Three-dimensional characterization of engineering surfaces using triangular motifs

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Abstract: The functional behaviour of manufactured surfaces is influenced by the surface topography. Encouraged by the results obtained in the implementation of two-dimensional motifs for practical profiles, a few attempts have been made to arrive at areal motifs for characterization of the surfaces. In this paper, an approach for establishing three-dimensional motifs based on triangular motif is presented. The triangular motif is formed with three peaks and a deep valley between them. To represent the surface by significant motifs, initial motifs are combined using four rules that are extended from the rules defined in ISO 12085 for combining two-dimensional motifs. Application of this technique to practical surfaces shows interesting results for characterizing the manufactured surfaces.

Keywords: surface roughness, three-dimensional motifs, triangulation, motif characterization

1 INTRODUCTION

Manufactured surfaces deviate from the ideal shape due to variations inherent in the process. These variations or errors arise from improper component setting, imperfections in the machine, and the nature of the manufacturing process itself. However, the wavelengths of errors introduced by these factors are found to be different. On the basis of wavelengths, errors present in the surface are classified into form error, waviness, and roughness. Generally, to characterize the surface and to study the behaviour of surface under different functional requirements, the errors mentioned earlier are separated by filtering. Several approaches are used for filtering, and methods based on envelope and motifs form alternative approaches [**1**, **2**]. Among these, the motif method of surface roughness evaluation has been found to be suitable for functional evaluation of surfaces [**3–6**]. Earlier characterization techniques were based on two-dimensional profile of the surface, primarily because of limitations of the instruments for the surface measurement. With the advances in measurement techniques and the perceived need for functional analysis of the three-dimensional surfaces, three-dimensional characterization of surfaces has gained importance. With this trend of three-dimensional analysis, many of the two-dimensional approaches have been extended to three-dimensional analysis. However, motif method of profile characterization does not have an obvious extension to three-dimensional surfaces.

A few attempts have been made to arrive at three-dimensional motifs for characterization of the surfaces. Zahouani [**7**] used the definition of a three-dimensional motif as the association of the lowest point of a scratch with two neighbouring summits located in a plane perpendicular to the scratch direction. Chen *et al.* [**8**] considered a three-dimensional motif as the association of four peaks with two three-dimensional peaks. The three-dimensional peak is defined as a point higher than four of its neighbours, whereas two-dimensional peak is defined as a point higher than two of its neighbours. For combination of three-dimensional motif elements, four combination rules are presented. In addition, this approach forces the motif shape to be a rectangle, oriented along the inspection direction. Therefore, measurement orientation must be along or orthogonal to preferred direction, where preferred direction is the

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direction of lay of machining. Morphological change tree approach has been used by Scott [**9**], which employs the concept of Maxwelian hills and dales. Barré and Lopez [**10**, **11**] have used the approach of watershed lines to identify the motif shapes. It is claimed that the two-dimensional motif is simply obtained as the intersection of the three-dimensional motif with a vertical plane. However, formation of motif is based on the watershed line and the combination of motifs is done on the basis of area alone without considering the depth characteristics. There is an obvious deviation from the concept of profile motifs envisaged originally by the automobile and related industries in France, which started with an attempt to evaluate depth and spacing of the irregularities considering significant peaks and valleys of the profile graphically. Two-dimensional motif does not take into account the nature of the profile within a motif.

As mentioned earlier, the present work attempts to characterize the surface by arriving at the triangular motifs connecting the peaks on the surface. These basic motifs are then combined by applying rules that are framed as an extension of two-dimensional motif combination rules to three-dimensional motifs. Taking hard-turned, ground, and stone-honed surfaces as examples of practical surfaces, three-dimensional motifs are established and the results are included in this paper. Finally, certain parameters have been defined for characterization of these motifs and their analysis is presented.

2 AREAL MOTIF METHOD

2.1 Definition of an areal motif

A motif can be defined as a design with recurring shapes and structures. The three-dimensional motif defined in this paper is based on a triangle with three vertices at three peaks. The triangular shape of the motif is chosen, because it forms the simplest planar figure and any complex surface can be approximated by triangles. Figure 1 shows a three-dimensional triangular motif. The dimensions Z_1, Z_2, and Z_3 indicate the heights of peaks from a datum that is taken as zero. It is important to know another dimension known as characteristic depth. The characteristic depth of a motif is the smallest of the relative heights of the peaks of a motif from a valley within it. Referring to Fig. 1, the relative heights are H_1, H_2, and H_3 and the characteristic depth CH is H_1.

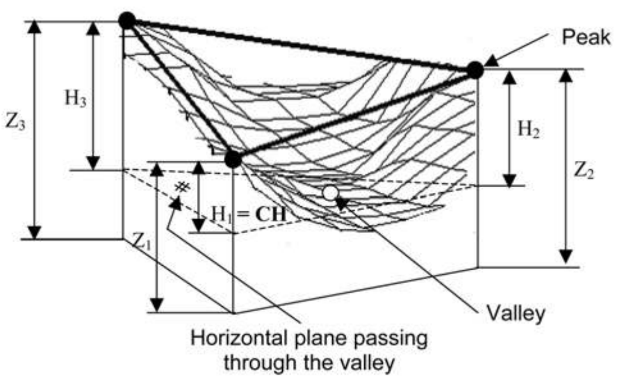

Fig. 1 Basic triangular motif

2.2 Identification of peaks

Scott [**12**] discusses the topological properties given by Euler and Kovalevsky, which need to be considered while identifying peaks on a three-dimensional surface. On the basis of this, definition for a peak in a three-dimensional surface is given as follows: 'there exists a neighbourhood which contains the critical point (peak) such that all closed paths within the neighbourhood which contain the critical point (peak) are lower in height than the critical point'. Here, it is important to define the neighbourhood of every discrete point on the surface. Earlier definition of four-point neighbourhood fails to meet the topological properties. Takashashi [**13**] defines the neighbourhood relation of a point by triangulating the surface, which is consistent with the topological properties mentioned by Euler and Kovalevsky. On the basis of this definition, all the critical points, which include the peaks, pits, and saddle points, can be extracted in a topologically correct way. It is shown that the peaks and valleys based on 'eight-point' neighbourhood remain as peaks and valleys throughout all possible triangulations [**12**]. Therefore, the peaks obtained by considering neighbourhood relation in a triangulated surface will hold true for the peaks defined with eight-point neighbourhood relation. With this consideration, this paper makes use of the eight-point neighbourhood relation for identification of peaks. Every point on the surface is checked against its eight neighbourhood points. If the point under consideration is higher than all of its neighbours, then the point is marked as a peak. This check is done for all the individual points on the surface except the boundary points.

2.3 Generation of motifs

For initial formation of triangles, the well-known Delaunay triangulation technique is used [**14**]. The Delaunay triangulation technique ensures that the minimum angle of any of the triangle is maximized. In other words, it tries to avoid the formation of long wedge-shaped triangles and triangles with obtuse angle. The Delaunay triangulation is unique (independent of the order in which the sample points are ordered) for all but a few trivial cases.

One such trivial case occurs when four points lie on the corners of a rectangle and the peaks are ordered differently. On the basis of ordering, the rectangle may be triangulated in one of two ways. In present work, the peaks are searched along parallel sections and ordering of peaks always starts from one end and hence such a situation is unlikely to occur.

3 MOTIF COMBINATION RULES

From the literature, it is seen that the French Standard allows different methods of motif combination for two-dimensional profiles depending on the surface property required. In the present work, four combination rules are extended from the two-dimensional motif rules defined in ISO 12085 [**3**]. This requires that the identification of the motifs to be combined has to be done by extracting their connectivity with other motifs and peaks. Thus, every peak will have the information about the peaks and motifs connected to it. For the combination rules discussed subsequently, all motifs connected to every peak are checked for possible combination. If all the four combination conditions are satisfied, then the peak under consideration is removed and new motifs are formed by re-triangulation. These four rules for combining the three-dimensional motifs are given subsequently.

3.1 Envelope condition

Any peak on the surface cannot be removed and its neighbour motifs cannot be combined, if this peak is larger than all the neighbour peaks. This condition will ensure that the important projecting peaks are not eliminated. However, if this peak is smaller than any of the neighbour peaks, then this peak can be considered for removal and new motifs can be formed with neighbour peaks. This situation where the combination is possible is shown graphically in Fig. 2.

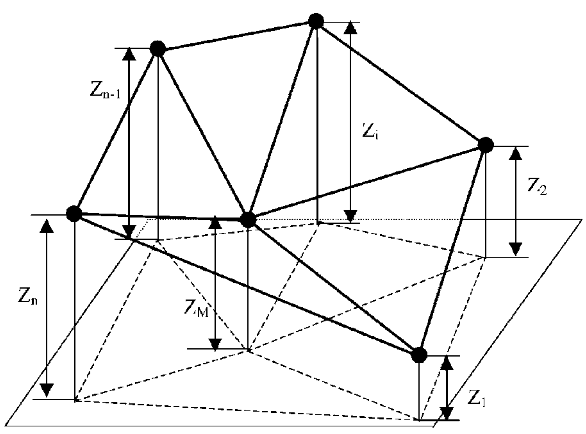

Fig. 2 Envelope condition (Rule 1): [$Z_M \leqslant \max(Z_i)$]

3.2 Size condition

Neighbour motifs of any peak can be combined by removing the peak if the lengths of projections on the datum for all edges of possible motifs are less than or equal to a size limit. This limit is conceptually same as the roughness cutoff used in the conventional filtering. Depending on the measurement condition and the surface details of interest, different size limits such as 20, 100, 500, and 2500 μm are recommended for two-dimensional motif combination. In the present work, 100 μm is taken as size limit to illustrate concepts involved in the motif combination rule on the basis of size condition. Figure 3 shows the condition under which combination of motifs can be done.

3.3 Magnification condition

A peak can be removed and the neighbour motifs are combined, if the largest characteristic depth of all the

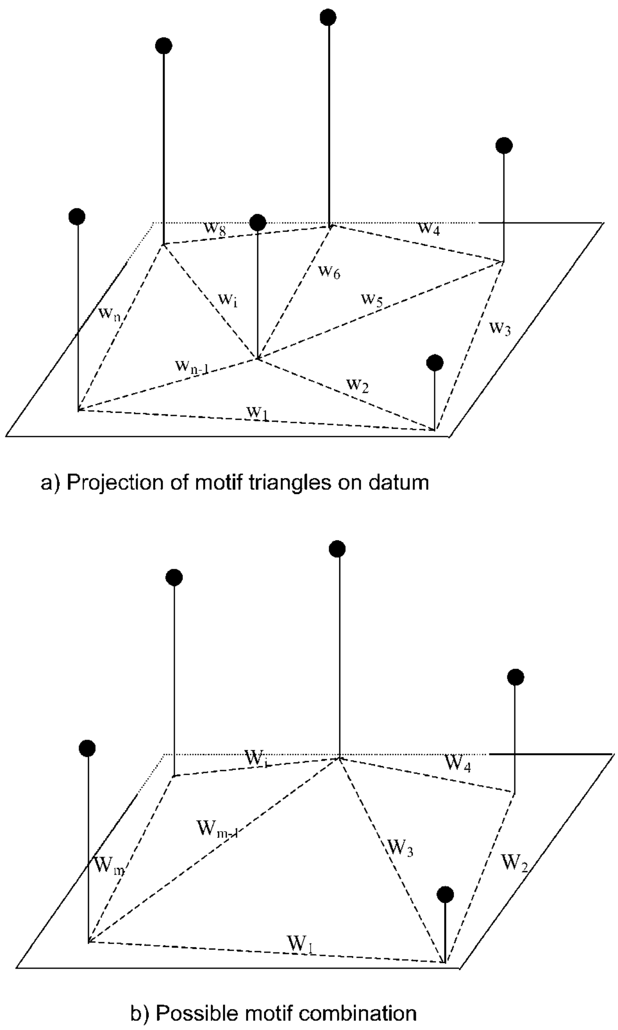

Fig. 3 Size condition (Rule 2): [$\max(W_i) \leqslant 100$ μm]. (a) Projection of motif triangles on datum and (b) possible motif combination

neighbour motifs is less than or equal to the largest characteristic depth of all the possible motifs. This condition eliminates a small peak that is relatively insignificant. The situation in which the combination is possible according to this condition is shown in Fig. 4.

3.4 Relationship condition

Neighbour motifs of any peak can be combined after removing the peak, if at least one of the characteristic depths of the neighbour motifs is $\leqslant 60$ per cent of the local reference depth T_R. Similar to two-dimensional case, there can be two possible local reference depths. In the first sequence, combination of motifs inside the 'segments' is carried out taking the local reference depth, T_R, as the characteristic depth of the segment in which the peak and its neighbours fall. The definition of a segment is given in section 4.3.3. Figure 5 shows the condition for combining the motifs, where the local reference depth, T_R, is shown for a segment. In next sequence, the local reference depth, T_R, can be taken as the maximum characteristic depth of all the possible motifs.

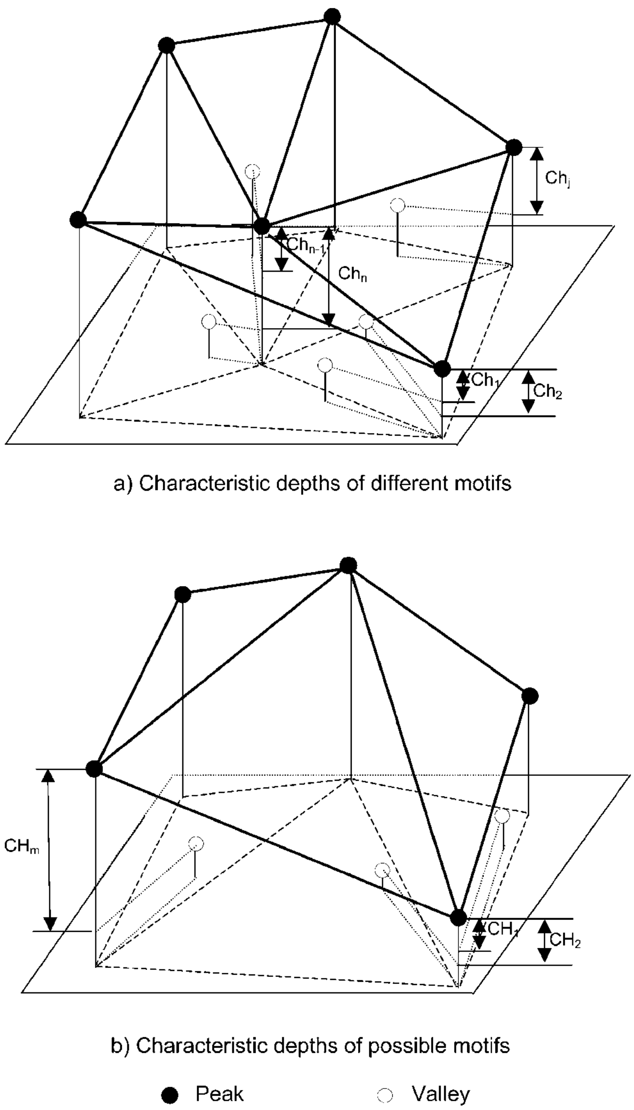

Fig. 4 Magnification condition (Rule 3): [max $(CH_i) > \max(Ch_j)$, $i = 1, 2, \ldots, m$; $j = 1, 2 \ldots, n$]. (a) Characteristic depths of different motifs and (b) characteristic depths of possible motifs

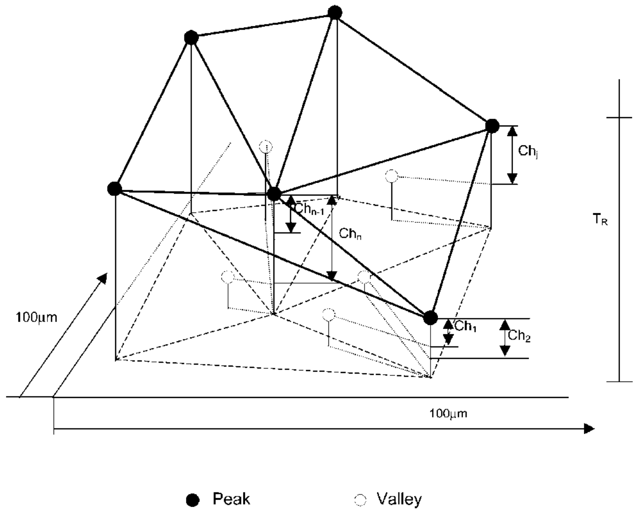

Fig. 5 Relationship condition (Rule 4): [$\min(Ch_j) \leqslant 0.6T_R$, $j = 1, 2 \ldots, n$] or ($\min(Ch_j \leqslant 0.6 \max(CH_i)$, $i = 1, 2, \ldots\ldots m$; $j = 1, 2, \ldots, n$)

4 IMPLEMENTATION

4.1 Identification of peaks

All the peaks on the surface are identified, if they satisfy the condition that the height of the peak is greater than that of its eight neighbour points as explained earlier.

4.2 Formation of triangular motifs

Initial formation of motifs is done using incremental algorithm for Delaunay triangulation, because it does not require the points to be in an ordered fashion. This technique ensures that triangles with shorter length of sides are formed by connecting closer peaks and hence maximizing the internal angles of triangles. The details of Delaunay triangulation technique are explained elsewhere [**14**].

4.3 Combination of motifs

4.3.1 Generation of peak–neighbour relation

In case of profiles, identification of a neighbour peak is very simple as there are only two adjacent peaks for a given peak. In case of surfaces, there are several peaks surrounding a given peak. To identify the neighbour peaks, triangular motifs formed in the initial phase by Delaunay triangulation are to be considered. For any peak p_i, its neighbour motifs are those motifs having this peak p_i as one of the vertices. Similarly, the neighbour peaks of a peak p_i are those peaks, which form the vertices of the neighbour motifs of peak p_i. Before combining the motifs, it is essential that the information about neighbour motifs and the neighbour peaks is generated for all the peaks. This neighbour information is stored for each peak on the surface.

4.3.2 General combination scheme

The motif combination rules are checked for every peak p_i on the surface. The first rule is fairly simple where the highest neighbour peak is compared with the peak p_i. Before proceeding to the second rule, a polygonal boundary is constructed with the peaks that are neighbours of peak p_i under consideration as shown in Fig. 6. Then all the possible triangular motifs with vertices that are the neighbours of peak p_i are formed excluding the peak p_i using Delaunay triangulation. It may be seen that peaks 3, 4, and 5 also form a motif, as Delaunay triangulation always forms a convex boundary. However, the motif formed by peaks 3, 4, and 5 does not fall within the polygonal boundary originally formed. To identify such motif, centroids of all possible motifs are obtained. If any of these centroids falls outside the polygonal boundary, the corresponding motif is removed from the newly formed list of motifs. Now, for checking the second rule, the lengths of projection of each edge of the possible motifs on datum plane are computed. If any length exceeds the size limit (say 100 μm), then second condition is violated. For the third rule, characteristic depth values are calculated for all the neighbour motifs and the possible motifs. With these characteristic depth values, third and fourth conditions are checked.

If a peak p_i and its neighbour triangles satisfy all the conditions, then the neighbour motifs of peak p_i are removed and the newly formed motifs are added to the list of motifs. In addition, the peak p_i is removed from the peak list. This process of checking individual peaks is continued for all peaks. However, peaks that are already associated with the motifs, which are removed from the triangle list, are not considered by suitably changing their status. This is done to ensure that previously generated neighbour information is not destroyed. After all the peaks have been checked, the motifs in the list of motifs and peaks in the peak list are stored into a file. In the second scanning, these motifs and peaks are read from the file and again the peak–neighbour information is established. With this information, the motifs are checked again for combination and this combination process is repeated until no more combination is possible with the motifs.

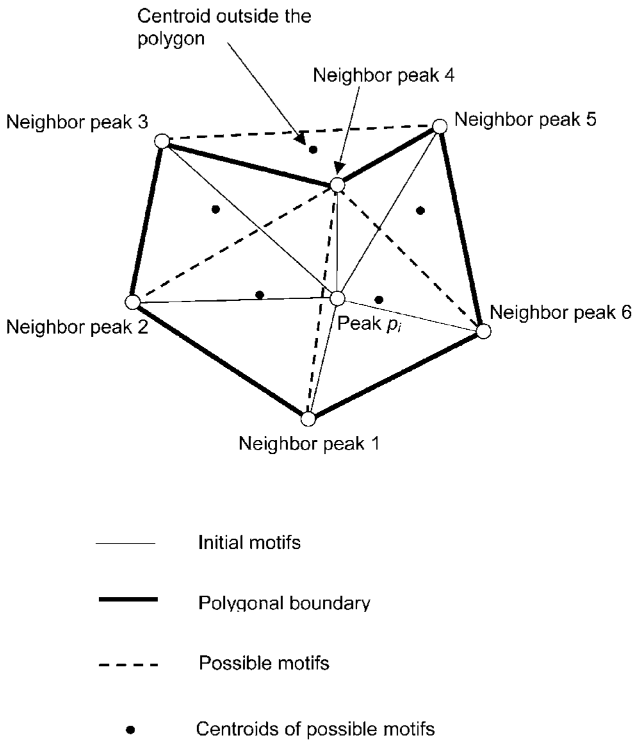

Fig. 6 Combining the motifs without changing the polygonal boundary

4.3.3 Combination within segments

The combination of motifs lying inside every segment has to be carried out first to have reproducible results [**3**]. The definition of a segment in two-dimensional case is defined in ISO 12085 on the basis of three conditions. These conditions have been extended to the three-dimensional case in this paper. A segment in three-dimensional case is defined as a triangular area connecting three peaks and satisfying the following conditions.

1. The projected distance between these three peaks is maximum.
2. The projected distance is smaller than or equal to size limit (100 μm).
3. There are no peaks inside the triangular area higher than these three peaks.

Figure 7 shows such a triangular segment formed by three peaks, namely, p_1, p_{11}, and p_{17} such that all the peaks lying inside the segment are lower than these three peaks. The local reference depth T_R, which is

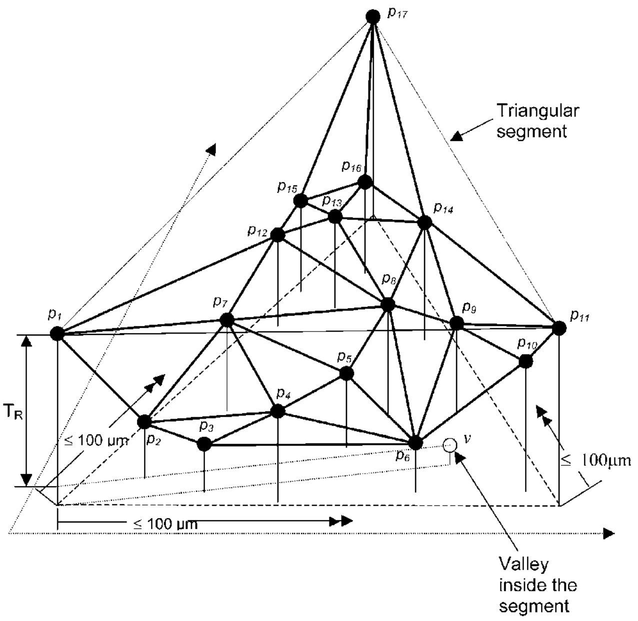

Fig. 7 Formation of a triangular segment

the characteristic depth of the segment is also shown. The characteristic depth is obtained by considering the difference between the peak p_1 (which is minimum among p_1, p_{11}, and p_{17}) and a deep valley v within the segment.

The formation of segments is again done with the Delaunay triangulation technique. All the peaks are arranged in descending order. Then, a super-triangle is formed such that it encloses all the peaks. Starting from the tallest peaks, the peaks are inserted one by one to form new triangles. If any triangle has length of edges less than the size limit, then no more points are inserted inside that triangle. The pseudocode of the algorithm is given in Appendix 2. While implementing this algorithm, care should be taken to see that insertion of a peak does not break down the segments formed already satisfying all the three conditions. This is particularly important for the triangles that share a common vertex with the super-triangle formed in Delaunay triangulation and have a peak lying on one edge.

For combination of motifs inside each segment, it is sufficient to check for three conditions, namely, envelope, magnification, and relationship conditions. During this combination process, the local reference depth is taken as the characteristic depth of the segment in which the peak under consideration falls. The combination of motifs is carried out in all segments of the surface until no further combinations are possible within each segment.

4.3.4 Combination over the whole area

In the second sequence, all the motifs resulting from the previous combination are considered for combination. For every combination, all the four conditions are tested. During this combination process, the local reference depth is taken as the maximum characteristic depth of the possible motifs. This combination process is continued until no further combination is possible on the whole area. This will result in a complete transformation of the surface into the largest possible motifs.

5 CHARACTERIZATION OF TRIANGULAR MOTIFS

For characterizing the triangular motifs, a parameter defining the depth of the motif is taken as

$$S = \frac{H_1 + H_2 + H_3}{3} \tag{1}$$

where H_1, H_2, and H_3 are the heights of peaks from the valley of respective motifs. This parameter is similar to the depth of the motif parameter defined in ISO 12085.

Apart from this, new parameters for shape and size differentiation are proposed here. For characterizing the size of the motifs, the area of the individual motifs can be taken. This is obtained by

$$A = \left\| \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right\| \tag{2}$$

where (x_1, y_1), (x_2, y_2), and (x_3, y_3) are the coordinates of the three vertices of a triangular motif.

To differentiate triangular motifs of different shapes, a shape factor is defined taking the area and perimeter.

$$\mathrm{SF} = k\frac{A}{P^2} \tag{3}$$

where A is the area of the motif computed using equation (2) and P is the perimeter given by

$$P = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} + \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2} + \sqrt{(x_1 - x_3)^2 + (y_1 - y_3)^2} \tag{4}$$

The value of k is calculated from P^2/A for an equilateral triangle as 20.785. With this value of k, SF is one for an equilateral triangle. A factor far less than one indicates acicular (long wedge shaped) triangle.

The predominant lay direction on the surface can be identified with a parameter given by

$$\alpha = \tan^{-1}(s_{\max}) \tag{5}$$

where $s_{\max}$ is the slope of the maximum altitude of the triangular motif. In the motif shown in Fig. 8,

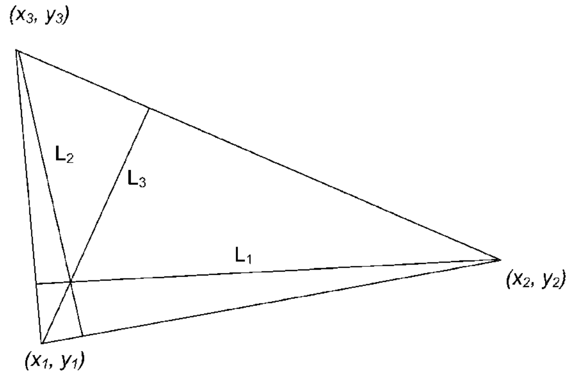

Fig. 8 Altitudes of triangular motif

the maximum altitude is L_1 and its slope can be obtained as

$$s_{\max} = -\frac{1}{m}$$

where $m = (y_3 - y_1)/(x_3 - x_1)$ is the slope of the edge perpendicular to the maximum altitude.

6 APPLICATION TO PRACTICAL SURFACES

The proposed three-dimensional triangular motif method is applied on practical surfaces, namely,

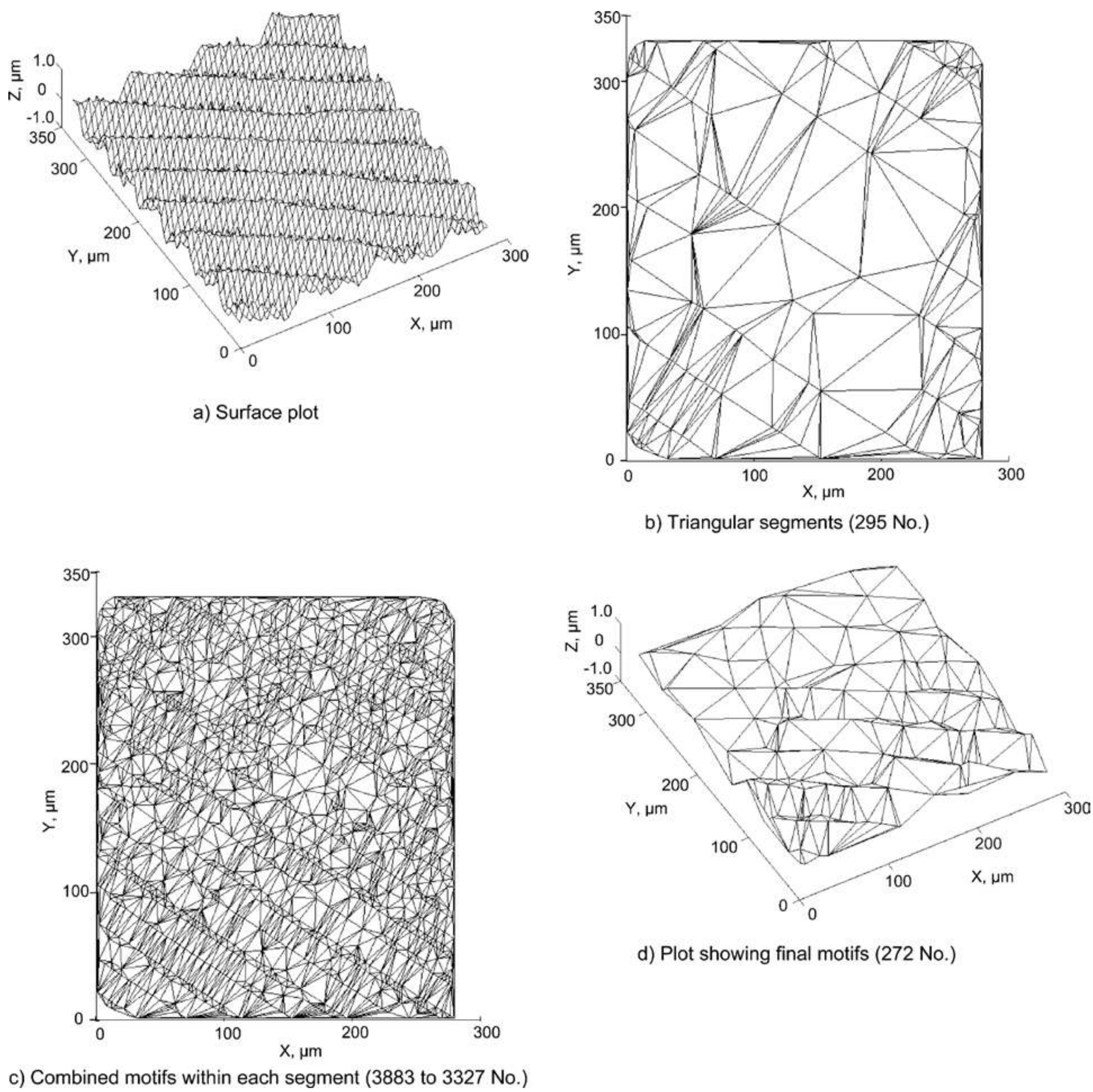

Fig. 9 Hard-turned surface: (a) surface plot, (b) triangular segments (No. 295), (c) combined motifs within each segment (No. 3883 to 3327), and (d) plot showing final motifs (No. 272)

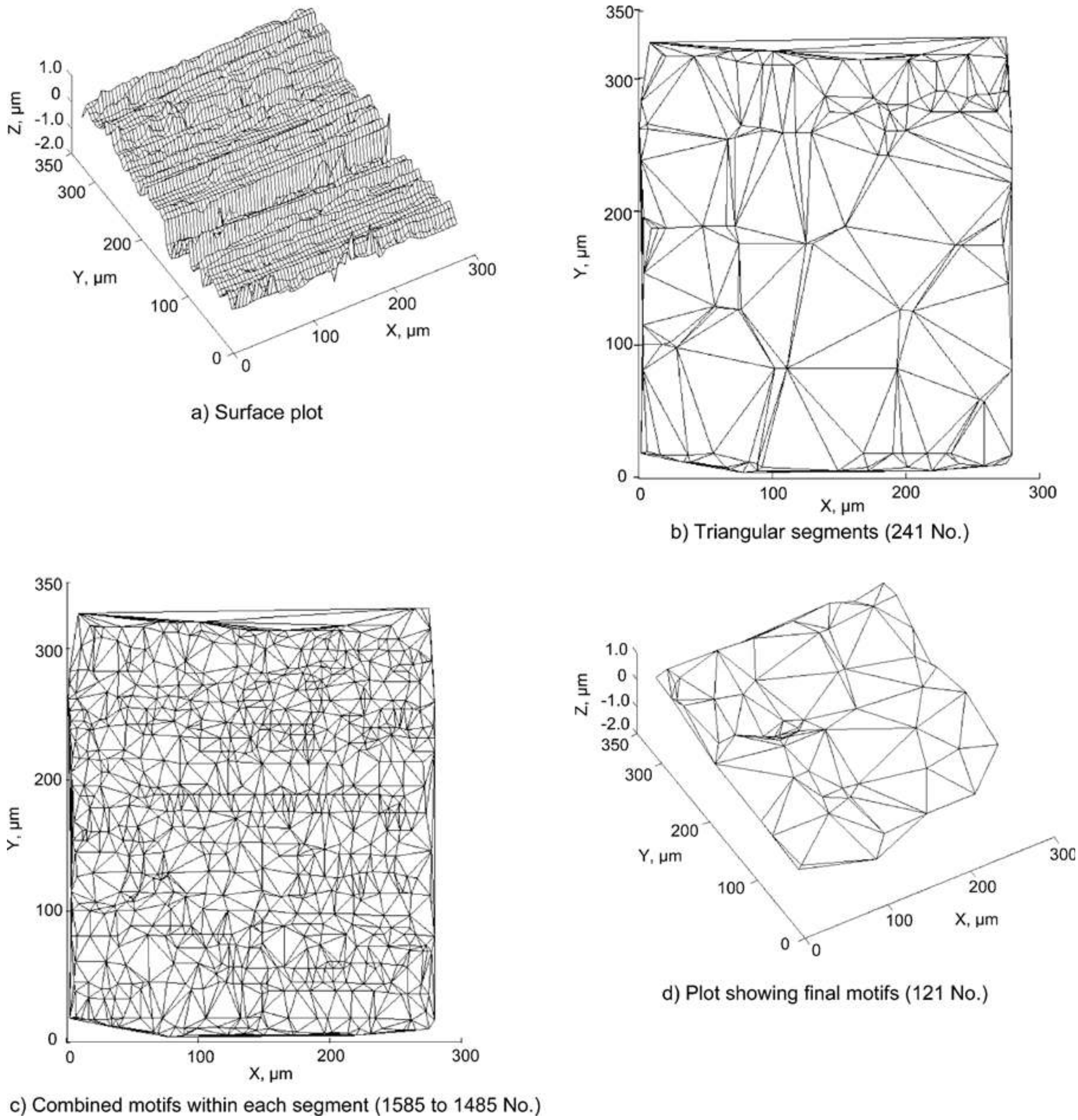

Fig. 10 Ground surface: (a) surface plot, (b) triangular segments (No. 241), (c) combined motifs within each segment (No. 1585 to 1485), and (d) plot showing final motifs (No. 121)

hard-turned, ground, and stone-honed surfaces. The three-dimensional measurement of the surfaces has been carried out using a tracing stylus instrument with a sampling interval of 1.1 μm along *x*-direction and with an interval between sections as 1.3 μm in *y*-direction. A stylus tip radius of 2.5 μm and a tracing speed of 0.5 mm/s are used. Figures 9(a), 10(a), and 11(a) show these three surfaces with 256 points in *x*-direction and 256 points in *y*-direction. For this area taken for analysis, the size limit is taken as 100 μm as shown in Figs 3 and 7. Figures 9(b), 10(b), and 11(b) show the triangular segments formed initially. The motifs obtained after combination within the segments are shown in Figs 9(c), 10(c), and 11(c). The reduction in the number of motifs is given in the corresponding figures. Figures 9(d), 10(d), and 11(d) show the final motifs formed on the hard-turned, ground, and stone-honed surfaces, respectively. The histograms of the motif parameters, namely, motif height, area of motifs, shape factor, and direction are shown in Figs 12, 13, and 14.

7 RESULTS AND DISCUSSION

From the final motifs formed on the surfaces (Figs 9(d), 10(d), and 11(d)), it is observed that

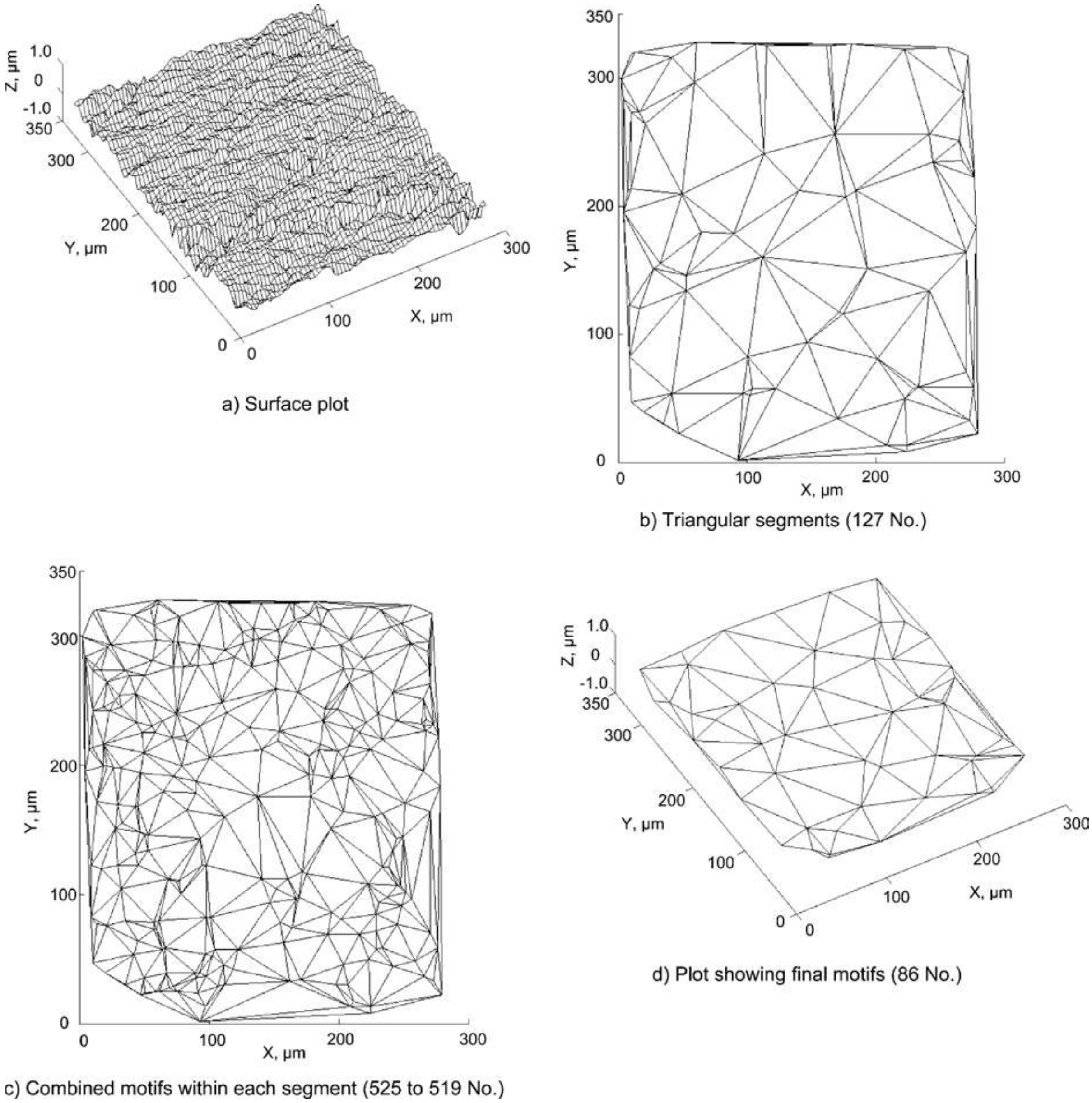

Fig. 11 Stone-honed surface: (a) surface plot, (b) triangular segments (No. 127) (c) combined motifs within each segment (No. 525 to 519), and (d) plot showing final motifs (No. 86)

hard-turned surface has many smaller motifs, whereas ground and stone-honed surfaces have much larger motifs. In addition hard-turned and ground surfaces show a number of acicular motifs. However, the height of motifs formed is not distinctly seen in these figures. More information is obtained from the histograms presented in Figs 12 to 14. Figure 12(a) shows the histogram of height of the motifs formed on the hard-turned surface. It can be seen that the average height of motifs is 0.97 μm, whereas for ground and stone-honed surfaces, the average heights of motifs are 0.96 and 0.71 μm, respectively, as shown in Figs 13(a) and 14(a). The spread of height values is larger in case of ground surface when compared with hard-turned and stone-honed surfaces. From this, it can be inferred that the surface height variation is uniform in case of hard-turned and stone-honed surfaces, because smaller spread indicates the presence of motifs with nearly same heights. In the case of ground surface, many of the motifs are in the range of 0.5–1.2 μm and few are ~2.0 μm and >2.0 μm. These motifs with heights >2.0 μm differ greatly from the average height value of 0.96 μm. It is therefore clear that there are some deep grooves or high peaks present on the ground surface, which tend to increase the height of some motifs much above the average value.

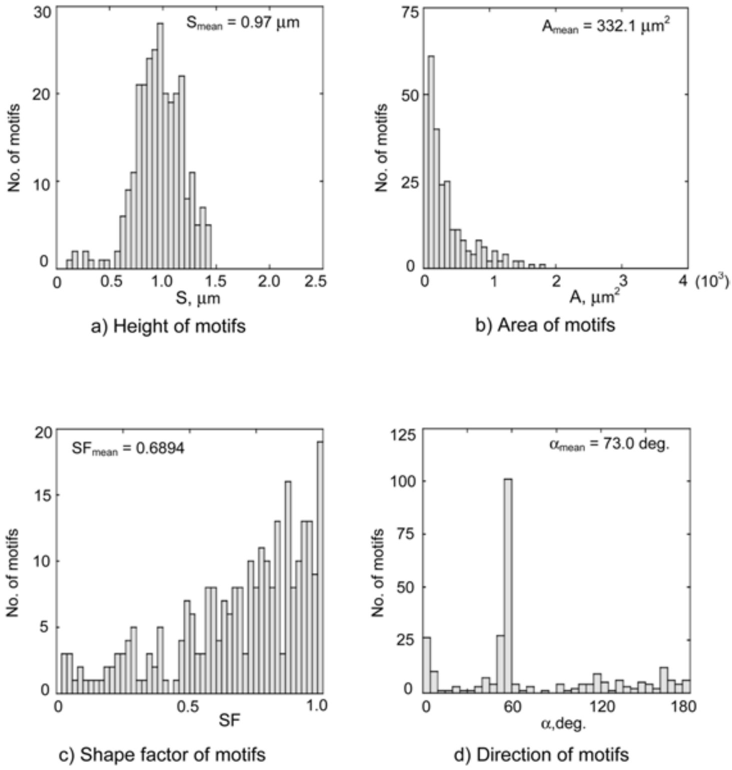

Fig. 12 Histogram of motif parameters for hard-turned surface: (a) height of motifs, (b) area of motifs, (c) shape factor of motifs, and (d) direction of motifs

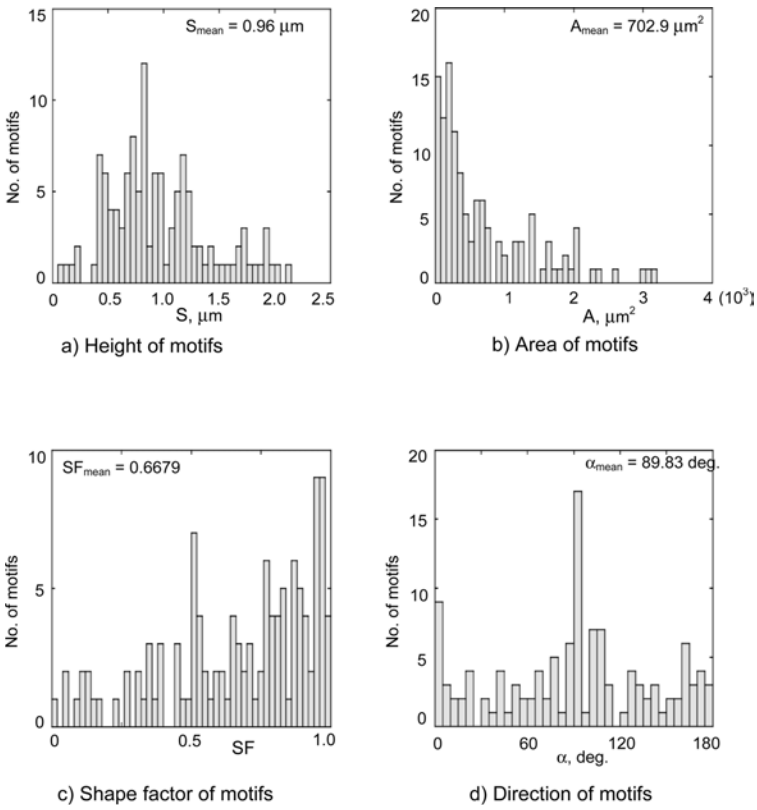

Fig. 13 Histogram of motif parameters for ground surface: (a) height of motifs, (b) area of motifs, (c) shape factor of motifs, and (d) direction of motifs

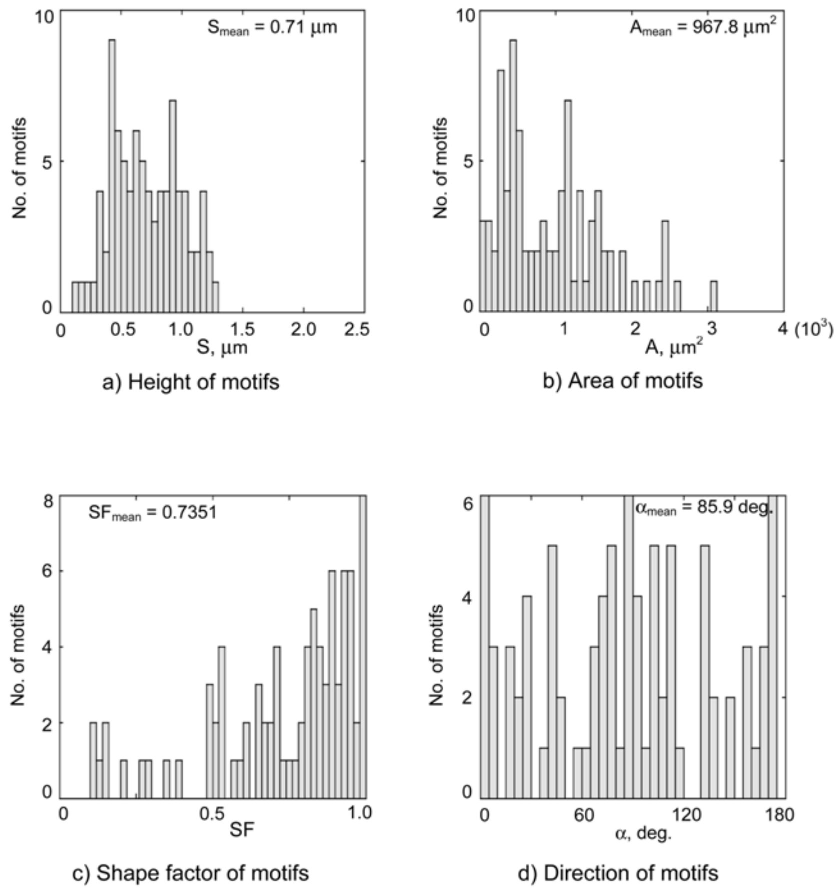

Fig. 14 Histogram of motif parameters for stone-honed surface: (a) height of motifs, (b) area of motifs, (c) shape factor of motifs, and (d) direction of motifs

The histogram of area of the motifs for hard-turned surface in Fig. 12(b) is more skewed to the left side and 215 motifs out of 272 have area $<500\ \mu m^2$. This happens as a number of significant peaks are lying closer and larger motifs are not formed by combination of motifs. In contrast, the histogram of area of motifs for ground and stone-honed surfaces shows the presence of both smaller and larger motifs. This is also seen from the mean area of motifs for all the three cases. However, the mean area of stone-honed surface ($967.8\ \mu m^2$) is larger than that for hard-turned and ground surfaces. Therefore, the density of significant peaks is lesser in case of stone-honed surface when compared with other surfaces.

Considering an ideal three-dimensional motif to be a pyramid, which has a base area equal to the mean area and a height equal to the mean height of motifs, the volume can be calculated as 106.96, 224.62, and $227.72\ \mu m^3$ for hard-turned, ground, and stone-honed surfaces, respectively. From these values, it can be inferred that the stone-honed surface exhibits better capability for lubricant retention. Load bearing capacity is influenced by factors such as number of peaks, peak curvature, and slope. In the present work, the number of motifs connecting the significant peaks for the hard-turned surface is 272 as against 121 and 86 for ground and stone-honed surfaces, respectively, for a given area. It is to be recalled that the peaks have been identified by considering eight neighbours. However, the peaks identified as dominant summit by radius of curvature and height would give a more realistic picture of load carrying capacity [**15**].

The histogram of the shape factor brings out the nature of different types of triangular motifs formed on the surface. It can be observed that the motifs present in the range of 0.9–1.0 show motifs which are near equilateral in shape. In addition, there are some motifs with values falling from 0.9 to near 0, which are more acicular. Presence of acicular motifs can be attributed to the fact that some peaks are lying very closer. With the present definition of peaks and the method/rules used for triangulation and motif combination, it is seen from Figs 9(d), 10(d), and 11(d) that most common triangular motifs are equilateral for all the surfaces. However, more number of acicular triangles is formed in the case of hard-turned and ground surfaces (as evident from mean SF values), showing that these are more anistropic than stone-honed surface.

The triangular motifs also align in the direction of lay, as angles are not constrained in the motif formation. The formation of obtuse angle is, however, avoided. Taking the altitude to indicate the direction, Fig. 12(d) shows the histogram of the direction of motifs formed on hard-turned surface. Most of the motifs are oriented along a direction at $\sim 60^\circ$ with respect to the x-direction, as shown in the figure. This can also be seen from the surface plot of hard-turned surface in Fig. 9(a) where a significant lay can be seen along a direction close to 60°. This is due to the fact that the measurement direction is inclined to the direction of lay present on the surface. In spite of this, triangular motif is able to pick up the direction of lay. The histogram of direction of motifs for ground surface in Fig. 13(d) also shows the direction of lay. However, Fig. 14(d) for stone-honed surface does not show any significant direction, bringing out the fact that there is no significant lay observed.

8 CONCLUSIONS

The application of triangular motifs for three-dimensional analysis of surfaces has been discussed in this paper. It is well known that triangles can represent any surface by simplest planar figures with a fair degree of accuracy. The motifs are formed by connecting the peaks present on the surface, which interact with external surface in contact situations. Using the motif combination rules proposed in this paper, insignificant motifs are combined to form larger significant motifs.

It is observed from the results that the surface features can be studied through the final motifs formed on the surface. The acicular (long wedge shaped) motifs predominantly oriented in a particular direction indicate that the peaks are closely located in one direction and sparsely distributed in other directions, thus highlighting the presence of edges corresponding to lay along that direction. The height distribution shows the presence or absence of isolated high peaks or deep grooves that may not be conducive for better functional performance of the surface. Similarly, the distribution of peaks on the surface can be studied from the histogram of the area of motifs. The mean height and area of motifs when combined can show lubricant retention property. In addition, the orientation of motifs can be identified and hence the presence or absence of lay can be decided. Hence, it can be concluded that three-dimensional motif method can characterize the surface in a better way, especially when the surface has to be used in contact situations.

This work considers the peaks based on eight neighbourhood points and only four combination rules outlined in ISO standard. The depth discrimination given for two-dimensional profile has not been implemented. The triangulation based on Delaunay method may be replaced by any other method of forming the triangles depending on the nature of the surface.

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APPENDIX 1

Notation

A	area of the motifs
Ch	characteristic depth of motif before combination
CH	characteristic depth of possible motif
H_i	relative height of a peak i
k	constant used in shape factor
p	peak
P	perimeter of the motif
s_{max}	slope of the maximum altitude of triangular motif
S	depth of the motif
SF	shape factor
T_R	local reference depth
v	valley
w	width of projection of the motif before combination
W	width of projection of the possible motif
Z_i	height of the peak i from a zero datum
α	motif orientation angle

APPENDIX 2: PSEUDO-CODE FOR SEGMENT FORMATION

```
1. Arrange all the peaks in descending order.
2. Take the first peak (i = 1) and insert into Delaunay
   triangulation.
3. Consider the next peak (i = i + 1)
4. If i > Total number of peaks, go to step 6
5. Check if this peak is inside any of the triangles
   If the point is inside a triangle j, check the length of
   sides of this triangle
     If the length of all edges <= WIDTH
       Go to step 3
     Else
       Insert the point into Delaunay triangulation
   mesh and form triangles
       Go to step 3
     If the point is outside any of the triangles
       Insert the point into the Delaunay triangu-
       lation mesh
       Go to step 3
6. Stop
```