# Complete Force Balancing of Spatial Linkages:[1]

**G. G. LOWEN[2] AND R. S. BERKOF.[3]** The authors are to be complimented on their fine work on the balancing of certain spatial linkages. The stretch-rotation operator concept is a valuable tool for describing the location of the total mechanism center of mass.

The discussers would like to interpret the authors' results concerning the RSSR mechanism somewhat differently and show that, whenever the center of mass of the coupler link lies on the line connecting the spherical joints, the mechanism can be completely force balanced.

With the coupler masses $m_2$ and $b_2$ on the link centerline, the expressions in the parentheses of equations (14) and (15), which are defined in equations (17) and (18) as $(-\lambda_1)$ and $(-\lambda_2)$, can be rewritten as follows:

$$m^*_2\left(1 - \frac{r_2^*}{a_2}\right) = -\lambda_1 = m_2', \tag{1}$$

$$m_2^* \frac{r_2^*}{a_2} = -\lambda_2 = m_2'', \tag{2}$$

where $m_2^*$ represents the combined mass of the coupler link, and $r_2^*$ represents the position of this center of mass (generally, there is no reason for adding a mass $b_2$ to the coupler link), such that:

$$m_2^* = m_2 + b_2, \tag{3}$$

$$r_2^* = \frac{m_2 r_2 + b_2 r_{b_2}}{m_2 + b_2}. \tag{4}$$

When considered in the aforementioned form, $m_2'$ and $m_2''$ are the static replacement masses[4] for the coupler link at the spherical joints of the input and output links, respectively.

Thus, equations (19) and (20) represent spatial vector systems:

$$\frac{m_1}{a_1}\mathbf{r}_1 + \frac{b_1}{a_1}\mathbf{r}_{b_1} = -m_2'\frac{\mathbf{a}_1}{a_1}, \tag{5}$$

$$\frac{m_3}{a_3}\mathbf{r}_3 + \frac{b_3}{a_3}\mathbf{r}_{b_1} = -m_2''\frac{\mathbf{a}_3}{a_3}, \tag{6}$$

which can be written finally as:

$$b_1\mathbf{r}_{b_1} = -\left[m_1\mathbf{r}_1 + m_2\left(1 - \frac{r_2^*}{a_2}\right)\mathbf{a}_1\right], \tag{7}$$

$$b_3\mathbf{r}_{b_3} = -\left[m_3\mathbf{r}_3 + m_2^*\frac{r_2^*}{a_2}\mathbf{a}_3\right]. \tag{8}$$

Consequently, if counterweights $b_1$ and $b_3$ are arranged according to equations (7) and (8), the "combined" centers of mass of links 1 and 3 will lie on their respective axes of rotation, and full static force balance is attained. Equations (21) and (22) represent the special inline case of this solution in which the vector expressions reduce to scalar form.

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[1] By R. E. Kaufman and G. N. Sandor published in the May, 1971, issue of THE JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 93, No. 2, pp. 620–626.

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[4] Timoshenko, S., and Young, D. H., *Advanced Dynamics*, McGraw-Hill, New York, 1948, p. 137; and Maxwell, R. L., *Kinematics and Dynamics of Machinery*, Prentice-Hall, Englewood Cliffs, N. J., 1960, pp. 270–273.

# Tool and Engineering Materials With Hard and Wear-Resistant Infusions[1]

**S. VAIDYANATHAN.[2]** The idea of combining the excellent shock resisting properties of high-speed steel with the hardness and wear-resistance of materials like sintered carbides has been tried out since the late 1950's and it is rather surprising that no mention of this process has been made in the paper by A. O. Schmidt. Some brief details of the process and the performance of HSS tools so treated are mentioned here.

The process by which carbides, graphites, tungsten, and other hard materials are deposited on the HSS tool surfaces is known as "spark-hardening" or "spark-toughening" or "carbide impregnation." Special purpose equipment for spark-hardening was developed in Russia in 1950 and by 1958 there were commercial machines of British and continental makes employing this principle. According to Russian sources [1][3] the method of spark-hardening is quite extensively used to improve the life of cutting tools, the erosion resistance of turbine blades, and the wear resistance of rock drills, plough shares, rolling stock wheel rims, and other various machine members.

Spark-hardening is a process in which a metallic surface is hardened as a result of the action of electric sparks. A hard metal serves as the anode and the softer metal as a cathode. Basically an R-C relaxation circuit (similar to the one used in earlier spark erosion machines) is employed to supply the energy for the sparks. Each spark melts a small amount of metal on the cathode surface and this volume is transferred through the spark channel. The material is deposited on the cathode material.

Spark-hardening has been applied to various types of cutting tools made of HSS and the published results of the tool performance were favorable. Some of the results are given in Table 1.

It is interesting to note that electron beams and lasers could also be applied to infuse hard materials on HSS tools to improve their performance. But it is felt that unless much better performance than what has been reported is possible, the spark-hardening method may still prove to be a simplified and economical process.

In spark-hardening of cutting tools occasional unfavorable results have been reported [3, 4]. This has been attributed to the formation of retained austenite in an intermediate layer. The prime cause for this is said to be the high temperature of spark followed by rapid quenching. Such thermal effects may be present particularly while employing high heat energies such as those encountered with electron beams and lasers.

**Table 1**

| Type of tool | Work material | Cutting conditions | Increase in tool life |
|---|---|---|---|
| Drill [1] | Steel 55 ($H_B$ = 220) | $v$ = 19 m/min<br>$f$ = 0.15 mm/rev<br>$D$ = 20 mm | 5 to 8 times |
| Drill [1] | Steel 45 (hardened to give $H_B$ = 270) | $v$ = 25 m/min<br>$f$ = 0.15 mm/rev<br>$D$ = 20 mm | 3 to 6 times |
| Turning [1] | Steel 45 ($H_B$ = 200) | $v$ = 60 m/min<br>$f$ = 0.23 mm/rev<br>$t$ = 2 mm | 4 to 10 times |
| Turning [2] | Steel CK60 ($H_B$ = 220) | $v$ = 45 m/min<br>$f$ = 0.20 mm/rev<br>$t$ = 2 mm | 4 to 16 times |

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[1] By A. O. Schmidt, published in the August, 1969, issue of the JOURNAL OF ENGINEERING FOR INDUSTRY, TRANS. ASME, Series B, Vol. 91, No. 3, p. 549.

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[3] Numbers in brackets designate References at end of discussion.

Another question regards the ease with which the process mentioned could be applied to infuse material on tools like drills, milling cutters and form tools. This is made possible in spark-hardening by fixing the hard material electrode in a hand held holder, which can be easily manipulated.

## References


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2 Böhme, W., "Standzeiterhöhung von Werkzeugen Mittels des Elektrofunkenverfahrens," *Fertigungstechnik* Vol. 14, No. 12, Dec. 1964, pp. 757–760.

3 Welsh, N. C., "Spark-Hardening of Tools," *International Research in Production Engineering, Proceedings of the Pittsburgh Conference*, ASME, 1963.

4 Reumscüssel, K., "Elektrofunkenbehandlung von Schneidwerkzeugen zum Zwecke der Standzeiterhöhung," *Fertigungstechnik*, Vol. 15, No. 9, Sept. 1965, pp. 545–549.

5 Welsh, N. C., "Spark-Hardening of Cutting Tools and Retained Austenite Formation," *Journal of Iron and Steel Institute*, Vol. 198, No. 5, May 1961, pp. 30–37.


## Authors' Closure

The discussion of Mr. S. Vaidyanthan concerns other techniques to improve high-speed-steel tools. There are several of these types commercially available in this country, but their application has been limited. The laser infusion of carbide in HSS tools has brought about marked improvement in shop performance when other treatments did not work. These tests are still running and will be reported at another time.