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TUNING OF PID CONTROLLERS FOR TIME DELAY UNSTABLE SYSTEMS WITH TWO UNSTABLE POLES

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ABSTRACT - An improved continuous cycling method is proposed for PID controllers for unstable systems with two unstable poles and time delay. The method involves the determination of the controller settings by solving the magnitude and the phase angle criteria for the system with a proportional controller. Subsequently, incorporating the Proportional-Derivative-Integral (PID) controller transfer function, with unity proportional gain and pre-determined values of reset time and derivative time in the first step, with the system model and again solving the amplitude and the phase angle criteria, we get the updated gain of the controller. The method is applied by simulation on (i) a second order system with time delay and two unstable poles and (ii) a non-linear model of a CSTR with complex conjugate unstable poles and time delay. The controller settings significantly enhance the performances of both the servo and the regulatory problems and give robust performances.

Keywords: improve dPID, time delay unstable systems, Two unstable poles, Ziegler-Nichols method

I. INTRODUCTION

The use of Proportional-Integral-Derivative (PID) controllers for industrial process control is the most popular technique. With the invention of PID controller in 1910 [1] and the Z-N tuning rules in 1942 [2] the popularity of the PID controllers increased immensely. The widespread use of PID controllers owe to its simple structure and the ease of on-line retuning.

To determine the parameters of the controller, many design methods, such as gain margin/phase margin method [3-6], pole placement technique [7-8], optimization technique [9-11], direct synthesis method [12-15], internal model control method [16-17], equating coefficient method [18] and robust loop shaping [19], have been reported in the last few decades. One of the earliest methods of tuning PID controllers is the Ziegler-Nichols method [2]. It is a heuristic method of determining the ultimate values of the controller. At the ultimate value, the system is at the point of marginal instability and gives sustained oscillations in the output. The ultimate gain and ultimate frequency are used to get the PID controller settings. The PID settings proposed by Ziegler – Nichols results in a large overshoot and an oscillatory response. The correlation between the ultimate period, the reset time and the derivative time was based on simulation of a large number of processes. The key criterion is a quarter decay ratio. Many other researchers have modified the ZN method to obtain significant performance improvement. Dwyer [20] has reviewed the methods. Tyreus-Luyben [21] proposed settings for PI and PID controllers, but the method results in a long settling time. Smith [22] and Yu [23] proposed modification in the tuning formulae based on the ultimate values.

Furthermore, most of the proposed methods, based on ultimate values of controllers, are implemented mostly on stable processes. Controller design for the unstable process with time delay is difficult. Some of the methods are modified based on achieving the desired closed loop response [24]. Many methods involving use of PID controller in series with the lead lag compensator are also proposed. Nikita and Chidambaram [25,26] proposed a method to improve the performance of a PID controller for First order unstable systems and for a second order plus time delay systems with one stable and one unstable poles.

In the present work, this method is extended to unstable time delay systems with two unstable poles. The method is simple, analytically derived and can be applied mostly to all the different classes of processes. Maximum sensitivity, phase margin and gain margin criteria are used for the robustness analysis of the proposed method.

II. CONTROLLER DESIGN METHOD

In this paper, the single loop feedback controller structure is used as shown in Figure(1), where G_p is the process transfer function, G_c is the PID controller transfer function of the form given by $G_c(s) = K_c + \frac{K_I}{s} + K_D s$

where, $K_I = \frac{k_c}{\tau_{I,}}$ $and\ K_D = k_c * \tau_D$

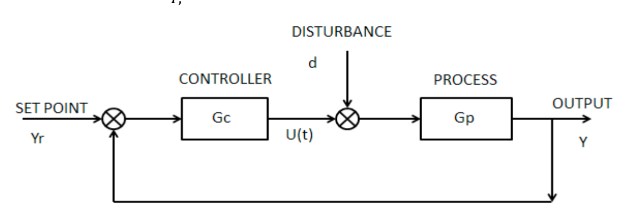

Figure 1: Feedback controller structure

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It should be noted that for an unstable system, along with the maximum value of controller gain ($K_{c,max}$), the minimum value of controller gain ($K_{c,min}$) also plays an important role. Ziegler and Nichols [2] proposed a tuning rule for the PID controllers based on the ultimate values of the system. However, the method gives oscillatory responses particularly for the control of the unstable systems. In the present work, a method is proposed for updating the value of the controller gain once integral and derivative actions are put into effect. The amplitude ratio (A_r) and cross over frequency (ω_c) for a system are obtained by solving the amplitude and the phase angle criteria. To make the overall loop gain equal to unity at phase lag equals to 180 deg, the controller gain is set equal to $1/A_r$. At this value, the closed loop system will become marginally stable and will give sustained oscillations with frequency equal to ω_{max}. To eliminate the offset we must include the integral mode to the controller. To speed up the response of the system, a derivative mode is added to the controller.

It is observed that on addition of the integral and derivative modes, the value of controller gain changes and its new value is to be calculated. For stable systems, Douglas [26] proposed a trial and error procedure to determine the values of controller settings based on the Bode plots. He proposed that the best results are obtained when the phase lag of approximately 10 deg (by the integral mode) and phase lead of approximately 45 deg (by the derivative mode) are added at the original cross over frequency. For this, the integral and derivative times are considered respectively as: $5/\omega_c$ and $1/\omega_c$. Around these values, the parameters are tuned to get a satisfactory performance. However, the examples considered are stable processes.

A. *Time integral performance*

To compare the performance of the system based on proposed method over the other methods present in the literature, Integral of the Square Error (ISE), Integral of the Time weighted Absolute Error (ITAE) and Integral of the Absolute error (IAE) values are considered for both the cases of unit step change in the input and unit step change in the disturbance. The criteria are defined as

$$ISE = \int_0^\infty \epsilon^2(t)dt; \quad IAE = \int_0^\infty |\epsilon(t)|dt ;$$

$$ITAE = \int_0^\infty t|\epsilon(t)|dt$$

$where, \epsilon(t) = y_r(t) - y(t)$ is the deviation of response from desired set point.

III. SIMULATION STUDIES

Example 1: Consider the open loop unstable second order plus time delay system with two unstable poles and a negative zero

$$G_p(s) = \frac{K_p(5s+1)\exp(-s)}{(10s-1)(5s-1)} \qquad (1)$$

where, $K_p = 1$

The amplitude ratio and the phase angle of the system can be written as

$$A_r = \frac{1}{\sqrt{10^2\omega^2+1}} \qquad (2)$$

$$\varphi = -\omega + 2\tan^{-1}5\omega + \tan^{-1}10\omega - 2\pi \qquad (3)$$

To determine the ultimate values of the controller, the system is made to approach the marginal instability at cross over frequency. The phase angle criterion [eq.(3) with $\varphi = -\pi$] is numerically solved to obtain the minimum value of frequency ($\omega_{c,min}$) and maximum value of frequency ($\omega_{c,max}$) corresponding to which minimum and maximum amplitude ratio of the system are calculated using eq.(2). To make the overall gain of the system unity, the ultimate value of controller gain (K_c) set equal to inverse of amplitude ratio (A_r).

Solving numerically eq.(3b) with $\varphi = -\pi$, we get,

$$\omega_{c,min} = 0.4009 \qquad (3a)$$

$$\omega_{c,max} = 1.1335 \qquad (3b)$$

On substituting the values of ω_{min} and ω_{max} in eq.(2), we get

$$A_r|_{min} = 0.2420 \qquad (4a)$$

$$A_r|_{max} = 0.0878 \qquad (4b)$$

As stated above, $K_c = \frac{1}{A_r}$, the minimum and maximum values of controller gain can be determined as

$$K_{c,min} = 4.1322 \qquad (5a)$$

$$K_{c,max} = 11.3792 \qquad (5b)$$

For unstable systems, the minimum value of controller gain plays an important role. It is seen that if the minimum value of controller gain is ignored while designing the PID controller, many systems cannot be stabilized. The ultimate period of oscillation is calculated based on the frequency [eq.(3b)]. The reset time and derivative time are determined using the Ziegler Nichols tuning rule.

$$P_u = \frac{2\pi}{\omega_{c,max}} = 5.5431 \qquad (6a)$$

$$\tau_I = \frac{P_u}{2} = 2.7716 \qquad (6b)$$

$$\tau_D = \frac{P_u}{8} = 0.6929 \qquad (6c)$$

The PID controller parameters for the ZN continuous cycling method are given in Table (1). Here, $P_u = 2\pi/\omega_c$,max.

For the proposed method, the values of reset time and derivative time are calculated using the formula given by

$$\tau_I' = \frac{5}{\omega_{c,max}}; \quad \tau_D' = \frac{0.8}{\omega_{c,max}} \qquad (7)$$

The proposed method includes determining the new ultimate value of the controller once the integral and derivative mode are put into effect.

For determination of new ultimate values, the controller transfer function, with proportional gain equal to 1 and with the calculated values of reset time and derivative time, are added to the system. The value of τ_I' and τ_D' calculated in eq.(7) are for the series form PID controller. These values are then transformed to parallel form for PID [23] by:

$$\tau_{I_p} = \tau_I' + \tau_D' \qquad (8a)$$

$$\tau_{D_p} = \frac{\tau_I' * \tau_D'}{\tau_I' + \tau_D'} \qquad (8b)$$

The updated system is given by

$$G_p(s) = \left(1 + \frac{1}{\tau_{I_p} s} + \tau_{D_p} s\right) \frac{K_p(5s+1)\exp(-s)}{(10s-1)(5s-1)} \quad (9a)$$

where, K_p = 1. For calculating the new ultimate values, the amplitude ratio and the phase angle criteria of the new system [eq. (9a)] are written as:

$$A_r = \frac{\sqrt{\left(\tau_{D_p}\omega - \frac{1}{\tau_{I_p}\omega}\right)^2 + 1}}{\sqrt{10^2\omega^2+1}} \quad (9b)$$

$$\varphi = \tan^{-1}(\tau_{D_p}\omega - \frac{1}{\tau_{I_p}\omega}) - \omega + 2\tan^{-1} 5\omega + \tan^{-1} 10\omega - 2\pi \quad (9c)$$

The phase angle criterion is solved numerically to get the new value of maximum frequency. At the crossover frequency, the phase lag of the system is π and the system generates a sustained oscillation.The minimum and maximum values of the frequency and the corresponding amplitude ratio are obtained as:

$$\omega_{c,min} = 0.4882 \quad (10b)$$
$$\omega_{c,max} = 2.2613 \quad (10c)$$
$$A_r|_{min} = 0.2017 \quad (11a)$$
$$A_r|_{max} = 0.0720 \quad (11b)$$

To make the overall system gain equal to 1,the controller gain is calculated ($K_c = \frac{1}{A_r}$) as:

$$K_{c,max} = 13.8717 \quad (12a)$$

$$K_{c,min} = 4.9569 \quad (12b)$$

The design value of the proportional gain of the controller is calculated as the average of the minimum and the maximum values of the controller. The controller parameters calculated are given in Table 1 for both the Ziegler Nichols method (eq. 6) and the proposed method.

Based on the controller settings (given in Table 1), the performance of the process is evaluated for a unit step change in the set point. An improved performance is obtained for the proposed method as seen in Figure 2(a). The improved performance is supported by the time integral performance analysis given in Table (2). For servo response, reduction of 59%, 83% and 59% are obtained in ISE, ITAE and IAE values for proposed method, in comparison with the ZN method. The performance of the process for a unit step change in the load (regulatory response) of the process is also shown in Figure 2(b). The transfer function model for the load is assumed to be same as that of the process transfer function. For the regulatory response, reduction of 50%, 81% and 59% in ISE, ITAE and IAE values are realized (Table 2).

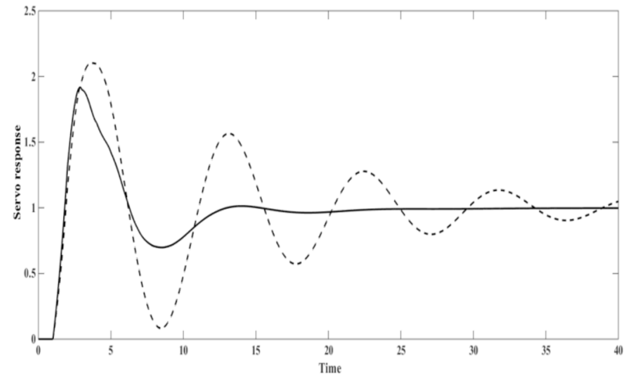

Figure 2(a): Servo response of the system (example 1)
Legend: Dash – ZN method; Solid: Proposed method

Table 1: Controller Settings

Controller parameters	ZN method	Proposed method
EXAMPLE 1		
$K_{c,des}$	7.7557	9.4143
τ_I	2.7716	5.1169
τ_D	0.6929	0.6084
EXAMPLE 2		
$K_{c,des}$	-6.5021	-9.0913
τ_I	0.5237	0.9668
τ_D	0.1309	0.1150

*τ_I = reset time; *τ_D = derivative time

*$K_{c,des} = \frac{K_{u,min} + K_{u,max}}{2}$ = Proportional Gain

Table 2: Time Integral Performance

Response	Method	ISE	ITAE	IAE
Example 1				
Servo	ZN	7.745	171.80	13.14
	Proposed	3.151	27.99	5.29
Regulatory	ZN	0.118	20.65	1.531
	Proposed	0.058	3.77	0.627
Example 2 (Non-Linear CSTR)				
Servo	ZN	0.187	1.357	0.751
	Proposed	0.284	0.746	0.701
Regulatory (10% disturbance added)	ZN	0.004	0.130	0.092
	Proposed	0.002	0.056	0.055

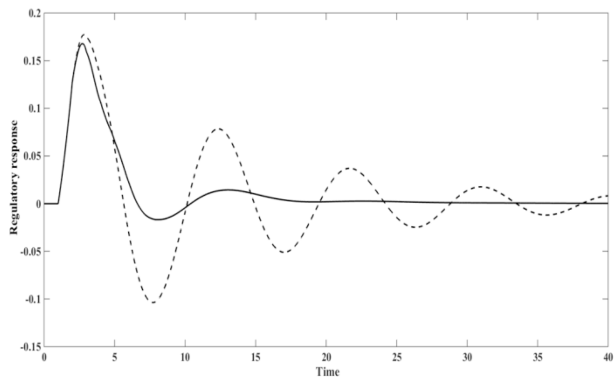

Figure 2(b): Regulatory response of the system (Example 1) Legend same as Figure 2(a)

The controller is designed based on the transfer function of the process, but there are always some chances of uncertainty in the process parameters. It is important that the controller parameters are designed in such a way that parameters are least affected by the change in the process dynamics. To ensure the robustness of the controller, the maximum sensitivity is determined.

The maximum sensitivity is defined as [27]:

$$Ms = \max_{\omega} |S(\omega j)| = \max_{\omega} \frac{1}{1+P(j\omega)C(j\omega)} \quad (13)$$

The shortest distance from the Nyquist curve of the loop transfer function to the critical point (-1, 0) is being equal to 1/Ms. In order to obtain a high robustness, small values of Ms are of interest. The maximum sensitivity can be related to the phase margin and gain margin as it simultaneously ensures both the following constraints [28]:

$$A_m \geq \frac{Ms}{Ms-1}, \text{ and } \varphi_m \geq 2\sin^{-1}(1/Ms) \quad (14)$$

The proposed method is compared with ZN method based on the maximum sensitivity, phase margin and gain margin as given in Table 3. It can be clearly seen in the Table 3 that the proposed method is more robust than the ZN method. The values for Maximum sensitivity (3.1582), phase margin (36.9196) and gain margin (1.4634) obtained for proposed method are better than ZN method indicating that the closed loop system is less sensitive to variations in the process dynamics.

Table 3: Maximum Sensitivity, Gain Margin and Phase Margin Comparison (Example 1)

Performance measure	ZN method	Proposed method
Maximum sensitivity	5.5782	3.1582
Phase margin	20.6545	36.9196
Gain margin	1.2184	1.4634

Example 2: Consider the following locally linearised transfer function model of non-linear continuous stirred tank reactor (CSTR) derived for measurement of the reactor temperature [28]. The system is unstable with 2 complex conjugate unstable poles (0.1851 ± 0.8457i) and a stable pole (-47.6559).

$$G_p(s) = -\frac{1.77(0.3186s+1)\exp(-0.15s)}{(0.028s^3+1.324s^2-0.479s+1)} \quad (15)$$

The same procedure, for the design of PID controller, is followed as in example 1. The amplitude ratio and the phase angle criteria equations are formulated as:

$$A_r = \frac{1.77\sqrt{(0.3186^2\omega^2+1}}{\sqrt{(1.324\omega^2-1)^2+(0.028\omega^3+0.479\omega)^2}} \quad (16)$$

$$\varphi = -0.15\omega + \tan^{-1}(0.3186\omega) - \pi - \tan^{-1}\frac{0.028\omega^3+0.479\omega}{1.324\omega^2-1} \quad (17)$$

To obtain a sustained oscillation, the system is brought on the verge of stability. On analysing the sustained oscillations, ultimate value of the proportional gain and ultimate frequency can be determined.

At the cross over frequency; $\varphi = -\pi$ (17a)

Eq.(17) can be written as:

$$-0.15\omega + \tan^{-1}(0.3186\omega) - \tan^{-1}\frac{0.028\omega^3+0.479\omega}{1.324\omega^2-1} = 0 \quad (17b)$$

Eq.(17b) is solved numerically to get the minimum value of frequency and maximum value of frequency:

$$\omega_{c,min} = 0 \quad (18a)$$

$$\omega_{c,max} = 5.9990 \quad (18b)$$

The values are substituted in eq. (16). The values of amplitude ratio at minimum and maximum frequencies are calculated as:

$$A_r|_{min} = -1.7699 \quad (19a)$$

$$A_r|_{max} = -0.0804 \quad (19b)$$

As mentioned above, in order to make the overall system gain unity, the minimum and maximum value of controller gain are set equal to the inverse of the amplitude ratio. The calculated values and the ultimate period of oscillation are given as: $K_{c,min} = -0.5650$ (20a)

$$K_{c,max} = -12.4392 \quad (20b)$$

$$P_u = 1.0474 \quad (21)$$

Based on which the reset time and the derivative time are calculated as:

$$\tau_I = \frac{P_u}{2} = 0.5237 \quad (22a)$$

$$\tau_D = \frac{P_u}{8} = 0.1309 \quad (22b)$$

Using the values obtained in above equations, controller based on ZN method is designed.

For the proposed method, the values of reset time and the derivative time are determined using the eq. (7) which are then converted into the corresponding parallel form using the eqs. (8a) and eqs. (8b). For determining the new ultimate value, the PID controller transfer function, with K_c =1 and with the calculated τ_{I_p} and τ_{D_p} is added to the system. The new system thus formed is given by Eq.(23).

$$G_p(s) = \left(1 + \frac{1}{\tau_{I_p}s} + \tau_{D_p}s\right)\left(\frac{-1.77(0.3186s+1)\exp(-0.15s)}{(0.028s^3+1.324s^2-0.479s+1)}\right) \quad (23)$$

The equations for the amplitude and the phase angle criteria are formulated for the new system [eq.(23)] as

$$A_r = \left(\sqrt{\left(\tau_D\omega - \frac{1}{\tau_I\omega}\right)^2 + 1}\right)\left(\frac{1.77\sqrt{(0.3186^2\omega^2+1}}{\sqrt{(1.324\omega^2-1)^2+(0.028\omega^3+0.479\omega)^2}}\right) \quad (24)$$

$$\varphi = \tan^{-1}(\tau_D\omega - \frac{1}{\tau_I\omega}) - 0.15\omega + \tan^{-1}(0.3186\omega) - \pi - \tan^{-1}\frac{0.028\omega^3+0.479\omega}{1.324\omega^2-1} \quad (25)$$

At crossover frequency: $\varphi = -\pi$, On solving eq.(25), we get,

$$\omega_{c,min} = 0.7133 \quad (25a)$$

$$\omega_{c,max} = 13.3988 \quad (25b)$$

corresponding to which we get

$$A_r|_{min} = -6.4097 \quad (26a)$$

$$A_r|_{max} = -0.0555 \quad (26b)$$

To make the overall loop gain of the system equal to one, the controller gain is set equal to $1/A_r$. This leads to the determination of the updated minimum and maximum controller gain respectively as -0.1560 and -18.0266.

The controller is designed based on the parameters calculated for both the cases as shown in Table 1. The performance of the controller is evaluated for the Non-Linear CSTR equation (Appendix) for a step change in the input (x_2 from 3.20 to 3.52) and for a step change in disturbance (h_c from 1.5 to 1.65). Figure 3(a) and Figure 3(b) illustrate the appreciable performance of the proposed method over ZN continuous method for the Non-Linear CSTR. The enhanced performance is supported by the time integral performance analysis given in Table (2). Based on Table (2) reduction of approximately 45% and 40% in ITAE and IAE values is obtained for the servo problem. The improved performances with the reduction of 50%, 57% and 40% in ISE, ITAE and IAE values are realized for step change in load disturbance. For Non-linear system, the overshoot is more (Figure 3a) so there is a need to add set point filter or slightly detune the parameter. However, for linearised system (eq. 15) reduction of 76% in ISE value and 15% in overshoot are obtained for a step change in input.

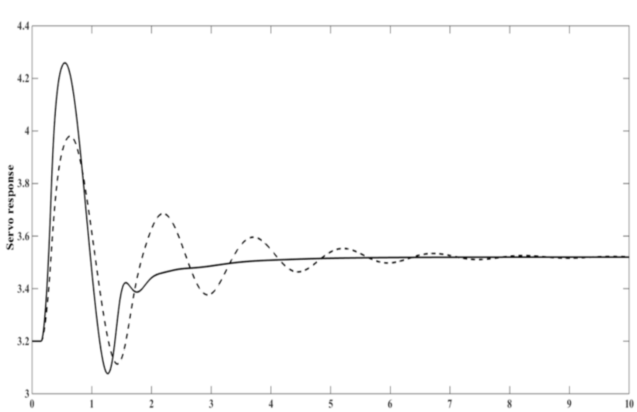

Figure 3(a): Servo response of Non-linear CSTR (example 2); Legends same as Figure 2(a)

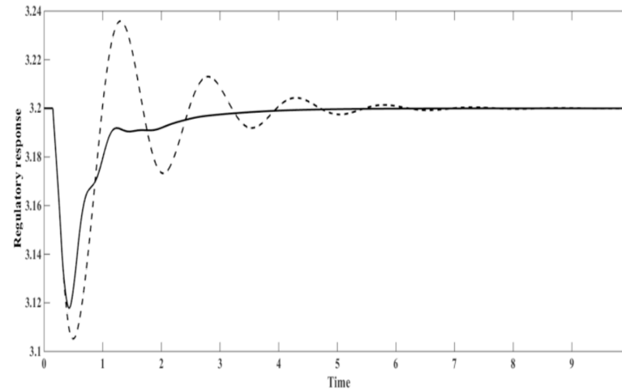

Figure 3(b): Regulatory response of Non Linear CSTR when 10% disturbance is added; Legends same as Figure 2(a).

IV. CONCLUSION

An improved continuous cycling method for tuning the PID controllers is proposed. The simulation results of unstable SOPTD model with two unstable pole and on a non linear CSTR model with time delay and complex conjugate unstable poles are given. Improved dynamic performances along with robustness are obtained with the proposed method. Significant improvements are obtained in the time integral performance analysis for both the servo and the regulatory responses.

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Appendix:

The Non-Linear equation of CSTR [29,30] is given by

$$\frac{dx_1}{dt} = \left[-D_a x_1 \exp\left(\frac{x_2}{1+\left(\frac{x_2}{E_a}\right)}\right)\right] + (1 - x_1) \quad \text{(A1)}$$

$$\frac{dx_2}{dt} = \left[QD_a x_1 \exp\left(\frac{x_2}{1+\left(\frac{x_2}{E_a}\right)}\right)\right] - [(1+h_c)x_2] + h_c x_3 \quad \text{(A2)}$$

$$\frac{dx_3}{dt} = 10[q_c(-1 - x_3) + h_c(x_2 - x_3)] \quad \text{(A3)}$$

x_1, x_2 and x_3 are the dimensionless concentration, reactor temperature and cooling jacket temperature. The other parameter values are heat transfer coefficient $(h_c) = 1.5$, Damköhler number $(D_a) = 0.135$,
Heat of reaction $(Q) = 11$, Coolant flow rate $(q_c) = 3.2$
and Activation Energy $(E_a) = 20$.

Using these parameter values and taking x_2 as the controller variable and q_c as the manipulated variable, the three equilibrium points (steady state) are

$$[x_{1s}\ x_{2s}\ x_{3s}] = [0.6861\ \ 1.3\ -0.1355] \quad \text{(A4)}$$

$$[x_{1s}\ x_{2s}\ x_{3s}] = [0.5460\ \ 2.0\ \ 0.0036] \quad \text{(A5)}$$

$$[x_{1s}\ x_{2s}\ x_{3s}] = [0.3195\ \ 3.2\ \ 0.3429] \quad \text{(A6)}$$

The system is considered to be at steady state condition (A6) at t = 0. For the given condition of operating point, the locally linearised model is given by eq. (15).

Nomenclature:

A_r amplitude ratio
K_c controller gain
K_P process gain
Pu period of oscillation
τ time constant
τ_I, τ_D integral and derivative time (parallel form)
τ_I', τ_D' integral and derivative time (series form)
ω frequency
ω_c cross over frequency
φ phase angle

subscript:

min minimumvalue
max maximum value