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Procedia Engineering 144 (2016) 414 – 424

Procedia Engineering

www.elsevier.com/locate/procedia

12th International Conference on Vibration Problems, ICOVP 2015

Vibration Control of Frame structure

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Abstract

The in-plane vibration analysis of frames connected at an arbitrary angle is performed using wave propagation method as proposed in [1]. The earlier work done by Mei in [1] and [2] were limited to frames having orthogonally connected members. In the present work, the procedure is extended for interconnecting members at an arbitrary angle. The wave propagation based method used in this work is computationally efficient compared to finite element method. Thus, optimal angular orientation of the intermediate member is found for minimal vibration transmission from the source to the target structure.

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Keywords: Wave propagation method; Finite Element Method, separated by semicolons; frames; optimization

1. Introduction

Planar frames comprise a wide variety of engineering structures. Many applications of frames have interconnecting members at an arbitrary angle. Though, numerical solution for such structures is routine, finding the exact solution for the vibration analysis of such frames is a challenging task.

Mei [1, 2] formulated the vibration analysis of such planar frames which considers the coupling effect of both transverse and longitudinal vibration with members connected at right angles. This approach uses wave propagation method for solving the equilibrium and continuity equation at the joint. In the present work, this approach is extended to frames which are connected at an angle. It is found that the wave approach is far more computationally efficient than Finite element Analysis as implemented in commercial software such as ANSYS. Owing to the

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Peer-review under responsibility of the organizing committee of ICOVP 2015
doi:10.1016/j.proeng.2016.05.151

efficacy of the wave-based approach, it can be used in structural optimization. In the present work, this is demonstrated for arriving at an optimal angular orientation between a one dimensional source and target structure. The objective in the optimization exercise is to minimize the vibration at the target structure for a point harmonic forcing applied at the source structure.

2. Wave Propagation Method

2.1. Governing Equations

Euler Bernoulli theory is used to model transverse vibrations and classical linear rod theory is used to model longitudinal vibration. The equations of motion for bending and longitudinal vibrations are

$$EI\left(\frac{\partial^4 y(x,t)}{\partial x^4}\right) + \rho A\left(\frac{\partial^2 y(x,t)}{\partial t^2}\right) = q(x,t) \tag{1}$$

$$\rho A\left(\frac{\partial^2 u(x,t)}{\partial t^2}\right) - EA\left(\frac{\partial^2 u(x,t)}{\partial x^2}\right) = p(x,t) \tag{2}$$

Where x is the position along the axis, t is time, $y(x,t)$ and $u(x,t)$ are transverse and longitudinal deflections, respectively, $q(x,t)$ and $p(x,t)$ are externally applied transverse and longitudinal forces, respectively, E and ρ are Young's modulus and density, respectively, I is the moment of inertia of the cross section, A is the cross-sectional area.

Suppressing harmonic time dependence ($e^{i\omega t}$), solution to equation (1) can be written as

$$y(x) = a_1^+ e^{-ik_1 x} + a_2^+ e^{-k_2 x} + a_1^- e^{ik_1 x} + a_2^- e^{k_2 x} \tag{3}$$

Where $k_1 = k_2 = \sqrt[4]{\rho A \omega^2 / EI}$ (4)

a_1^+ = amplitude of propagating flexural wave in + x direction
a_1^- = amplitude of propagating flexural wave in − x direction
a_2^+ = amplitude of near field flexural wave in + x direction
a_2^- = amplitude of near field flexural wave in − x direction

Solution to equation (2) can be written as

$$y(x) = c^+ e^{-ik_3 x} + c^- e^{ik_3 x} \tag{5}$$

Where $k_3 = \sqrt{\frac{\rho}{E}}\omega$ (6)

$c^+ = amplitude\ of\ propagating\ longitudinal\ wave\ in + x\ direction$
$c^- = amplitude\ of\ propagating\ longitudinal\ wave\ in - x\ direction$

2.2. Propagation Matrix

Consider a wave moving from point A to B. If there is no discontinuity and there is a uniform structural element between these two points, the waves at these points are related by a propagation matrix as given in [1]. Waves at A and B are related as $b^- = f(x)a^-;\ b^+ = f(x)a^+$, where

$$f(x) = \begin{bmatrix} e^{-ik_1 x} & 0 & 0 \\ 0 & e^{-k_2 x} & 0 \\ 0 & 0 & e^{-ik_3 x} \end{bmatrix}$$

is the propagation matrix.

a^+, b^+ are forward moving wave vectors and a^-, b^- are backward moving wave vectors, which contains transverse propagating, transverse near field and longitudinal propagating components.

$$a^+ = \begin{bmatrix} a_1^+ \\ a_2^+ \\ c^+ \end{bmatrix}; a^- = \begin{bmatrix} a_1^- \\ a_2^- \\ c^- \end{bmatrix}; \; b^+ = \begin{bmatrix} b_1^+ \\ b_2^+ \\ d^+ \end{bmatrix}; \; b^- = \begin{bmatrix} b_1^- \\ b_2^- \\ d^- \end{bmatrix}.$$

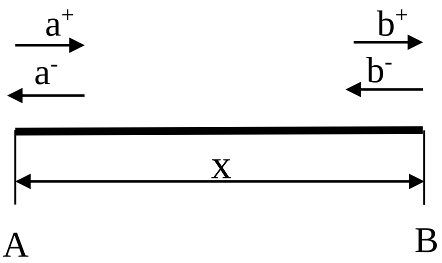

Figure 1. Waves in uniform beam.

2.3. Equilibrium conditions at joint

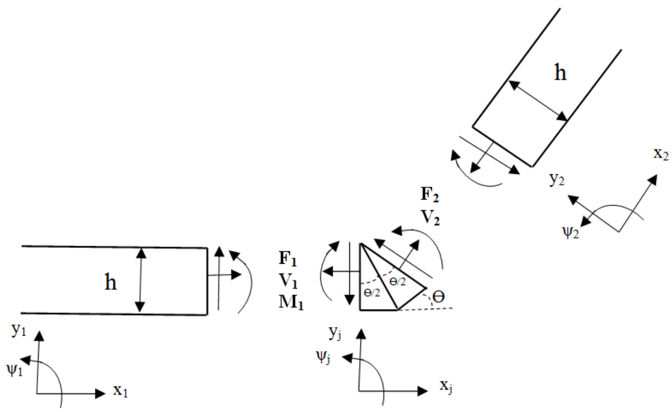

Figure 2. Free body diagram of angular joint.

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. The free body diagram at the angular joint is shown in Figure 2. Imposing the equilibrium conditions, the equations of motion of the joint are obtained as follows

$$-V_1 + F_2 \, sin\,\theta + V_2 \cos\theta = m\,\ddot{y}_j$$
$$-F_1 + F_2 \cos\theta - V_2 \sin\theta = m\,\ddot{u}_j$$
$$-M_1 + M_2 + V_1 \frac{h}{2}\tan\frac{\theta}{2} + V_2 \frac{h}{2}\tan\frac{\theta}{2} = J\,\ddot{\psi}_j \tag{7}$$

The continuity equations at the joint are

$$u_1 = u_j;\ u_2 = u_j \cos\theta + y_j \sin\theta$$
$$y_1 = y_j - \frac{h}{2}\tan\frac{\theta}{2}\psi_j, y_2 = -u_j \sin\theta + y_j \cos\theta + \frac{h}{2}\tan\frac{\theta}{2}\psi_j$$
$$\psi_1 = \psi_j\,, \psi_2 = \psi_j \tag{8}$$

Where F, V, M are axial force, shear force and bending moment respectively, y, u, ψ are translational deflection, axial deflection and bending slope respectively, subscripts 1 and 2 refers to beam 1 and beam 2 respectively, j indicates joint.

The axial force, shear force and bending moment are related to the displacements through

$$V = -EI\frac{\partial^3 y(x,t)}{\partial x^3}$$
$$M = EI\frac{\partial \psi(x,t)}{\partial x}$$
$$F = -EA\frac{\partial u(x,t)}{\partial x} \tag{9}$$

Also, $\boldsymbol{\psi} = \frac{\boldsymbol{dy}}{\boldsymbol{dx}}$ according to Euler beam theory.

2.4. *Reflection and Transmission at joints*

At joints waves undergo reflection and transmission, the incident wave is related to the reflected and transmitted wave through reflection and transmission matrices, respectively. The reflection and transmission matrices are derived following a similar procedure as given in [1].

Consider a set of positive going waves moving from beam 1 to beam 2. Incident wave A+ give rise to reflected wave A-and transmitted wave B+.

$$B^+ = t_{12}A^+,\ A^- = r_{11}A^+ \tag{10}$$

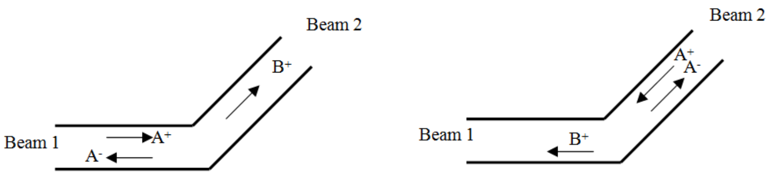

Figure 3. Wave reflection and transmission at an angular joint

Similarly for a set of positive going wave from beam 2 to beam1

$$B^+ = t_{21}A^+,\ A^- = r_{22}A^+ \tag{11}$$

Where $A^+ = \begin{bmatrix} a_1^+ \\ a_2^+ \\ c^+ \end{bmatrix}$; $A^- = \begin{bmatrix} a_1^- \\ a_2^- \\ c^- \end{bmatrix}$; $B^+ = \begin{bmatrix} b_1^+ \\ b_2^+ \\ d^+ \end{bmatrix}$

Now considering the incident wave from beam 1 to beam 2 we have,

$$y_1 = a_1^+ e^{-ik_1x_1} + a_2^+ e^{-k_2x_1} + a_1^- e^{ik_1x_1} + a_2^- e^{-k_2x_1}$$

$$u_1 = c^+ e^{-ik_3x_1} + c^- e^{ik_3x_1}$$

$$\psi_1 = -ik_1a_1^+ e^{-ik_1x_1} - k_2a_2^+ e^{-k_2x_1} + ik_1a_1^- e^{ik_1x_1} - k_2a_2^- e^{-k_2x_1}$$

$$y_2 = b_1^+ e^{-ik_1x_2} + b_2^+ e^{-k_2x_2}$$

$$u_2 = d^+ e^{-ik_3x_2}$$

$$\psi_2 = -ik_1b_1^+ e^{-ik_1x_2} - k_2b_2^+ e^{-k_2x_2} \tag{12}$$

For a set of waves moving from beam 1 to beam 2, substituting (12) in (8) gives

$$\begin{bmatrix} ik_1\frac{h}{2}\tan\frac{\theta}{2} & k_2\frac{h}{2}\tan\frac{\theta}{2} & \csc\theta \\ 1+ik_1\frac{h}{2}\tan\frac{\theta}{2} & 1+k_2\frac{h}{2}\tan\frac{\theta}{2} & -\cot\theta \\ ik_1 & k_2 & 0 \end{bmatrix} B^+ - \begin{bmatrix} 1 & 1 & \cot\theta \\ 0 & 0 & -\cot\theta\cos\theta-\sin\theta \\ -ik_1 & -k_2 & 0 \end{bmatrix} A^- = \begin{bmatrix} 1 & 1 & \cot\theta \\ 0 & 0 & -\cot\theta\cos\theta-\sin\theta \\ ik_1 & k_2 & 0 \end{bmatrix} A^+ \tag{13}$$

Substituting (11) and (9) in (7) gives

$$\begin{bmatrix} ik_1^3 EI\cos\theta & -k_2^3 EI\cos\theta & ik_3\mathrm{EA}\sin\theta - m\omega^2\cos\theta \\ -ik_1^3 EI\sin\theta & k_2^3 EI\sin\theta & ik_3\mathrm{EA}\cos\theta \\ -k_1^2 EI - ik_1^3 EI\frac{h}{2}\tan\frac{\theta}{2} - ik_1 J\omega^2 & k_2^2 EI + k_2^3 EI\frac{h}{2}\tan\frac{\theta}{2} - k_2 J\omega^2 & 0 \end{bmatrix} B^+ \\ - \begin{bmatrix} -ik_1^3 EI & k_2^3 EI & -m\omega^2\cot\theta \\ 0 & 0 & -ik_3\mathrm{EA} + m\omega^2 \\ -k_1^2 EI - ik_1^3 EI\frac{h}{2}\tan\frac{\theta}{2} & k_2^2 EI + k_2^3 EI\frac{h}{2}\tan\frac{\theta}{2} & 0 \end{bmatrix} A^- \\ = \begin{bmatrix} ik_1^3 EI & -k_2^3 EI & m\omega^2\cot\theta \\ 0 & 0 & ik_3\mathrm{EA} + m\omega^2 \\ -k_1^2 EI + ik_1^3 EI\frac{h}{2}\tan\frac{\theta}{2} & k_2^2 EI - k_2^3 EI\frac{h}{2}\tan\frac{\theta}{2} & 0 \end{bmatrix} A^+ \tag{14}$$

Solving (13) and (14) together with (10) will give transmission matrix t_{12} and reflection matrix r_{11}.

For a wave moving from beam 2 to beam 1, the equations the continuity and equilibrium conditions becomes

$$\begin{bmatrix} -ik_1\frac{h}{2}\tan\frac{\theta}{2} & -k_2\frac{h}{2}\tan\frac{\theta}{2} & \cot\theta\cos\theta + \sin\theta \\ 1 + ik_1\frac{h}{2}\tan\frac{\theta}{2} & 1 + k_2\frac{h}{2}\tan\frac{\theta}{2} & \cot\theta \\ ik_1 & k_2 & 0 \end{bmatrix} B^+ \\ - \begin{bmatrix} -1 & -1 & \cot\theta \\ 0 & 0 & \csc\theta \\ -ik_1 & -k_2 & 0 \end{bmatrix} A^- = \begin{bmatrix} -1 & -1 & \cot\theta \\ 0 & 0 & \csc\theta \\ ik_1 & k_2 & 0 \end{bmatrix} A^+ \tag{15}$$

$$\begin{bmatrix} 0 & 0 & ik_3\mathrm{EA} - m\omega^2 \\ ik_1^3 EI & -k_2^3 EI & m\omega^2\cot\theta \\ k_1^2 EI + ik_1^3 EI\frac{h}{2}\tan\frac{\theta}{2} + ik_1 J\omega^2 & -k_2^2 EI - k_2^3 EI\frac{h}{2}\tan\frac{\theta}{2} + k_2 J\omega^2 & 0 \end{bmatrix} B^+ \\ - \begin{bmatrix} ik_1^3 EI\sin\theta & -k_2^3 EI\cos\theta & -ik_3\mathrm{EA}\cos\theta \\ -ik_1^3 EI\cos\theta & k_2^3 EI\cos\theta & -ik_3\mathrm{EA}\sin\theta + m\omega^2\csc\theta \\ k_1^2 EI + ik_1^3 EI\frac{h}{2}\tan\frac{\theta}{2} & -k_2^2 EI - k_2^3 EI\frac{h}{2}\tan\frac{\theta}{2} & 0 \end{bmatrix} A^- \\ = \begin{bmatrix} ik_1^3 EI\sin\theta & k_2^3 EI\sin\theta & ik_3\mathrm{EA}\cos\theta \\ ik_1^3 EI\cos\theta & -k_2^3 EI\cos\theta & ik_3\mathrm{EA}\sin\theta + m\omega^2\csc\theta \\ k_1^2 EI - ik_1^3 EI\frac{h}{2}\tan\frac{\theta}{2} & -k_2^2 EI + k_2^3 EI\frac{h}{2}\tan\frac{\theta}{2} & 0 \end{bmatrix} A^+ \tag{16}$$

Solving (15) and (16) together with (11) will give transmission matrix t_{21} and reflection matrix r_{22}.

2.5. Reflection at supports

At boundaries waves undergo reflection, the incident wave a^+ is related to reflected wave a^- as $a^- = ra^+$. The reflection matrices for common boundary conditions are given in [2] as

$$r_s = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix};\ r_c = \begin{bmatrix} -i & -1-i & 0 \\ -1+i & i & 0 \\ 0 & 0 & -1 \end{bmatrix};\ r_f = \begin{bmatrix} -i & -1+i & 0 \\ 1-i & i & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

where r_s, r_c, r_f refers to reflection matrices of simply supported, clamped and free end conditions respectively.

2.6. External Excitation

When an external excitation is present, the waves on either side of the external excitation point are related as

$$b^+ - a^+ = q + m - f;\ b^- - a^- = -q + m + f$$

Where a^+ and a^- are the forward and backward moving wave vectors just before the excitation point, respectively and b^+ and b^- are the forward and backward moving wave vectors just after the excitation point, respectively. q, m, f are vectors given in [1] as

$$q = \begin{bmatrix} -i \\ -1 \\ 0 \end{bmatrix} \frac{\overline{Q}}{4EIk_1^3};\ m = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \frac{\overline{M}}{4EIk_1^2};\ f = \begin{bmatrix} 0 \\ 0 \\ i \end{bmatrix} \frac{\overline{F}}{2EAk_3}$$

where $\overline{Q}$,$\overline{F}$, $\overline{M}$ are point transverse force, axial force and moment respectively.

3. Analysis of Angular Frame

The vibration analysis of an angular frame as shown in figure 4 is performed. The example frame used has Young's modulus E =206 GN/m^2, Poisson's ratio =0.29, mass density ρ=7800 kg/m^3 and cross section is 1.27 cm by 1.27 cm.

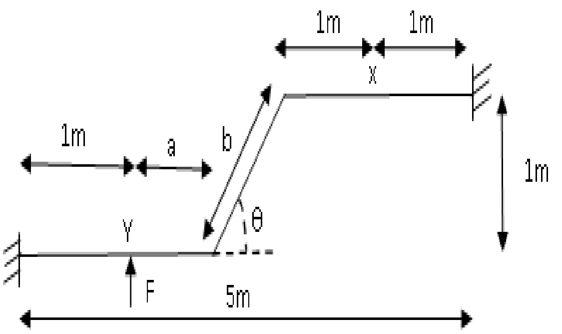

Figure 4. Vibration analysis of frame performed in the present work

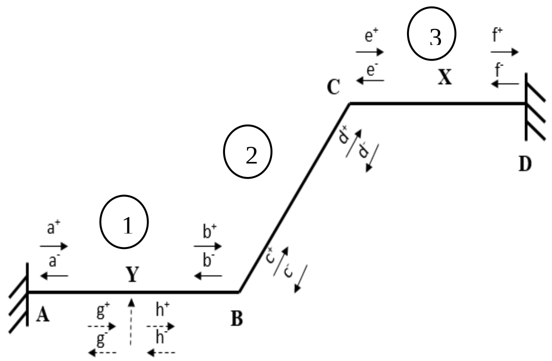

Figure 5. Waves in angular frame

3.1. Waves in angular frame

The reflection and transmission relations are given as explained above.
For free vibration
At angular joint B

$$c^+ = r_{22}c^- + t_{12}b^+;\ b^- = r_{11}b^+ + t_{21}c^-$$

At angular joint C

$$e^+ = r_{33}e^- + t_{23}d^+;\ d^- = r_{22}d^+ + t_{32}e^-$$

At boundaries A and D

$$a^+ = r_f a^-;\ f^+ = r_f f^-$$

Along AB, along BC and along CD

$$b^+ = f(AB)a^+; a^- = f(AB)b^-; d^+ = f(BC)c^+; c^- = f(BC)d^-;$$

$$f^+ = f(CD)e^+; e^- = f(CD)f^- .$$

Now there are 12 equations and each of these have 3 component waves. Thus, system of 36 X 36 system of homogeneous equations is obtained. Writing the above set of equations in matrix form gives AZ=0; Where A is the coefficient matrix of size 36x36, Z is the wave component vector of size 36x1. The Eigen values of matrix A gives natural frequencies and Eigen vector gives mode shapes.

For forced vibration

At excitation point Y

$$g^+ - h^+ = q + m - f;\ g^- - h^- = -q + m + f$$

Propagation equations along AB are modified as

$$g^+ = f(AY)a^+; a^- = f(AY)g^-;\ b^+ = f(YB)h^+; h^- = f(YB)b^-$$

For forced vibration this will become a matrix in the form $A_f Z_f$=F; Where A_f is the coefficient matrix of size 48x48, F is the excitation vector of size 48x1. Solving the above equations gives response.

3.2 Natural frequency and Mode shape

Using the above formulation, the natural frequency and mode shapes for $\Theta=45^0$frame are found. Also FEM simulation through ANSYS was conducted to obtain the natural frequencies and mode shapes and the results are compared. The results agree well.

Table 1. Natural Frequency of $\Theta=45^0$ frame.

Mode No	Wave method	ANSYS
1	2.3	2.3
2	12.5	12.5
3	14.4	14.4

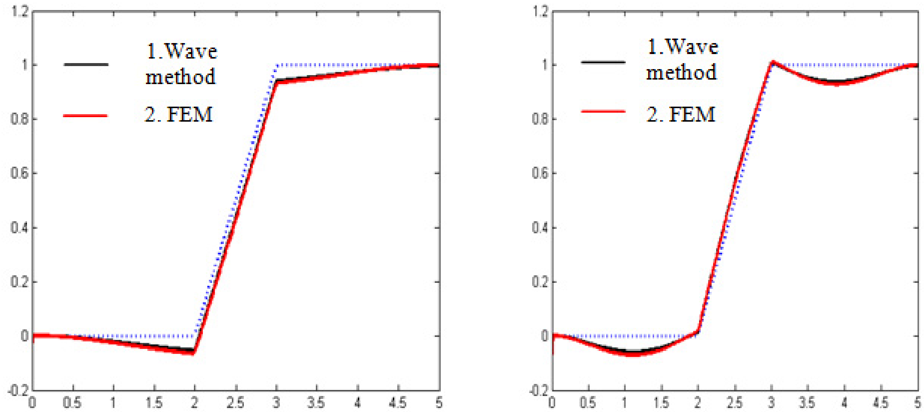

Figure 6. Comparison of first two modes shapes of ϴ=45^0 frame. The unreformed shape is shown as dotted line.

3.3 Frequency Response

The frequency response function for ϴ=45^0 frame with force applied at point Y and response measured at X is found out using wave method. Also FEM simulation through ANSYS is also conducted to find the same and the results are compared. The results agree well.

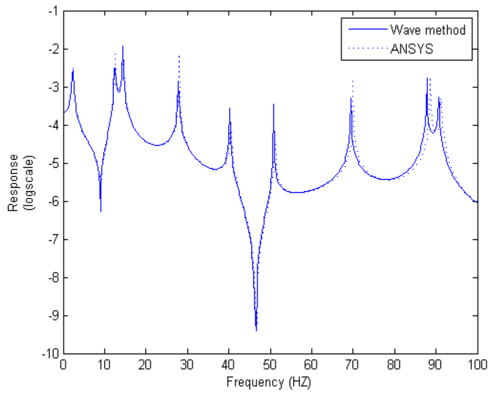

Figure 7. The response at X with force applied at Y

4. OPTIMIZATION OF ANGULAR FRAME

As discussed above, the wave method owing to its small matrix sizes yield an efficient computational method for structural dynamic simulation of planar frames. In the next part of the work, this method used to arrive at optimal structural design.

Two horizontal cantilever beams to be interconnected through an intermediate member as shown in figure 4. The cantilever ends of the two beams are at a fixed distance apart as shown in figure. The objective is to find the orientation ϴof the intermediate member such that minimal vibration response is observed at point X in the target structure due to harmonic point forcing applied at point Y in the source structure. The constraints given in the problem are vertical distance between the source and target structure is 1m (bsinθ=1m) and horizontal distance between the source and target structure is 2m (a+bcosθ=2m).

4.1 Results for optimization

Wave analysis is performed on the structure as shown in figure 4 for different values of ϴ. The optimum orientation of the frame is found for minimal vibration at frequencies 200 Hz and 2000 Hz. The results areas shown in figure 9 and figure 11. At 200 Hz, ϴ=35^0 gives minimum vibration response at X and at 2000Hz,ϴ=50^0 gives minimum vibration response at X .The FRF of ϴ=35^0 frame and ϴ=50^0 frame are plotted using wave method and FEM as shown in figure 10 and 12. As is expected, the FRFs shows a minima at 200 Hz and at 2000 Hz for ϴ=35^0 frame and ϴ=50^0 frames, respectively. This shows that the optimization procedure done by wave method is valid.

As is evident from the plots the FRF value using FEM deviates from the wave method results at higher frequencies, this is because at higher frequencies the accuracy of FEM method decreases whereas wave method gives exact solution at all frequencies. Thus wave method can be used as an efficient tool for optimization of frame structures.

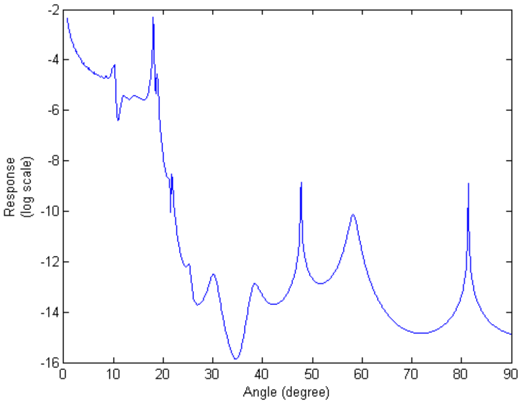

Figure 8. Response amplitude at x plotted as a function of θ at frequency=200 Hz

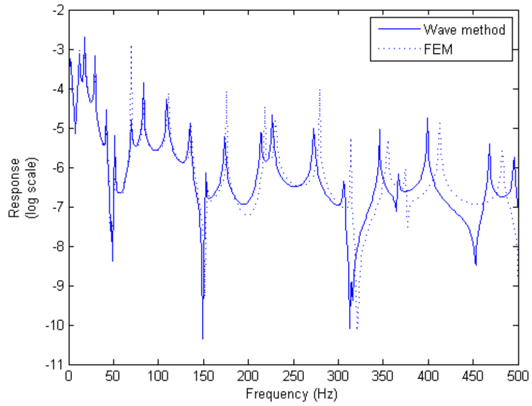

Figure 9. Response amplitude at x with force applied at Y for ϴ=35^0 frame

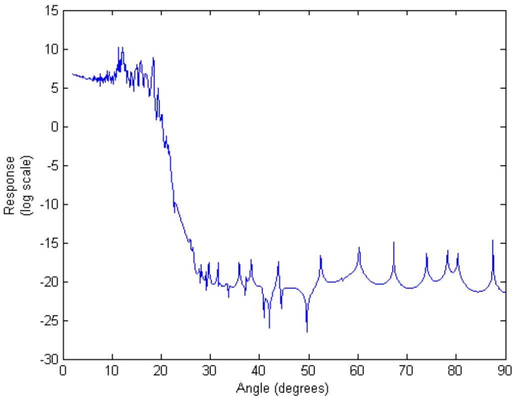

Figure 10. Response amplitude at x plotted as a function of θ at frequency=2000Hz

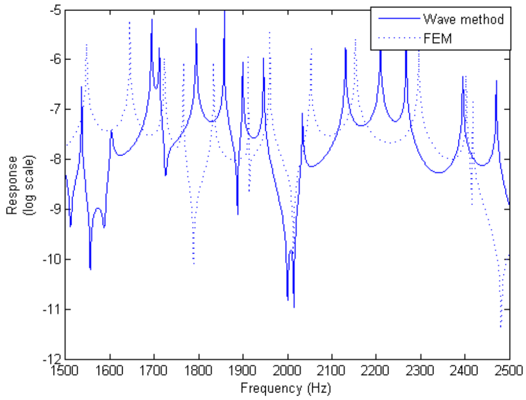

Figure 11. Response amplitude at x with force applied at Y for ϴ=50^0 frame

5. Conclusion

An exact solution for the vibration analysis of angular frame has been found using wave propagation method. The reflection and transmission matrices at the angular joint are derived as function of angle ϴ. The wave propagation method is computationally efficient as compared to finite element method. Further, unlike FEM which demands finer meshing for higher frequencies, the size of the system of equations involved in the wave method is uniform for all frequencies. Being analytical procedure, the accuracy of the wave method is not compromised at higher frequencies. The efficacy of this method is used in structural optimization exercise for determining the optimum angle of an intermediate member for minimum vibration transfer in a planar frame structure. The results show that the angular orientation for minimum vibration transfer depends on exciting frequency. Thus, if nature of excitation is known *a priori* an optimal structural design can be arrived at. However, for broad band excitation such optimization exercise remains challenging.

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