of cosmic inflation after Planck, arXiv:1502.05733.

[87] D. Wands, Duality invariance of cosmological perturbation spectra, Phys. Rev. D **60**, 023507 (1999).