

# Use of spatial phase shifting technique in digital speckle pattern interferometry (DSPI) and digital shearography (DS)

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**Abstract:** Digital speckle pattern interferometry (DSPI) and digital shearography (DS) are well known optical tools for qualitative as well as quantitative measurements of displacement components and its derivatives of engineering structures subjected either static or dynamic load. Spatial phase shifting (SPS) technique is useful for extracting quantitative displacement data from the system with only two frames. Optical configurations for DSPI and DS with a double aperture mask in front of the imaging lens for spatial phase shifting are proposed in this paper for the measurement of out-of-plane displacement and its first order derivative (slope) respectively. An error compensating four-phase step algorithm is used for quantitative fringe analysis.

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## 1. Introduction

Digital speckle pattern interferometry (DSPI) and digital shearography are two powerful non-contact, whole-field optical techniques for the measurement of displacement and its derivatives under various loads [1-5]. Both the techniques use phase shifting method for extracting quantitative displacement data from the system with the removal of phase ambiguity and increased accuracy of measurement [6]. Temporal phase shifting (TPS) is widespread method in DSPI and DS, in which the phase-shifted data are acquired in a temporal sequence of camera frames [4,5,7]. But this method is susceptible to external disturbances like vibration, temperature fluctuation, or rapid motion of the test object itself. Spatial phase shifting (SPS) technique is a simple way to eliminate the external disturbances [8-13]. In this method images are recorded either simultaneously by several CCD cameras with an appropriate static phase shift between each of the images or by a single CCD target with the introduction of spatial carrier fringes whose spacing should correspond to the distance that covers at least three pixels of the CCD. The former method is not cost effective, as it requires expensive CCD cameras (at least 3), as well as it requires precise alignment of the cameras (to fraction-of-a-pixel accuracy) [12]. The later method is convenient and attractive to implement SPS technique for speckle and speckle shear fringe analysis [8].

In this paper we present two arrangements for DSPI and DS that employ a double aperture mask in front of an imaging lens for spatial phase shifting for out-of-plane displacement and slope measurements. It is well known that the double aperture arrangements are intrinsically sensitive to in-plane displacement [3]. The optical configurations proposed here for implementation of spatial phase shifting are insensitive to in-plane displacement components of a deformation vector. The diameter of the apertures and their separation are designed in such a way that it produces a carrier fringe pattern as to provide phase shift of  $90^\circ$ /column of the CCD pixels as it has some additional advantages over  $120^\circ$ /column [13]. As explained in Sec. 4, this results in a minimum speckle size of 5 pixels. The first arrangement described is for the measurement of out-of-plane displacement. The object and reference waves enter through two separate apertures and are combined coherently at the image plane [14-15]. The reference wave is made diffuse by a ground glass placed in front of the appropriate aperture. The interference pattern between the object and reference waves, contains spatial carrier fringes in each speckle. The second arrangement is for the first order displacement derivative (slope) measurement, for which an in-plane insensitive double aperture [16] digital shearography arrangement is used. Waves from the object are sheared as they enter the

apertures and the interference pattern between the sheared images produces a spatial carrier fringes with in the speckle that is recorded by the CCD camera.

Conventional way to calculate phase change due to object displacement is the “difference of phases” method [4,7], where speckle phases are calculated before and after object displacement using the spatially phase shifted frames. Extraction of the phase shifted correlation speckle fringe patterns is possible from the stored spatially phase shifted frames. Phase change due to displacement of the object can be also analyzed from these fringes by using the “phase of difference” method [4]. Since the former method is superior to later method [4] for static speckle fringe analysis, we followed the analysis using the difference of phases method. Error compensating four-phase algorithm is used for phase calculation that has the advantage that linear reference phase error is eliminated [17]. Speckle noise is reduced by individually filtering the numerator and the denominator of the arctangent function used for phase calculation using phase filter before recombining them [5]. Experimental results are presented for a centrally loaded diaphragm with its edge rigidly clamped for both out-of-plane displacement and slope measurement.

## 2. Out-of-plane displacement measurement:

Figure 1 shows the schematic of the optical arrangement for out-of-plane displacement measurement. The arrangement essentially contains a double aperture mask (A) for spatial phase shifting. The diffuse object is illuminated by a laser beam from a He-Ne laser (20 mW) after collimation. A reference mirror (RM) is placed very near to the object so that the laser beam is also incident on the reference mirror. The scattered object wave enters through one of the apertures with the help of mirror ( $M_1$ ) and the front surfaces coated right angle prism (P). Similarly the reflected smooth wave from the reference mirror is incident on a ground glass plate via mirror ( $M_2$ ) and the front surfaces coated right angle prism (P). The ground glass is attached to one of the apertures so that the scattered light from it will act as reference wave [15]. The double aperture mask is placed very near to an imaging lens (L). Both the waves enter the apertures independently and they combine coherently at the CCD plane (image plane).

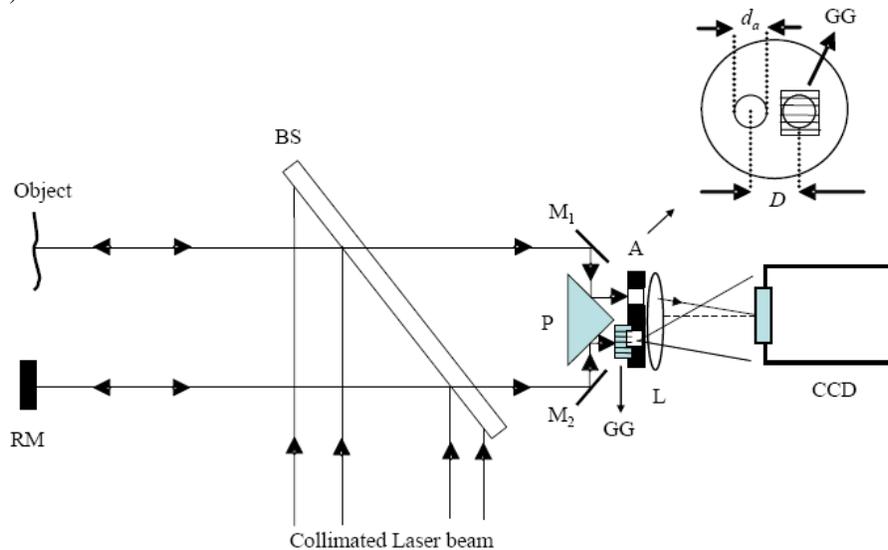


Fig. 1. Schematic of an out-of-plane sensitive digital speckle pattern interferometry arrangement: O, Object; RM, Reference mirror; BS, Beam splitter; M, Mirrors; P, Front surfaces coated right angle prism; A, double-aperture mask; GG, Ground glass; L, Imaging Lens.

The phase change due to object displacement for normal illumination and observation can be directly related to out-of-plane displacement ( $w$ ) as [3, 15]

$$\Delta\phi(x, y) = \frac{4\pi}{\lambda} w(x, y) \quad (1)$$

### 3. Slope measurement:

Figure 2 shows the schematic arrangement for slope measurement. The diffuse object is illuminated by a collimated laser beam. The scattered wave from the object is divided into two waves at the beam splitter  $BS_2$ . Both the apertures in A, in front of an imaging lens, receive the scattered wave independently via the set of mirrors ( $M_1$ - $M_3$ ) and ( $M_4$ ,  $M_5$ ), and the front surfaces coated right angle prism (P) respectively. Both the waves with identical path length combine coherently at the image plane of the imaging system. One of the mirrors is adjusted for shear ( $\Delta x$ ) along the  $x$ -direction.

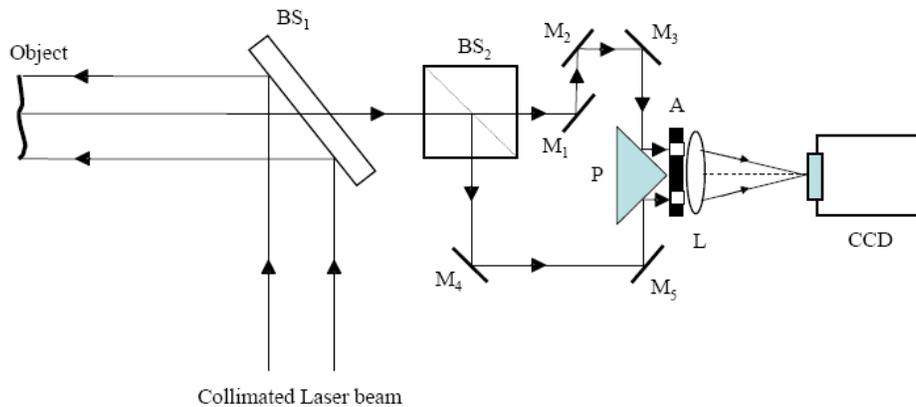


Fig. 2. Schematic of an in-plane insensitive digital shearography arrangement: O, Diffusely reflecting surface;  $BS_1$ , Beam splitter;  $BS_2$ , Cube beam splitter;  $M_1$ - $M_5$ , Mirrors; P, Front surfaces coated right angle prism; A, Two-aperture mask; L, Lens.

The phase change due to object displacement,  $\Delta\phi(x, y)$ , can be expressed as [4-5, 16]

$$\Delta\phi = \frac{4\pi}{\lambda} \frac{\partial w}{\partial x} \Delta x \quad (2)$$

The phase difference  $\Delta\phi$  provides the partial  $x$ -derivative of the out-of-plane displacement component, i.e. slope only. It may be noticed from the optical geometry that in the absence of the shear between the two object beams, this configuration is insensitive to object deformation, because the scattered beams entering through the apertures undergo identical phase changes and hence the net phase change,  $\Delta\phi$  is zero.

### 4. Spatial phase shifting (SPS) and phase calculation:

The fields passing through the apertures interfere at the image plane and thus produce a spatial carrier fringe pattern within the speckles. It is customary to assume the fields to be emanating from the centers of the apertures. Let the phases of the waves passing through the apertures have a spherical part plus a speckled part. The phase difference ( $\Delta\phi$ ) between the two spherical parts of the wavefronts at the image plane ( $x, y$ ) is given by [12]

$$\Delta\phi = \frac{2\pi D}{\lambda V} x \quad (3)$$

where  $D$  is the aperture separation and  $V$  is the image distance from the lens. Therefore the image field is modulated by a fringe pattern with a constant spacing of  $\sim \lambda V / D$ . Each speckle thus carries the fringe pattern. The average size of the speckle ( $d_s$ ) is given by  $d_s = 1.22 \lambda V / d_a$ , where  $d_a$  is the aperture diameter. Since the aperture sizes  $d_a$  can be no larger than the separation of their centers,  $D$ , we have [12,13]

$$d_a \leq D \quad \text{i.e.} \quad \frac{d_s}{1.22} = \frac{\lambda V}{d_a} \geq \frac{\lambda V}{D} = \frac{2\pi}{\omega_0} \quad (4)$$

where  $\omega_0$  is the spatial angular frequency of the carrier fringe (in radians per pixel). Hence, if we adjust  $\omega_0 = \pi/2$ , the smallest speckle size we can get is  $d_s \cong 5d_p$ , where  $d_p$  is the pixel size. The fringe width is thus equal to the width of four pixels ( $4d_p$ ) of the CCD sensor for the  $90^\circ$  phase-shift per column. The intensity distribution  $I_n(x_k, y)$  at a pixel in the  $k$ th column of the CCD along the  $x$ -direction is given by

$$I_n(x_{k+n}, y) = I_b(x_{k+n}, y) + \gamma(x_{k+n}, y) \cos[\phi(x_{k+n}, y) + \omega_0(k+n)] \quad (5)$$

where  $n$ , the number of phase sample and  $n \in \{-1, 0, 1, 2\}$  and  $1 < k < M$ ,  $I_n$ , intensity of the spatially phase shifted frame,  $I_b$ , bias intensity (due to superposition of two wave-fronts),  $\gamma$ , intensity modulation,  $\phi$ , speckle phase.

To find the phase at a pixel in a particular column of the image, some neighbouring pixels are needed to provide the phase-shifted interference data. The above equation then imposes the restriction that  $\phi(x_k, y)$  should be uniform over the speckle dimension, and the same applies to  $I_b(x_k, y)$  and  $\gamma(x_k, y)$ ; that is, spatial fluctuations of these quantities should be as small as possible.

The phase  $\phi(x_k, y)$  can be calculated by using error compensating four-phase algorithm given by [17]

$$\phi(x_k, y) + \omega_0 k = \tan^{-1} \left( \frac{(I_1 + I_2 + I_4) - 3I_3}{3I_2 - (I_1 + I_3 + I_4)} \right) = \tan^{-1} \left( \frac{N_s}{D_s} \right) \quad (6)$$

where  $N_s$  and  $D_s$  are the numerator and denominator of the arctangent function and  $s$  represents the state of the object i.e. either before or after object displacement.

As a result of the object displacement, the initial phase,  $\phi_i(x, y)$ , changes to  $\phi_f(x, y) = \phi_i(x, y) + \Delta\phi(x, y)$ , where is  $\Delta\phi(x, y)$  the phase change due to object displacement. This phase change can be calculated either by phase-of-difference method or by difference-of-phases method [4]. The former method involves generation of correlation fringes. One can obtain correlation fringes from SPS interferograms obtained in Eq. (5), according to the relation given by [13]

$$I_{n,c}(x, y) = \left| I_f(x_{k+n}, y_l) - I_i(x_k, y_l) \right| \quad (7)$$

where the subscript  $c$  denotes correlation fringes and  $n \in \{-1, 0, 1, 2\}$ .

The phase difference,  $\Delta\phi(x, y)$  can be obtained with these phase shifted speckle correlation fringes. But the lateral image shift of course causes lower speckle correlation between  $I_f(x_{k+n}, y)$  and  $I_i(x_k, y)$  than between  $I_f(x_k, y)$  and  $I_i(x_k, y)$ , resulting in non-constant fringe contrast within the set of the  $I_{n,c}(x_k, y)$ , and consequently, unnecessary errors in the phase calculation. Therefore, correlation fringes from SPS are even less suitable for the phase of differences method than are those from temporal phase shifting (TPS) [13]. In the difference of phases method, the phase difference  $\Delta\phi(x, y)$  can be obtained by subtracting the evaluated phase

maps before and after object displacement and thus the speckle phase noise ( $\phi_i$ ) will be eliminated [4,7]. The phase map can be obtained by [18]

$$\Delta\phi(x, y) = \tan^{-1} \left( \frac{N_f D_i - N_i D_f}{N_f N_i + D_f D_i} \right) = \tan^{-1} \left( \frac{N}{D} \right) \quad (8)$$

The resulting phase map obtained in this method is still noisy, thus filtering is necessary in order to improve it. For the present case, we have twice filtered individually the numerator ( $N$ ) and the denominator ( $D$ ) of the arctangent function using  $3 \times 3$  windowed phase filtering [5]. The filtered numerator and the denominator are then used in Eq. (8) to get filtered phase map. The filtered phase distribution,  $\Delta\phi(x, y)$  is the wrapped or modulo- $2\pi$  phase map, which range from  $-\pi$  to  $\pi$ . Unwrapping is the procedure, which removes these  $2\pi$  phase jumps (discontinuities) and the result is converted into desired continuous phase function [19].

## 5. Results and Discussion

The double aperture mask used for spatial phase shifting has two circular holes of diameter 3mm and they are separated along x-direction by an amount 3.25mm. The separation between the apertures is a critical parameter as it determines the phase shift per column of the CCD sensor. For the present case phase shift per column is  $90^\circ$ . We have used a Jai CV-A1 CCD camera, with matrix 1384x1035 and  $4.65 \mu\text{m}$  pixel size. It has been investigated in ref. [13] that  $90^\circ/\text{column}$  is more appropriate choice as it yields better performance over whole range of fringe densities. Also it can tolerate large phase mis-calibration. Further, the error compensating four-phase algorithm technique that uses  $90^\circ$  phase steps, reduces the linear reference phase errors [18]. The double aperture is placed in front of an imaging lens of focal length 89mm. We have chosen the lens in order to image full object on to the CCD sensor. The object is a centrally loaded diaphragm with its edge rigidly clamped and having diameter of 60mm, coated with white matt paint. A right-angle prism with its faces coated for high reflectivity is carefully aligned with respect to the aperture mask.

For out-of-plane displacement measurement, a small ground glass is attached to one of the aperture so that the smooth reflected light from the reference mirror will scatter through the ground glass. This will act as reference wave and in the image plane interfere with the object wave, thus form spatial carrier fringe. Figure 3 shows such an interference patterns after magnification, revealing the carrier fringe inside the speckle. The reference mirror is placed very near to the object so that collimated laser beam illuminates both the object and the reference mirror at normal incidence. The normal illumination and observation arrangement has the advantage of forming near common path for both object and reference waves. Precise alignment of the mirrors, the front surfaces coated right angle prism and also the CCD is essential for proper superposition of the waves onto the CCD plane. Software based on LabVIEW is developed in order to record the images before and after object displacement. The software also displays the real time subtraction of the images, thus the correlation speckle fringe can be controlled during loading. Figure 4 shows first two spatially phase shifted correlation fringe patterns obtained using Eq. (7), namely  $-90^\circ$  and  $0^\circ$ . As mentioned in Sec. 4, the contrast of the correlation fringes is non-constant and hence not suitable for phase calculation [13]. Therefore the analysis is carried out by the difference of phases method as represented by Eq. (8).

Figure 5(a) shows the raw phase map corresponding to the out-of-plane displacement obtained by phase of difference method. In order to reduce noise, the numerator and denominator of the arctangent function as given in Eq. (8) are twice filtered individually using  $3 \times 3$  windowed phase filtering [5]. Figure 5(b) shows the noise improved phase map.

Unwrapped 2D and 3D plots of the phase map after converting to out-of-plane displacement are shown in Fig. 5(c) and (d), respectively.

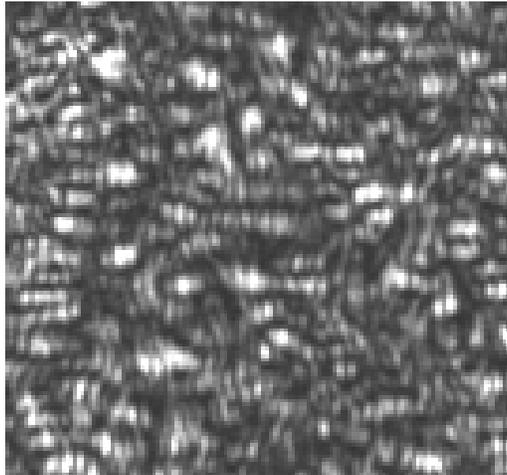
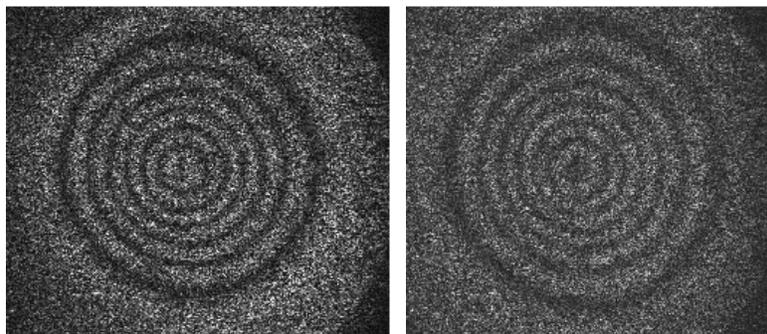


Fig. 3. Magnified portion of the speckle pattern with double aperture arrangement revealing the spatial carrier fringe



(a)

(b)

Fig. 4. Speckle correlation fringes with spatial phase shift: (a)  $-90^\circ$  and (b)  $0^\circ$

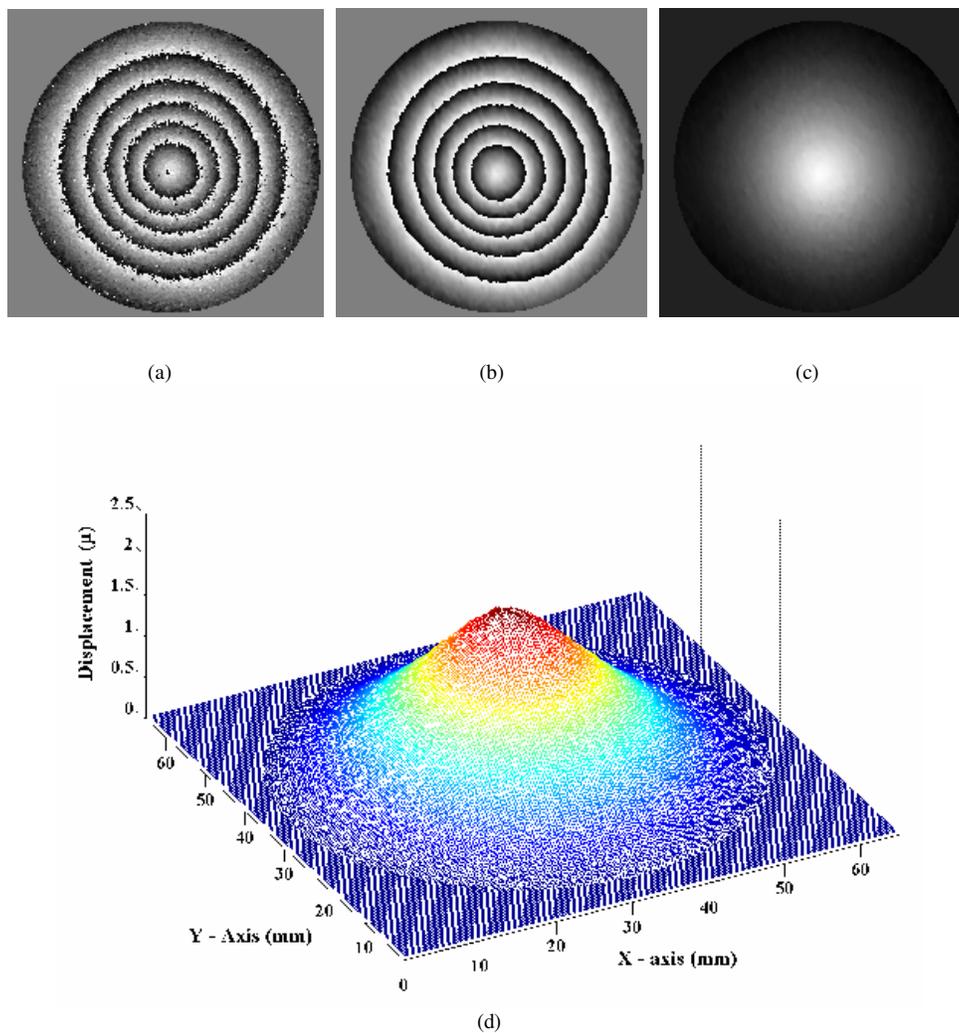


Fig. 5. Out-of-plane displacement measurement: (a) raw and (b) filtered phase maps, (c) unwrapped 2D and (d) 3D plots

For slope measurement, the mirrors in the set-up (Fig. 2) are aligned to superpose both the object waves with identical magnification at the CCD plane. Shear can be introduced by tilting one of the mirrors in the set-up. The sheared waves enter the apertures independently and interfere at the CCD plane to form spatial carrier fringe. For the present experiments the object plane shear is 4 mm along  $x$ -direction. Similar processing has been done here in order to get noise improved phase map. Figure 6(a)-(b) show the raw and noise-improved phase maps, respectively, corresponding to the first order displacement derivative or slope. Unwrapped 2D and 3D plots of the phase map after converting to slope are shown in Fig. 6(c) and (d), respectively.

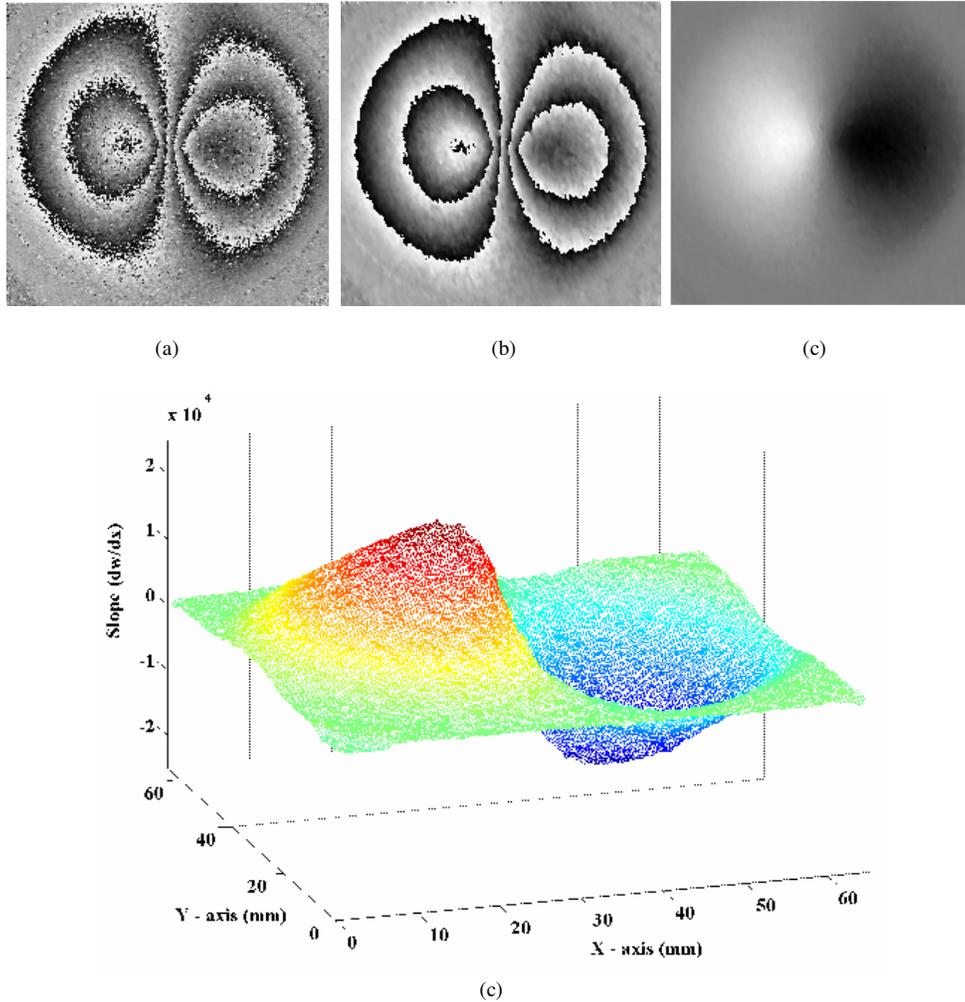


Fig. 6. Slope measurement: (a) raw and (b) filtered phase maps, (c) unwrapped 2D and (d) 3D plots

## 6. Conclusion

We have presented quantitative analysis of out-of-plane displacement and slope using spatial phase shifting technique with double aperture DSPI and DS arrangements. The advantage of technique is that speckle phase can be measured from a single frame. This has an advantage when the deformation is varying continuously. Further, the proposed spatial phase shifting technique is simple and cost effective as it requires only a double aperture mask in front of the imaging lens instead of a conventional PZT driven phase shifting unit. The limitation of the method is that the spatial carrier frequency is fixed by the optical geometry and the double aperture mask needs to be redesigned every time the geometry is changed to accommodate different sizes of objects. Also it requires a large (comparable to the object under test) beam-splitter in order to illuminate the object. But with the use of slightly diverging beam, the beam-splitter size can be reduced. We have tested 60 mm diameter object with our setup. The arrangement is well suited for small size objects such as MEMS.

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