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Directivity based noise control for an obliquely incident transmission problem

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Abstract

The objective of the present work is to improve the transmission loss offered by a barrier separating two acoustic spaces. The barrier is subjected to a low frequency obliquely incident acoustic pressure over one side. A novel concept of achieving this is proposed through addition of point mass at the optimal location over the barrier surface. This allows for a local control of the transmitted noise at the target location through a control on the directivity pattern. It is noted that for some target directions high reduction in sound pressure is possible as compared to the unloaded plate whereas, for other target directions only marginal reduction in sound pressure is achievable by point mass attachment. This is in contrast to the normal incidence transmission problem where it is equally possible to achieve noise control in any direction *viz.* normal, tangential and oblique direction to the plate by attachment of a point mass at the optimal location on plate.

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Keywords: Barrier design; Acoustic directivity control; Local noise control; Design optimization.

1. Introduction

Noise control from an acoustic source, as we understand, could be of two types i.e. global and local. Global noise control aims to reduce noise at all the receiver locations over the source, whereas, local noise control aims to minimize noise at some specific receiver location(s). It is well known that, in general passive techniques for global noise control work well for high frequency vibration. Active control techniques span the low and medium frequency

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vibration [1]. However, active control techniques are costly, difficult to implement and work efficiently for a narrow frequency band. Thus, in such a scenario local control of noise at specified receiver locations could be an alternative. The significance of local noise control lies in the fact that in various applications the objective of a noise control engineer is to minimize the audible noise at some specific locations. For example, in automotive application the target is to reduce the noise at the driver and the passenger ear locations. It is acceptable even if this implies increased noise levels at non-critical locations. In this work, the possibility of redistributing the available acoustic energy with aim to reduce acoustic levels in target direction is studied for obliquely incident transmission problem.

The study of acoustic directivity has received wide attention in the literature from various perspectives. Chatillon[2] studied the influence of source directivity in noise levels inside an industrial hall through simulation and experiment. Noise level was reported to be changed by 4 dB with a change in the source directivity. Zhang [3] observed different horizontal and vertical directivity pattern of railway noise through measurement. Though in the above works aim was not to control the directivity pattern, there are other papers in which the aim was to control directivity for different applications. Juarez et al.[4] developed an acoustic transducer system for long distance echo-ranging application. Directivity controlled piezo-electric transducer for sound reproduction was developed by Bedard and Berry [5]. Seo and Kim [6] studied directional radiation pattern in structural acoustic coupled system. Three typical directivity patterns i.e. steered, focused and omnidirected were presented. Thus, numerous literature is available on the study and control of directivity. However, the use of directivity of the transmitted noise in achieving noise control at specific target directions from the source has not been reported in the literature. The objective of the present work is to demonstrate the above concept through calculations for a simple transmission problem - namely a square simply supported plate on an infinite rigid baffle. The directivity control is achieved by point mass attachments on to the structure. Wu et al.[7] combined analytical and numerical methods for free vibration analysis of a rectangular plate with any number of point masses and springs. The effect of distributed mass on acoustic radiation behavior of plates was studied by Li et al.[8]. It was reported that the effect of mass on the acoustic radiation of the plate was large when the mass loading was concentrated. Thus, in the present work we have preferred the attachment of point mass over distributed mass. Also, it is easier to work with point masses than distributed masses. It was also reported that the effect of mass loading on the acoustic radiation from the plate is large when it was placed in air than in water. In present work the acoustic medium over the plate is assumed to be air. This makes analytical formulation easier by avoiding the effect of fluid-structure interaction.

Thus, voluminous literature is available on the study of the vibration response and acoustic radiation from mass loaded structures. Sharma et al.[9, 10] studied the concept of directivity based control for local noise control at selected receiver direction over a simply supported square plate on rigid baffle. It was reported that with minimal mass addition (20% of the plate mass) at the optimal location, the sound pressure at the target location can be reduced by 3 orders of magnitude as compared to the unloaded plate. Sharma et al.[11] presented novel method of barrier design with the aim to reduce the transmitted noise at selected target locations over simple planar structure subjected to acoustic pressure excitation in the normal direction. Reduction in sound pressure up to 4 orders of magnitude, as compared to the unloaded plate, was reported. In this work, the concept of directivity based noise control is explored for a plate subjected to an oblique incidence acoustic pressure.

2. Model description and analysis methodology

A simplified model is chosen for demonstration of novel directivity based local noise control. Figure 1 shows the schematic of the model under analysis. It consists of a square plate, simply supported on all four sides, separating two acoustic spaces. The region $z > 0$ is considered as excitation side whereas, $z < 0$ is assumed as the transmission side. The plate is mounted on an infinite rigid baffle. The baffle assumption allows easy computation based on the Rayleigh integral formula [12]. The plate is harmonically excited by a low frequency acoustic plane wave over its one side. The incident acoustic pressure is inclined in both polar and azimuthal directions to the plate. It is to be noted that transmission involves radiation in the half space $z < 0$.

In this work, the response of the unloaded and mass loaded plate subjected to the harmonic acoustic excitation is obtained using mode-superposition technique. It is noticed that with 50 modes of the unloaded plate modal series solution converges. Hence, 50 modes have been used to compute the vibration response of both unloaded plate and mass-loaded plate. The normal surface velocity is calculated from the obtained response which is used to calculate the sound pressure at a receiver location and hence the directivity pattern. Finally, the optimal location of mass to minimize the sound radiation at a receiver location is obtained using a numerical optimization routine on MATLAB. The possibility of local noise control at a receiver direction in normal, tangential and oblique to the barrier is explored.

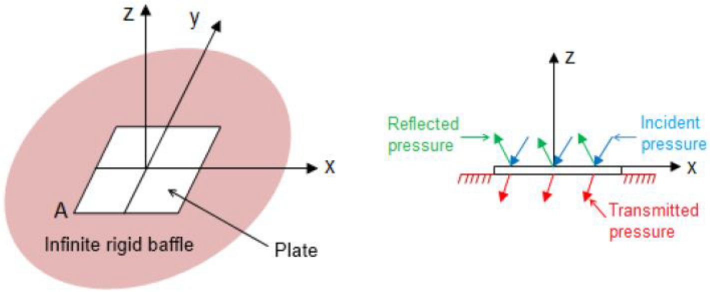

Fig 1. Schematic of a simply-supported plate mounted on a baffle and excited by plane acoustic wave over one side.

The principle of directivity control by point mass attachment relies on the fact that each mode of any structure has different radiation characteristics. It is also well known that the harmonic forced response of a structure can be obtained as a sum of its modes [13]. Thus, it is inferred that directivity pattern can be altered by altering the modal contribution. This can in turn be achieved by attaching points mass(es) on the structure. Again, the forced response of the mass-loaded structure can be decomposed in terms of the modes of the unloaded structure. This is because the effect of local mass addition can be effectively modeled as an additional frequency dependent point force on the unloaded structure. Thus, the contributions of the modes of the baseline structure can be controlled by appropriate placement of the point mass over the structure.

3. Formulation

The analytical formulation to compute vibration and acoustic response of a simply supported plate on an infinite rigid baffle and subjected to obliquely incident plane acoustic wave is presented in this section.

3.1. Vibration response

The response of any structure to a harmonic excitation can be obtained as a weighted sum of the modes of the structure as [13]

$$w(x,y,t) = \sum_i \phi_i(x,y) q_i(t) \tag{1}$$

In the above equation, $w(x, y, t)$ is the displacement response at location (x, y), q_i's are the mode participation factors and ϕ_i's are the modes of the structure. In this work, the vibration response of a plate loaded with a point mass is derived using the modes of the unloaded plate. This is accomplished by considering the additional point mass as a frequency dependent force. The response of the unloaded plate can be obtained by simply substituting the magnitude of the point mass to zero.

For a plate loaded with a point mass m_0 at (r, s) and excited by a harmonic plane acoustic wave on one side of it, the values of generalized coordinates (q_i's) can be obtained by from the following equation

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) = \frac{1}{M_i}\left(\int_S \{p_i(x,y,t) + p_r(x,y,t) - p_t(x,y,t)\}\phi_i(x,y)dxdy + F(t)\phi_i(r,s)\right)$$

In the above equation, M_i is the modal mass, $F(t)$ is the force exerted by the point mass on the plate, p_i, p_r and p_t are the incident, reflected and transmitted pressure, respectively. These three pressures can be written as the sum of blocked pressure (the pressure that occur on the incident side when the plate is considered as rigid wall) and the re-radiated pressure. The blocked pressure is two times of the incident pressure ($p_b(x,y) = 2p_i(x,y)$). The re-radiated pressure is negligible as compared to the blocked pressure [15]. Substituting the values of the force exerted by the point mass on the structure and oblique incidence pressure, the above equation can be rewritten as

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) = \frac{1}{M_i}\left(\int_S \sum_m \sum_n p_{mn}\left(\sin(\frac{m\pi x}{l})\sin(\frac{n\pi y}{l})\right)^2 dxdy + m_0\phi_i(r,s)\sum_j \ddot{q}_j\phi_j(r,s)\right) \quad (2)$$

In the above equation, m and n define the modes and p_{mn} is the generalized pressure given by

$$p_{mn} = \frac{8P_i}{l^2}\int_{x=0}^{l}\int_{y=0}^{l} \exp[-ik\sin\theta_i(x\cos\phi_i + y\sin\phi_i)]\sin(\frac{m\pi x}{l})\sin(\frac{n\pi y}{l})dxdy$$

where, P_i is the amplitude of the incident pressure. Since the excitation is harmonic, Eq. (2) can be solved by substituting $q_i = q_i e^{i\omega t}$. The equation is solved for q_i 's and normalized as [11]

$$q_{i\ non-dim} = \frac{\bar{q}_i}{l} = \frac{m_0(\omega/\omega_1)^2}{M((\omega_i/\omega_1)^2 - (\omega/\omega_1)^2)}\left(\frac{p_{mn}l}{4m_0\omega_1^2}\left(\frac{\omega_1}{\omega}\right)^2 + \phi_i(r,s)\sum_j \frac{\bar{q}_j}{l}\phi_j(r,s)\right) \quad (3)$$

In the above equation, ω_1 is the fundamental structural natural frequency of the plate. The parameters for the vibration response are (1) Ratio of excitation frequency to first structural natural frequency (ω/ω_1) (2) Non-dimensional force ($P_i l/4m_0\omega_1^2$) (3) Percentage of plate mass added as point mass (m/M_0).

3.2. Transmitted sound

The sound radiation from planar radiator lying on an infinite rigid baffle is given by Rayleigh integral as [12]

$$p(r,t) = \frac{j\omega\rho_0}{2\pi}e^{j\omega t}\int_S \frac{\tilde{v}_n(r_s)e^{-jkR}}{R}dS$$

where k is the acoustic wave number given by ω/c, r is the position vector of the observation point, r_s is the position vector of the elemental surface δS having normal velocity amplitude $v_n(r_s)$ and R is the magnitude of the vector r - r_s : $R = |r - r_s|$. Since the point forcing is harmonic the plate displacement will also be harmonic, the surface velocity is obtained from displacement as $v_n = j\omega w(x,y,t)$. Thus, from the displacements computed in the previous section normal velocity to the plate surface can be obtained. The obtained velocity is used in Rayleigh integral formula to calculate sound pressure at a receiver location. Such sound pressure calculation is done for various receiver locations on a hemispherical grid to obtain the *directivity pattern.* The Rayleigh integral is non-dimensionalized and dimensionless pressure is calculated as

$$\frac{p}{\rho_0 c l \omega_1} = \frac{-k^2 l^2}{2\pi}\left(\frac{\omega}{\omega_1}\right) e^{j\omega t} \int_{S'} \bar{w}\frac{e^{-jkr}}{kr} d\bar{x} d\bar{y}$$

where *S'* represents integration over unit area. The term ($p/\rho_0 c l \omega_1$) is defined as non-dimensional pressure. The parameters for non-dimensional pressure are (1) Ratio of excitation frequency to the first natural frequency of the structure (ω/ω_1) (2) Non-dimensional frequency *kl* (3) Ratio of receiver distance by the length of the plate (*r/l).*

4. Results and discussion

Though the formulation for vibration response and sound radiation is generalized, the results in this paper are presented for a fixed value of the parameters as shown in Table 1. The incident acoustic wave is assumed to be inclined in both polar and azimuthal direction to the plate by π/4 ($\theta_i = \phi_i$ = π/4).

Table 1. Non-dimensional parameters used for vibration and acoustic response

S#	Parameter description	Value
1	Ratio of excitation frequency to first natural frequency (ω/ω_1)	2
2	Non-dimensional force ($P_i l/m_0\omega^2$)	1.61e-5
3	Magnitude of attached mass (m_0/M)	0.2
4	Non-dimensional frequency (*kl*)	1.84
5	Ratio of receiver distance by plate length (*r/l)*	10

4.1. Structural Vibration Response

The response of the unloaded plate and a mass loaded plate subjected to oblique acoustic excitation is presented in this section. For an oblique incident pressure the phase of the input pressure varies spatially over the plate (refer Eq. (4)). This is in contrast to the normally incident acoustic pressure wherein the pressure loading on the plate surface is in phase [11]. In case of oblique incidence, there is a phase difference to be accounted in the pressure load acting over the plate surface. The modal forcing (p_{mn}) in such a case is complex (refer Eq. (6)). Since the excitation is complex the mode participation factors obtained after solving Eq. (8) will also be complex. Hence, response is in present case is complex. This indicates that different modes are present with different phase. The response of the unloaded and mass loaded plate is obtained by analytical formula using mode-summation procedure as discussed earlier. The real and imaginary part of the response of the unloaded and mass-loaded plate (*x/l* = l, *y*/*l* = l) subjected to an oblique acoustic excitation is shown in Figure 2 and Figure 3, respectively. The x and y limits of response plots represents the length of the plate, normalized to unity for the purpose of plotting. This same convention will be used for subsequent response plots in this article.

4.2. Optimization of the Point Mass Location

In this section, the optimal location of the point mass over the plate to minimize sound transmission along a target direction is obtained using numerical optimization technique. Since large number of modes is required for response calculation analytical optimization is not amenable. The design variables in this optimization exercise are the x and y location of the point mass over the plate. While performing the optimization all structural and acoustic parameters are held constant. The constraints in this exercise is that the x and y location of the point mass should fall within the plate area. A constrained numerical optimization is performed using *fmincon* function in MATLAB. The following cases are presented for directivity based control of acoustic transmission

- Case 1 – Unloaded plate.

- Case 2 – Optimally loaded plate to minimize sound transmission in the normal direction to the plate.
- Case 3 – Optimally loaded plate to minimize sound transmission in a tangential direction to the plate.
- Case 4 – Optimally loaded plate to minimize sound transmission in an oblique direction to the plate.

The optimal location of the point mass over the structure for Case 2, 3 and 4 are shown in Table 3.

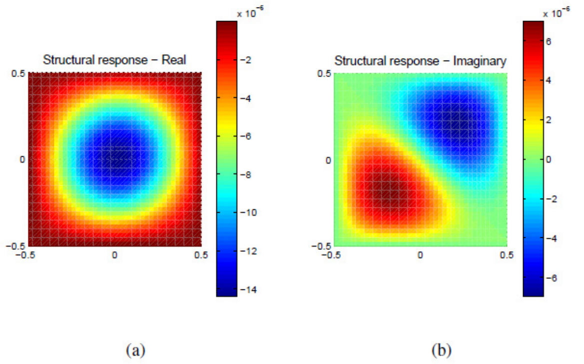

Fig 2. Response of the unloaded plate to acoustic excitation inclined in both polar and azimuthal directions (a) Real Part (b) Imaginary part.

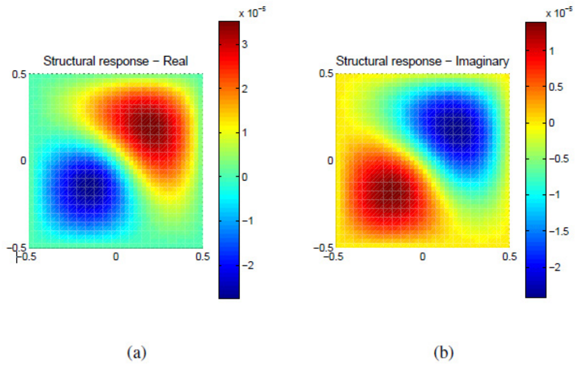

Fig 3. Response of a mass-loaded plate to acoustic excitation inclined in both polar and azimuthal directions (a) Real Part (b) Imaginary part.

Table 2. Optimal location of the point mass for different cases under study.

S#	X location (x/l)	Y location (y/l)
Case 2	0.0609	-0.0600
Case 3	0.3323	0.3692
Case 4	-0.0160	-0.0499

The real and imaginary part of structural displacement response and directivity pattern of the unloaded plate and a plate loaded with point mass to minimize sound transmission in the normal direction to the plate are shown in Figure 4. The x and y limits of the directivity plot represents the diameter of the imaginary hemisphere. The radius of the hemisphere is normalized to unity for plotting. This same convention will be followed for presenting the results for all subsequent cases.

Sound pressure along the target direction from the optimally loaded plate is reduced by four orders of magnitude as compared to that from the unloaded plate. The attained decrease in the sound transmission is attributed to the phase cancellation between regions of positive and negative vibration response. These regions of positive and negative response are created due to attachment of the point mass at the optimal location, forming a nodal line. In the present case the sound pressure is reduced in a very narrow region around the target receiver location. This is in contrast to the problems studied in [10 and 11] where the sound pressure was reduced at all the points above the nodal line. It was also noticed that uniform distribution of the point mass over the plate by increasing the thickness or density of the plate does not result in any major reduction on the transmitted noise.

The numerical values of the complex mode participation factor for first 4 modes for both these cases are shown in Table 3. On comparing the absolute values, it is noted that for the unloaded plate the dominant contribution is due to the first mode whereas for the mass loaded plate the dominant contribution is from the second and the third. The first model of a SS plate results in omni-directional radiation pattern in low frequency, whereas, the second and third mode have highly directional radiation pattern. This results in omni-directional radiation pattern for the unloaded plate and directive transmission pattern for the mass loaded plate.

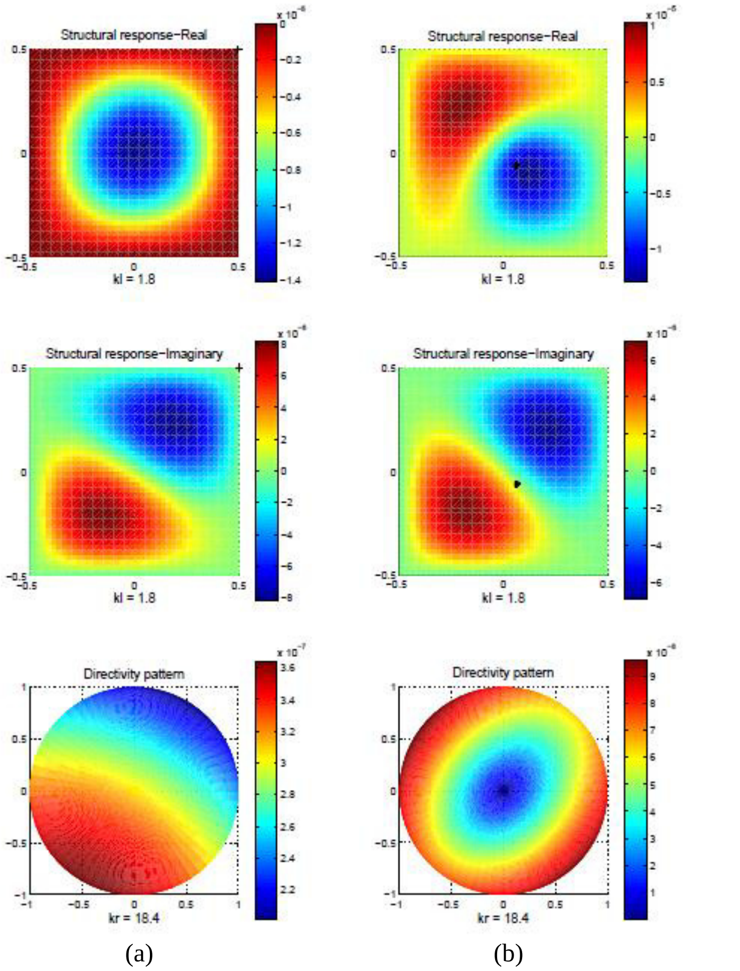

Fig 4. Response and directivity pattern of (a) the unloaded plate (b) a plate loaded with point mass to minimize transmitted noise in the normal direction to the plate.

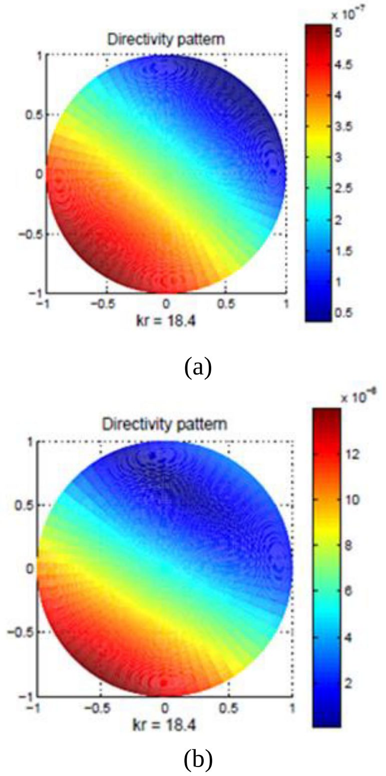

Fig 5. Directivity pattern of a plate loaded with point to minimize sound transmission in (a) tangential direction to the plate (b) oblique direction to the plate.

Table 3. Mode participation factor of first 4 modes to calculate response of the unloaded and mass-loaded plate (Case 2).

Mode#	Unloaded plate ($x10^{-4}$)	Mass loaded plate ($x10^{-5}$)
1	-0.1327+0i	-0.1373+0.002i
2	0+0.0455i	0.6697+0.4570i
3	0+0.0455i	-0.6596+0.4547i
4	-0.0018+0i	-0.02593+0i

The directivity pattern for case 3 and case 4 are shown in Figure 5. It was noticed that transmitted sound pressure for case 3 was reduced by 1 order of magnitude as compared to the unloaded plate. The reason behind this is poor phase cancellation of noise in the target direction due to asymmetry (oblique excitation). For case 4 again 4 orders of reduction in the sound pressure was observed. This unequal reduction in sound pressure for different target direction for oblique incidence transmission problem is in contrast to the normal incidence problem where it is equally possible to achieve sound reduction in any direction due to symmetry [11].

5. Conclusions

In this paper, the possibility of controlling acoustic directivity pattern of a simply-supported plate, subjected to oblique incidence acoustic pressure, for the benefit of local improvement of transmission loss is presented. Due to oblique incidence of acoustic pressure, there is a phase difference in the pressure loading acting on the plate surface which results in complex structural vibration response. The possibility of noise control along normal, tangential and oblique directions to the plate is presented by attachment of lumped mass at the optimal location. It is found that phase cancellation rather than vibration reduction to be the reason behind reduction in sound transmission along the target direction. This phase cancellation between regions of positive and negative vibration response is created due to mass placement at the optimal location forming a nodal line. In this work, we are essentially decreasing the contribution of monopoles and increasing the dipole contribution to obtain directive transmission characteristics. However, the transmission loss obtained varies (with orders of magnitude) between different directions. This implies phase cancellation is applicable to varying effect along different directions. This is in contrast to the case of normal incidence wherein all target directions show a possibility of phase cancellation. Moreover, in contrast to the problems studied in [10, 11] where sound pressure is reduced at all the points above the nodal line, in the present case no such effect is observed. The reason behind this is that due to complex response the phenomenon of phase cancellation is complicated. The uniform distribution of the point mass is found to result in a meagre reduction in the acoustic transmission and hence is an ineffective way of mass addition to control acoustic transmission in a target direction.

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