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Studies on Phase Shifting Mechanism in Pulse Tube Cryocooler

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Abstract. Pulse Tube cryocoolers (PTC) are being used extensively in spacecraft for applications such as sensor cooling due to their simple construction and long life owing to a fully passive cold head. Efforts at ISRO to develop a PTC for space use have resulted in a unit with a cooling capacity of 1W at 80K with an input of 45watts. This paper presents the results of a study with this PTC on the phase shifting characteristics of an Inertance tube in conjunction with a reservoir. The aim was to obtain an optimum phase angle between the mass flow ($\dot{m}$) and dynamic pressure ($\tilde{p}$) at the PT cold end that results in the largest possible heat lift from this unit. Theoretical model was developed using Phasor Analysis and Transmission Line Model (TLM) for different mass flow and values of optimum frequency and phase angles were predicted. They were compared with experimental data from the PTC for different configurations of the Inertance tube/reservoir at various frequencies and charge pressures. These studies were carried out to characterise an existing cryocooler and design an optimised phase shifter with the aim of improving the performance with respect to specific power input.

1. Introduction

The development of PTCs for space applications has been growing rapidly owing to their improving specific mass, compactness, reliability and versatility. They have been used in space missions such as AIRS and IMAS [1, 2]. PTCs can be mechanically configured in different ways viz: In-Line, U-tube or Co-axial. The configuration chosen in this paper is the U-tube to facilitate the integration of a forward looking detector / dewar assembly.

Phase shifting mechanisms are a distinguishing feature of Stirling type PTCs and are used to get the optimum phase between the flow parameters at the desired section to maximise the performance of the unit. This paper investigates the inertance type phase shifter, which consists of a small diameter tube (inertance tube) having a large l/d ratio. The use of inertance tube adds inductive impedance to the circuit, which, in combination with the reservoir capacitance causes the mass flow phasor to lag the pressure phasor at the warm end of the PTC. The inertance effect is dominant only when the cryocooler is operated at higher frequencies and hence is suited for operation in Stirling type of PTCs.

The pulse tube cryocoolers under consideration for this work are hermetic units that are filled with pure Helium gas. The only active moving component is the piston, which reciprocates in the compressor, thus generating periodic pressure waves in the working fluid. As this is an AC type of system one of the crucial parameters determining the performance of the system is the magnitude and phase relationship between the mass flow rate and pressure at the cold end of the pulse tube. This is achieved by design, characterising and optimising the Inertance type Phase shifting mechanism.

2. Phasor Analysis

Phasor analysis is used to derive a relationship between the flow parameters (i.e. mass flow rate and pressure) in the pulse tube.

The phasor analysis was proposed by Hoffman and Pan [3] is carried out based on the following assumptions [4]. A schematic of the PTC is given in figure 1.

- The process occurring in the Pulse tube is adiabatic. The other elements viz. aftercooler, cold and warm end heat exchangers are assumed to operate under isothermal conditions.
- Pressure is constant throughout the Pulse tube and the amplitude of the dynamic pressure is small compared to the charge pressure.
- The pressure and temperature variations are sinusoidal, with no phase difference between them.

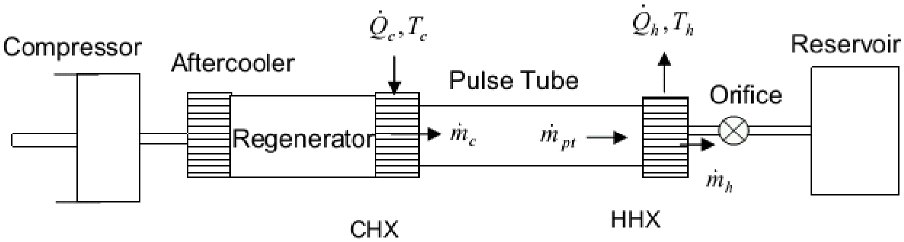


Figure 1. Schematic diagram of the Orifice Pulse Tube cryocooler for Phasor analysis.

Here $\dot{m}_c, \dot{m}_{pt}, \dot{m}_h, \dot{m}_o$ represent the mass flow rate at the Pulse tube cold end, Pulse tube, Pulse tube warm end and through the orifice respectively. Applying the conservation of mass condition:

$$\dot{m}_{pt} = \dot{m}_h - \dot{m}_c \quad (1)$$

Note that for an Orifice PTC, $\dot{m}_h = \dot{m}_o$. Considering the working fluid as an ideal gas and with the above assumptions, the following expression is obtained [4]:

$$\dot{m}_c = \frac{\omega P_1 V_{pt}}{\gamma R T_c} \cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c}\ \dot{m}_h \quad (2)$$

As $\dot{m}_h \propto \Delta P$, $\dot{m}_h$ will be in phase with the pressure difference or pressure vector.

The phasor representation of the above equation is as shown in figure 2. Note that $\omega = 2\pi f$, where, f is the frequency of operation, P_1 = amplitude of pressure fluctuation, V_{pt} = volume of the pulse tube, T_c = cold end temperature, T_h = hot end temperature, R is the ideal gas constant and γ = ratio of specific heats.

For the best performance the angle between mass flow rate ($\dot{m}_h$) and Pressure vector i.e. angle (θ) has to be minimised at the cold end. This is clearly achieved if $\frac{\omega P_1 V_{pt}}{\gamma R T_c}$ is as small as possible.

Extending the above analogy to include all components of the inertance tube based PTC; phasor diagram in figure 3 is obtained. A wide range of phase angles, lagging or leading are possible by using the inertance tube. From practical considerations, the following two configurations are narrowed down

- case (a) : for minimum regenerator loss($\dot{m}$ and $\tilde{p}$ are in phase within the regenerator).
- case (b) : for maximum heat lift at cold end ($\dot{m}$ and $\tilde{p}$ are in phase at the pulse tube cold end).

The next section discusses the electrical analogy of a phase shifting circuit. Subsequently, the inertance tube (with reservoir) is modeled using transmission line theory (TLM) to obtain its phase characteristics.

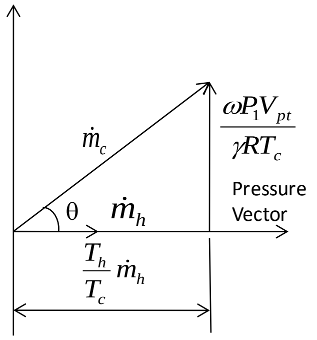


Figure 2. Phasor representation for flow through the Pulse tube.

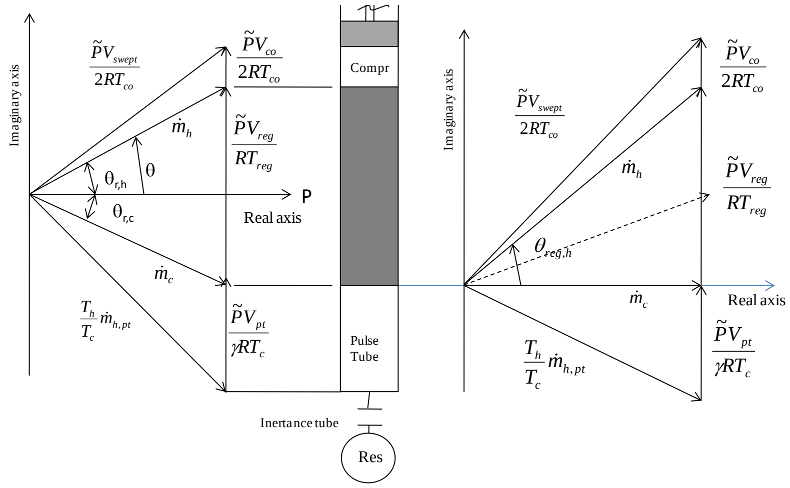
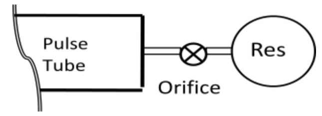
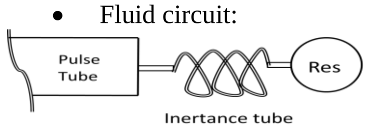
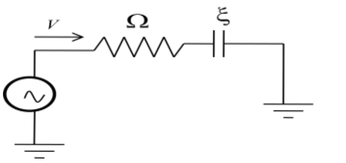
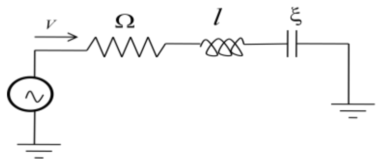


Figure 3. Phasor diagrams for the two practical cases (a) and (b) for the IPTC [4, 6].

3. Electrical Analogy

The electrical analogy of the reservoir is the capacitor and the analogy for the inertance tube is the inductor. The electrical equivalent of frictional flow (whether in an orifice tube or the frictional flow in a tube) is the resistor. It should also be noted that current is analogous to the mass flow rate and voltage is analogous to the pressure, thus mass flow rate leads pressure in a reservoir whereas the vice versa is true in an inertance tube. In purely frictional flow (as in an orifice), the mass flow rate and the pressure are in phase. This is depicted in table 1 below:

Table 1. Comparison of Orifice and Inertance PTC.

Orifice PTC	Inertance PTC
• Fluid circuit:	• Fluid circuit:
	
• Equivalent electrical circuit	• Equivalent electrical circuit
	
• Impedance (Z) : $Z = \Omega - \frac{1}{j\omega\xi}$	• Impedance (Z): $Z = \Omega + j\left(\omega l - \frac{1}{\omega\xi}\right)$

4. The Transmission Line Model [5]

From the phasor analysis it is observed that the phase shifter needed for maximising the performance is case (a) where $\dot{m}$ aligns with the pressure phasor at the center of the regenerator because in miniature cryocoolers regenerator losses dominate at low temperatures.

To arrive at the geometry of the inertance tube phase shifter the transmission line model developed by Radebaugh [6], for fluid circuits is used, schematic of which is as shown in figure 4.

The rate of change of voltage: $\frac{dV}{dx} = zI$, where $z = r + j\omega l$ (3)

The complex constant *(z)* is expressed in terms of resistance (*r)* and inductance (*l*) per unit length.

Similarly, the rate of change of current: $\frac{dI}{dx} = yV$, where $y = g + j\omega c$ (4)

The complex constant (*y*) is expressed in terms of the shunt conductance (*g)* and capacitance *(c)* per unit length. In the above relations $j = \sqrt{-1}$ and ω is the frequency of operation of the system.

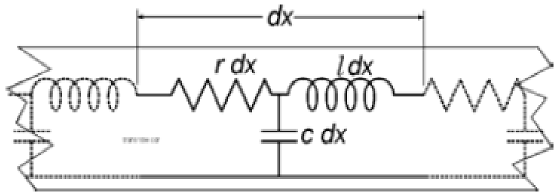


Figure 4. Simplified Transmission Line Network [6].

Expressions for *r*, *l* and *c* derived from first principles are now outlined. Resistance *(r)* per unit length of flow in the inertance tube arises from viscous drag and can be expressed as

$$r = \left(\frac{32 f_r \dot{m}}{\pi^2 \bar{\rho} D^5}\right)\left(\frac{2}{\pi}\right) \text{ where, } f_r = 0.046 Re^{-0.2}\text{; Re is the Reynolds no. given as } Re = \left(\frac{8\dot{m}}{\pi^2 D\mu}\right) \quad (5)$$

Inertance *(l)* per unit length results from inertia of the oscillating gas in the tube becomes significant at high frequencies. The tube's compliance results from the compressibility of the gas [7]

$l = \frac{4}{\pi D^2}$ and $c = \frac{\pi D^2}{4\gamma R\bar{T}}$; R is the ideal gas constant, $\bar{T}$ the average temperature and γ is the ratio of specific heats for Helium. (6)

The reservoir compliance (Z_r) is related to its volume V_r by $z_r = \frac{V_r}{\gamma R T_r}$ (7)

Substituting the fluid equivalents for current, voltage and impedance in the above differential equations that represent the electrical analogy of the inertance tube the following differential equations are obtained:

$$\frac{d\tilde{P}}{dx} = z\dot{m} = \left(\frac{64 f_r \dot{m}}{\pi^3 \bar{\rho} D^5} + j\omega\frac{4}{\pi D^2}\right)\dot{m} \quad \text{and} \quad \frac{d\dot{m}}{dx} = y\tilde{P} = \left(j\omega\frac{\pi D^2}{4\gamma R\bar{T}}\right)\tilde{P} \quad (8)$$

A key parameter defining the phase angle (θ) between $\dot{m}$ and $\tilde{P}$ of the inertance tube is the complex impedance which is defined as $Z = \frac{\tilde{P}}{\dot{m}}$. Solving the above differential equations after taking into account the reservoir capacitance (Z_r), the net (complex) impedance of the inertance tube and reservoir is [6]:

$$Z_{pt} = Z_0\left(\frac{Z_r + Z_0 \tanh(\sqrt{zy}\,L_e)}{Z_0 + Z_r \tanh(\sqrt{zy}\,L_e)}\right) \quad (9)$$

Thus with known characteristics of the inertance tube and reservoir it is possible to determine the complex impedance and hence phase angle (θ) between the mass flow rate and the pressure at the inertance tube inlet.

5. Description of experimental set–up

Three experiments were carried out to understand the behavior of the inertance tube/reservoir phase shifter:

1. Dual piston compressor connected directly to a reservoir with a short tube: the goal of this experiment was to estimate the mass flow rate.
2. Full PTC containing compressor, cold head and inertance tube/reservoir phase shifter: the goal here was to study the influence of the phase shifter on the cold head temperature of the PTC as a function of the system pressure, operating frequency and phase angle.
3. A single piston compressor connected to a reservoir through an inertance tube: the goal of this experiment was to estimate the natural frequency of the phase shifter.

Details on these experimental set-ups are outlined in this section below.

a) Initially mass flow experiments were carried out to evaluate the range of its output from the compressor using the isothermal mass conservation technique by connecting the compressor, shown in figure 5, directly to a reservoir of known volume [8].

b) Performance (cool down) tests were then conducted to evaluate the mass flow at the inertance tube inlet and study the cooling performance of the unit with respect to the natural frequency of the phase shifting circuit details of which are given in table 2.

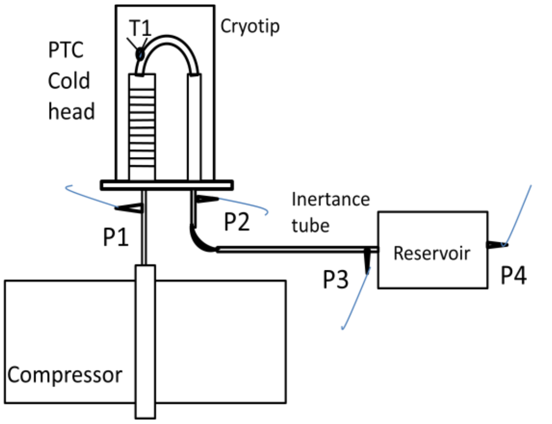


Figure 5. Experimental set-up for performance evaluation.

Table 2. Experimental set-up for performance evaluation (cool down) tests.

Compressor	Dual pistons; 1.81 cm^3
Regenerator	ϕ 6 x 40 mm
U tube	ϕ 1.2 x 39 mm
Pulse tube	ϕ 4 x 40 mm
Position sensors	Schaevitz – 250 MHR Range - ± 5 mm
Pressure sensor	Endevco 8510B Range 500 psi
Inertance tube	Sizes as given in Table 3
Reservoir	100 cm^3

c) Experimental set-up to evaluate the natural frequency of the phase shifting circuit.

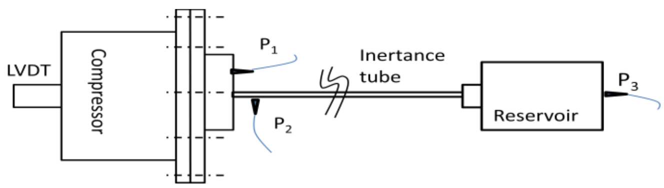


Figure 6. Natural frequency determination of phase shifting circuit.

Table 3. Natural freq measurement; set-up specifications.

Compressor	Single piston
Working vol.	3.14 cm^3
Position sensors	Schaevitz – 250 MHR Range: ± 5 mm
Pressure sensor	Endevco 8510B Range 500 psi
Operating freq.	54 Hz
Reservoir	100 cm^3

6. Results and discussions

Initially experimental determination of mass flow rate into the reservoir was carried out. Using these mass flow rates, it is sought to estimate the phase angles (i.e. of pressure with respect to mass flow rate) using TLM. The importance of natural frequency is then studied on the operating temperature of the cryotip by conducting the performance (cool down) tests on the cryocooler.

The natural frequency obtained from TLM has a direct functional relationship with the optimum operating frequency of the cooler obtained experimentally. In the light of the importance of this natural frequency an experimental method to evaluate the natural frequency of an inertance tube/reservoir is presented.

6.1 Mass flow measurement tests

The tests were performed as detailed in the experimental set-up section. Two reservoir combinations at four pressure values from 20 to 26 bar and repeated twice to ensure validity of the values obtained. From the above tests, it is observed that the mass flow out of the compressor varies from 0.3-1.3 g/s.

6.2 Cool down tests with variation in mass flow with different Inertance tube configurations

The mass flow at inertance tube inlet is as shown in table 4. It varies from 0.043 g/s to 0.095 g/s for the inertance tube lengths from 4.5 m - 2.75 m which were evaluated with a reservoir of 100 cm^3.

Table 4. Output data from cooling tests for different Inertance tube geometry.

Sl. no	Inertance tube		Frequency -Hz		Mass flow at IT entry (g/s)	Phase angle TLM (deg)	Pr. Ampl (bar)	Temp (K)
	Dia (mm)	Length (m)	Exptl Opt	TLM				
1	1.76	4.5	54	58	0.043	56.7	1.09	151
2.	1.76	4.0	59	65	0.055	59.9	1.14	132
3.	1.76	3.5	64	74	0.072	64.2	1.12	116
4.	1.76	3.0	71	86	0.091	67.2	1.14	103
5.	1.76	2.75	76	94	0.095	69.5	1.12	98
6.	1.12	2.0	64	124	0.096	49.1	1.096	119
7.	1.12	1.75	64	141	0.113	47.4	1.103	130
8.	1.12	1.5	64	165	0.125	46.8	1.034	141

In the tests carried out on the PTC with different inertance tube configurations, the lowest temperature achieved in the cryocooler (at no load) varied from 98 K to 151 K. The compressor, regenerator, pulse tube and reservoir remained same in all tests, which were conducted at 26 bar.

The results clearly show that the performance of the PTC is critically dependent on the configuration of the inertance tube. It can be observed that the phase angle varied from 56.7° to 70.9° for the different configurations of ϕ 1.76 mm tube and from 46.8° to 49.1° for ϕ 1.12 mm tube.

Table 4 also shows the natural frequency of the inertance tube as estimated by the mathematical model. It can be seen that the lowest temperatures were achieved with all configurations are somewhat close to the natural frequencies calculated by the model. The differences between them are due to the assumptions made in the mathematical model and the experimental uncertainties in mass flow measurement and restrictions by the experimental set-up on the maximum operational frequency.

In the case of the ϕ 1.12 mm Inertance tube, a drop in the performance is observed as the deviation between the Natural frequency of the phase shifting assembly and the operating frequency widens with decrease in the length of the tube.

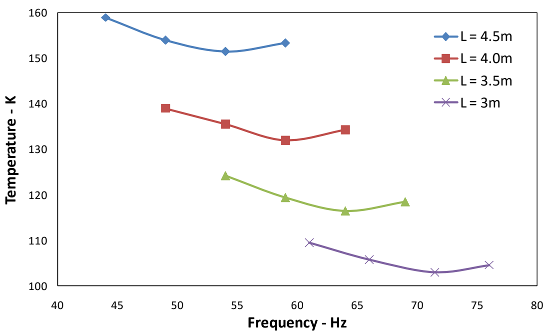


Figure 7. ϕ 1.76 mm Inertance tube; Temperature variation with Frequency.

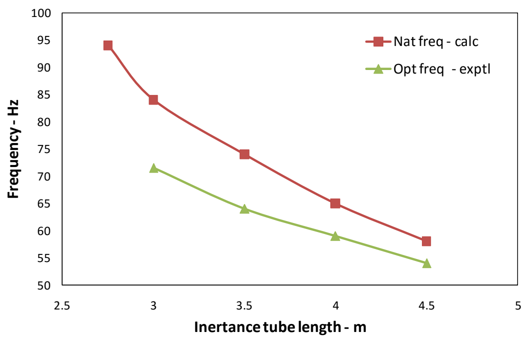


Figure 8. ϕ 1.76 mm Inertance tube Calculated vs. Optimum Expt. frequency.

Figure 7 shows the experimentally measured temperatures of the cryotip as a function of operating frequency. All these curves used a inertance tube (inner) diameter of 1.76 mm and a reservoir volume of 100 cm^3. Each curve was for a different length of inertance tube. For each curve the temperature first decreases and then increases with distinct minima. It is thus clear from this graph that.

- An optimum frequency (f_o) exists for each inertance tube/reservoir combination.
- The cryotip temperature (at f_o) decreases with an increase in the natural frequency of the PSM.

Figure 8 compares f_o (experimental frequency resulting in lowest temperature) with the natural frequency (as calculated by TLM). The following points can be observed:

- f_o is always lower than the natural frequency.
- As the natural frequency increases, f_o increases.

The correlation between f_o and natural frequency (calculated using TLM) plotted in figure 9 shows a linear correlation between the two, with a correlation coefficient of 0.997, for the range of tubes evaluated. This clearly indicates that if a higher operating frequency is desired for the PTC, the natural frequency of the PSM (i.e. inertance tube/reservoir) should be increased.

A key observation is that cryotip temperature (at f_o) decreases at higher frequencies. This occurs due to higher mass flow rates through the system. Another approach to increase mass flow rate through the system is to increase the operating pressure (i.e. increase the *density*) of the gas. Thus tests were conducted for determining the performance with variation in charge pressure P_0.

Data is plotted for different lengths of the inertance tube at their respective optimized frequency as derived from earlier test results are plotted in figure 10.

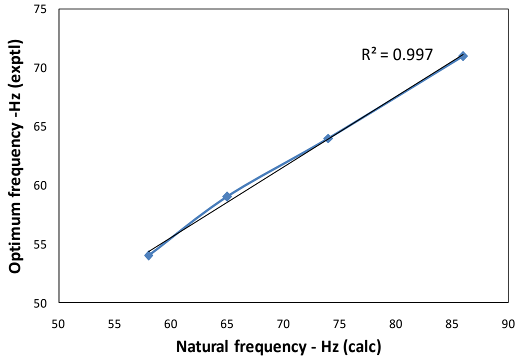


Figure 9. ϕ 1.76 mm Inertance tube Correlation between Calc and Opt frequency.

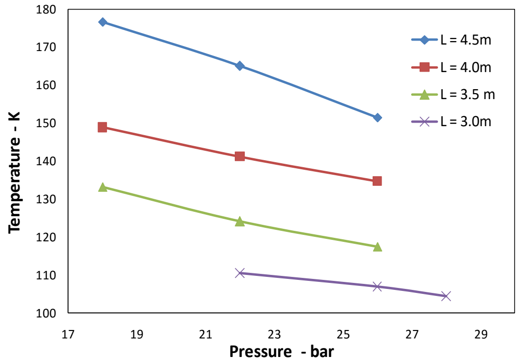


Figure 10. ϕ 1.76 mm Inertance tube Temperature variation with Charge Pressure.

6.3 Natural frequency of Phase Shifting Mechanism

The importance of natural frequency of the PSM prompted a search for a purely experimental approach to measure this natural frequency. The experimental set-up for the same is as shown in figure 6 with details as per table 3. Different geometries of inertance tube were connected between the compressor and reservoir and evaluated at different pressures as shown in figure 11.

Consider a system consisting of a spring, mass and damper. Such a system has a natural frequency given by $(1/2\pi)(\sqrt{k/m})$. Close to the natural frequency, even small excitations are amplified resulting in large oscillation (i.e. due to the high 'β'). Consider –

- Inertance tube: oscillating fluid slug is equivalent to the "mass". Mass flow rate of this slug is equivalent to the "velocity" of the mass.
- Reservoir: equivalent to the spring. The pressure rise over charge pressure is equivalent to the "displacement" of the mass (hence compression of the spring)
- Frictional resistance in inertance tube: equivalent to the "damper".

Thus at the natural frequency of the PSM (i.e. resonance), the oscillating mass in the inertance tube has high mass flow rate hence large change in pressure amplitude in the inertance tube inlet would be expected for a given excitation.

The approach taken (for a given inertance tube/reservoir combination) is to see at what frequency a minimum compressor displacement is required to achieve a fixed value of pressure amplitude at the inertance tube inlet. The results of this are presented in figure 11. It is seen that there exists a clear minima in the piston amplitude for fixed inertance tube inlet pressure amplitude of 1 x 10^4 Pa.

The natural frequency is experimentally observed to be 54 Hz for the 1.76 mm tube case, whereas for the 3 mm case it is 60 Hz. For both cases, TLM predicts a natural frequency that is 10 Hz higher than the experimentally determined value.

The TLM results are based on a constant mass flow rate whereas in reality the mass flow rate changes with the piston amplitude. This is one possible cause for the mismatch. This requires further study.

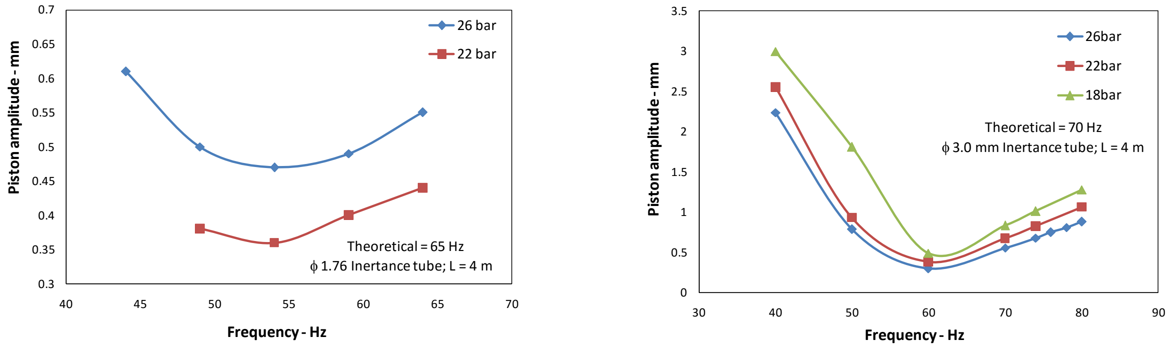


Figure 11. Natural frequency experimental data for ϕ 1.76 mm and ϕ 3 mm Inertance Tubes.

7. Conclusions

The results of the study carried out in this work shows the following:

a. The performance of a Pulse tube cryocooler is a strong function of the phase angle between the pressure and flow rate phasors.
b. The cooling capacity of the cryocooler is highest when the compressor frequency is close to the natural frequency of the inertance tube. Higher natural frequency of the inertance tube higher the cooling capacity or lower the temperature achieved.
c. The minima in temperature obtained in the PTC with respect to frequency variation is directly correlated with the natural frequency, obtained from the mathematical model, of the PSM.
d. Higher the average pressure higher the cooling capacity, however the highest pressure is limited by the presence of transition joints between dissimilar metals in the cryocooler.
e. Importance of natural frequency and an experimental approach to determine the same. When frequency is varied for constant pressure amplitude at the inertance tube inlet, the minimum value of the piston amplitude needed to achieve this, corresponds to the natural frequency.

8. Further work

1. Study for natural frequency determination of compound Inertance tubes and subsequent cooling performance of the PTC with these configurations to obtain lower temperatures.
2. As observed from the test results, improved performance is obtained at higher frequencies and pressures. Hence Design and realisation of a high frequency PTC is envisaged.

Acknowledgement

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