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Stability of shear flow of stratified fluids with fine dust

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The linear stability of plane parallel shear flows of a stably stratified incompressible fluid laden with uniformly distributed fine dust particles is studied. It is shown that the Miles criterion for stability, the Rayleigh–Synge criterion for instability, and Howard's semicircle theorem can be extended under the assumptions that the mass concentration is very small and that the relaxation time of the dust particles is very much less than the time scale characterizing the basic flow. The effect of fine dust is found to increase the region of instability. In addition, a semi-ellipse criterion for instability has been given as an improvement over Howard's semicircle criterion.

I. INTRODUCTION

Most of the analytical results that have been reported on the dynamics of a dusty gas are based upon Saffman's model in which a two-phase continuum is assumed. The dust particles are supposed to be uniform in size and shape. The bulk concentration of the dust is assumed to be very small so that there is no interaction between any two dust particles. The net effect of the dust on the fluid continuum is taken to be the drag force proportional to the relative velocity. Saffman[1] used this model to study the effect of dust particles on the stability of the laminar flow of a gas in order to see how dust affects the critical Reynolds number in transition from laminar to turbulent flow.

Much work has since been reported on the subject. To mention some of the recent works, Crooke and Walsh[2] have derived analytical solutions for both steady and transient flows through an infinitely long pipe of arbitrary cross section. Singh and Dube[3] have independently solved the problem of the steady flow through a circular pipe. Peddieson[4] has given analytical and numerical solutions for several unsteady parallel flows of particulate suspensions, under the assumption of a nonlinear drag force between the fluid phase and the particles in some of the problems studied and under the assumption of linear drag force (Saffman model) in the remaining. More recently, Dandapat and Gupta[5] have studied the nonlinear stability based upon the energy method formulated by Serrin. Nag *et al.*[6] have considered Couette flow.

In a series of papers Drew[7-9] has developed another model for the flow of particle fluid mixture in which a lifting force on the particulate is also taken into consideration along with the drag force and used it to study the stability of Couette flow and the stability of the Stokes' layer. For a bibliography of earlier publications on the dynamics of dusty gas we refer the interested readers to a review article by Marble.[10]

The study of two-phase flows derives its importance from its application to problems of fluidization in chemical technology, in pollution control problems, and in heat transfer technology. This apart, recent spacecraft observations have confirmed that dust particles play an important role in the dynamics of the Martian atmosphere as well as in the diurnal and surface variations in the temperature of the Martian weather.[11] We therefore hopefully believe that it will be of interest to study the effect of dust on the gravity waves. As a preliminary study of the gravity waves in a dusty gas, here we have considered the stability of parallel shear flow in a stably stratified, inviscid, incompressible fluid laden with small sized, uniform dust particles.

The stability of shear flow of a heterogeneous or stratified fluid has been considered by many earlier research workers.[12] However, we have closely followed Howard's[13] work in which he has derived a semi-circle theorem for unstable modes. Many of the later studies of the subject are based on this work. Howard and Gupta[14] have studied the stability of incompressible, axisymmetric swirling flows between two concentric cylinders and have derived analogous results. The semi-circle theorem has been extended to the case of compressible, stratified flows by Chimonas[15] and to that of perfectly conducting, incompressible, stratified flows by Narasimha Murthy *et al.*[16] Recently, Kochar and Jain[17] have improved on Howard's semi-circle theorem by proving a semi-ellipse theorem for unstable modes.

II. EQUATIONS FOR PERTURBATION

We consider the two-dimensional stability problem of plane parallel shear flow $[U(y), 0, 0]$ between two parallel planes $y=y_1$ and $y=y_2$ of an inviscid (except for the presence of dust particles), incompressible, stably stratified fluid laden with uniformly distributed fine dust particles of uniform size and shape. If m is the mass of each dust particle, g stands for the gravitational force per unit mass, S is the Stokes' drag coefficient, and U^* is a typical mean shear velocity, we make the assumption that $mg/SU^* \ll 1$ so that sedimentation effects can be neglected. In addition, we assume that $\tau = m/S$ which is a measure of time taken by the dust particles to adjust to the local fluid motion is very much less than the time characterizing the basic flow. More specifically, we assume that if L is a characteristic length of the fluid motion, $mU/LS \ll 1$. Under these assumptions it can reasonably be expected that the dust particles follow the fluid elements and are neutrally buoyant in the steady state.

The basic equations of flow for our analysis will then

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be the momentum equation for the fluid

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = -\nabla p + \rho \mathbf{g} + SN(\mathbf{v}-\mathbf{u}), \quad (1)$$

the momentum equation for the dust particles (considered as a continuum)

$$mN\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = SN(\mathbf{u}-\mathbf{v}); \quad (2)$$

the condition of incompressibility

$$\frac{\partial \rho}{\partial t} + (\mathbf{u}\cdot\nabla)\rho = 0; \quad (3)$$

and the equations of continuity for fluid and the dust

$$\nabla\cdot\mathbf{u} = 0, \quad (4)$$

$$\frac{\partial N}{\partial t} + \nabla\cdot(N\mathbf{v}) = 0. \quad (5)$$

In Eqs. (1)–(5), u and v are fluid and particle velocities and N, p, and ρ denote the number density of dust particles per unit volume, the fluid pressure, and density of fluid, respectively. Although the fluid is assumed to be inviscid, its viscosity (however negligible) will manifest itself in the Stokes' coefficient of drag which varies with the size and shape of the dust particles. In the case of the dust particles of spherical shape and of radius a, the Stokes' drag coefficient $S = 6\pi a\mu$, μ being the viscosity of the fluid.

Applying the standard procedure for linear stability problem for a two-dimensional perturbation of the basic flow

$$\mathbf{u} = [U(y) + u_x, u_y], \quad \mathbf{v} = [U(y) + v_x, v_y], \quad \mathbf{g} = -g\nabla y,$$
$$\rho = \rho_0(y) - \rho_0'\eta, \quad p = P + p^*, \quad N = N_0 + N^*, \quad (6)$$

Eqs. (1)–(5) give the following equations of perturbation:

$$\rho_0(Du_x + U'u_y) = -(p)_x + SN_0(v_x - u_x), \quad (7)$$

$$\rho_0(Du_y + \beta g\eta) = -(p)_y + SN_0(v_y - u_y), \quad (8)$$

$$m(Dv_x + U'v_y) = S(u_x - v_x), \quad (9)$$

$$mDv_y = S(u_y - v_y), \quad (10)$$

$$D\eta = u_y, \quad (11)$$

$$(u_x)_x + (u_y)_y = 0, \quad (12)$$

where the asterisks have been dropped, $D \equiv \partial/\partial t + U\partial/\partial x$, $(\varphi)_s$ denotes the partial derivative of φ with respect to s, and the prime denotes differentiation with respect to y. It may be noted that the initial distribution of dust particles being uniform, N_0 is a constant.

For a normal mode analysis of linear stability we choose a particular Fourier component of the perturbation and assume, for every field variable, the form

$$\psi(x,y,t) = \mathrm{Re}\{\psi(y)\exp[iK(x-ct)]\}. \quad (13)$$

Equations (7) to (13) then give the following relations for the amplitude functions:

$$u_x = -[(U-c)\eta]', \quad (14)$$

$$u_y = iK(U-c)\eta, \quad (15)$$

$$v_x = \frac{-S}{[S + iKm(U-c)]^2} \times \{(U-c)\eta'[S + iKm(U-c)] + U'\eta[S + 2iKm(U-c)]\}, \quad (16)$$

$$v_y = \frac{iKS(U-c)\eta}{S + iKm(U-c)}, \quad (17)$$

$$p = \rho_0(U-c)^2\left[\left(1 + \frac{mSN_0}{\rho_0[S + iKm(U-c)]}\right)\eta' + \frac{iKm^2SN_0U'}{\rho_0[S + iKm(U-c)]^2}\eta\right], \quad (18)$$

$$N = \frac{2iKmSN_0U'\eta}{[S + iKm(U-c)]^2}, \quad (19)$$

leading to the differential equation

$$\left\{\rho_0(U-c)^2\left[\left(1 + \frac{mSN_0}{\rho_0[S + iKm(U-c)]}\right)\eta' + \frac{iKm^2SN_0U'}{\rho_0[S + iKm(U-c)]^2}\eta\right]\right\}' - \rho_0\left[K^2(U-c)^2\left(1 + \frac{mSN_0}{\rho_0[S + iKm(U-c)]}\right) - g\beta\right]\eta = 0, \quad (20)$$

where $\beta = -\rho_0'(y)/\rho_0(y)$.

The differential equation (20) along with the boundary conditions $\eta(y_1) = \eta(y_2) = 0$ gives an eigenvalue problem for the determination of the eigenvalues c_{ei} and the eigenfunctions η. The basic flow is unstable for the particular eigenmode c_{ei} if $\mathrm{Im}c_{ei} > 0$ and the corresponding eigenfunction is nontrivial. This eigenvalue problem appears to be intractable for analytical solution and therefore, we simplify the equation under the assumption that for the fine dust the mass concentration $f(= mN_0/\rho_0) \ll 1$ along with the assumption $mU/LS \ll 1$ mentioned earlier.

III. MILES' THEOREM

Neglecting the higher order terms in f and mU/LS, Eq. (20) reduces to

$$[\rho_0(U-c)^2(1+f)\eta']' - \rho_0[K^2(U-c)^2(1+f) - g\beta]\eta = 0. \quad (21)$$

It is superfluous to state that in the absence of dust particles ($f = 0$) Eq. (21) reduces to the one originally derived by Miles.[12] It may also be noted that Eq. (21) cannot be obtained from Miles' equation just by replacing g by $g/(1+f)$ since the mass concentration f is not a constant.

We now seek a necessary condition under which the flow is unstable for a particular Fourier component of the perturbation. To this end we set $c = c_r + ic_i$ with $c_i > 0$ and suppose that η is the corresponding unstable solution of (21).

Introducing $W = U - c \neq 0$ and $G = W^{1/2}\eta$, where the same branch of $W^{1/2}$ is to be used throughout (y_1, y_2), Eq. (21) can be written

$$[\rho_0(1+f)WG']' - \{\tfrac{1}{2}[\rho_0(1+f)U']' + \rho_0(1+f)K^2W + \rho_0(1+f)W^{-1}[U'^2/4 - g\beta/(1+f)]\}G = 0 . \qquad (22)$$

Multiplication by the complex conjugate G^* of G and integration over (y_1, y_2) of this equation under the conditions $G(y_1) = G(y_2) = 0$ leads to

$$\int_{y_1}^{y_2} \rho_0(1+f)W(|G'|^2 + K^2|G|^2)dy + \frac{1}{2}\int_{y_1}^{y_2} [\rho_0(1+f)U']'|G|^2 dy + \int_{y_1}^{y_2} \rho_0(1+f)\left(\frac{U'^2}{4} - \frac{g\beta}{(1+f)}\right) W^* \left|\frac{G}{W}\right|^2 dy = 0 . \qquad (23)$$

Recalling that $c_i > 0$, the imaginary part of (23) implies

$$\int_{y_1}^{y_2} \rho_0(1+f)(|G'|^2 + K^2|G|^2)dy + \int_{y_1}^{y_2} \rho_0(1+f)\left(\frac{g\beta}{(1+f)} - \frac{U'^2}{4}\right)\left|\frac{G}{W}\right|^2 dy = 0 . \qquad (24)$$

This is clearly impossible if $g\beta/(1+f) - U'^2/4$ is non-negative throughout, so that a necessary condition for instability is that $g\beta/(1+f) - U'^2/4$ be somewhere negative. Taking the complement of this we have a sufficient condition for stability that $g\beta/(1+f) - U'^2/4$ should be everywhere non-negative. Defining the local Richardson number by

$$J(y) = g\beta/(1+f)U'^2 ,$$

the sufficient condition for stability of the parallel shear flow of a gas with fine dust can be stated as

$$J(y) \geq 1/4$$

throughout the domain (y_1, y_2).

IV. THE SEMI-CIRCLE THEOREM

To prove Howard's semi-circle theorem we multiply Eq. (21) by η^*, the complex conjugate of η, integrate the resulting equation between y_1 and y_2, and use the boundary conditions $\eta(y_1) = \eta(y_2) = 0$, to get

$$\int_{y_1}^{y_2} \rho_0(1+f)W^2(|\eta'|^2 + K^2|\eta|^2)dy - \int_{y_1}^{y_2} \rho_0 g\beta|\eta|^2 dy = 0 . \qquad (25)$$

Separation of the real and imaginary parts leads to

$$\int_{y_1}^{y_2} \rho_0(1+f)[(U-c_r)^2 - c_i^2](|\eta'|^2 + K^2|\eta|^2)dy \times - \int_{y_1}^{y_2} \rho_0 g\beta|\eta|^2 dy = 0 , \qquad (26)$$

$$2c_i\int_{y_1}^{y_2} \rho_0(1+f)(U-c_r)(|\eta'|^2 + K^2|\eta|^2)dy = 0 . \qquad (27)$$

With $Q = \rho_0(1+f)(|\eta'|^2 + K^2|\eta|^2)$ and $c_i > 0$ for instability, we have from (27)

$$\int_{y_1}^{y_2} UQ\,dy = c_r\int_{y_1}^{y_2} Q\,dy . \qquad (28)$$

Using this in (26) leads to

$$\int_{y_1}^{y_2} U^2Q\,dy = (c_r^2 + c_i^2)\int_{y_1}^{y_2} Q\,dy + \int_{y_1}^{y_2} g\rho_0\beta|\eta|^2 dy . \qquad (29)$$

If now U_{max} and U_{min} denote the maximum and minimum of $U(y)$ in $y_1 \leq y \leq y_2$, we have

$$\begin{aligned} 0 &\geq \int_{y_1}^{y_2} (U - U_{max})(U - U_{min})Q\,dy \\ &\geq \int_{y_1}^{y_2} U^2Q\,dy - (U_{max} + U_{min}) \times \int_{y_1}^{y_2} UQ\,dy + U_{max}U_{min}\int_{y_1}^{y_2} Q\,dy \\ &\geq [c_r^2 + c_i^2 - (U_{max} + U_{min})c_r + U_{max}U_{min}] \times \int_{y_1}^{y_2} Q\,dy + \int_{y_1}^{y_2} \rho_0 g\beta|\eta|^2 dy \end{aligned}$$

resulting in the inequality,

$$\{[c_r - \tfrac{1}{2}(U_{max} + U_{min})]^2 + c_i^2 - \tfrac{1}{4}(U_{max} - U_{min})^2\} \times \int_{y_1}^{y_2} Q\,dy + \int_{y_1}^{y_2} \rho_0 g\beta|\eta|^2 dy \leq 0 . \qquad (30)$$

Since for stable stratification $\beta > 0$ and $Q > 0$, the inequality (30) implies

$$[c_r - \tfrac{1}{2}(U_{max} + U_{min})]^2 + c_i^2 \leq [\tfrac{1}{2}(U_{max} - U_{min})]^2 . \qquad (31)$$

This gives the semi-circle theorem that a complex wave speed for any unstable mode lies within the semi-circle in the upper half-plane with the range of $U(y)$ as the diameter.

Since $f > 0$, we note that

$$\frac{1}{4}(U_{max} - U_{min})^2 - \frac{\int_{y_1}^{y_2} \rho_0 g\beta|\eta|^2 dy}{\int_{y_1}^{y_2} Q\,dy} > \frac{1}{4}(U_{max} - U_{min})^2 - \frac{\int_{y_1}^{y_2} \rho_0 g\beta|\eta|^2 dy}{\int_{y_1}^{y_2} (1+f)^{-1}Q\,dy} , \qquad (32)$$

which indicates that the bounding semi-circle of the complex wave speed for any unstable mode in the case of a clean gas lies wholly within that corresponding to the case of a gas with fine dust. This implies that for a gas with fine dust the region of instability increases which is in agreement with Saffman's finding that fine dust destabilizes the flow.

V. THE GROWTH RATE FOR UNSTABLE MODE

We now show that the growth rate Kc_i for an unstable mode is also bounded along with the local Richardson number and the complex wave velocity. Writing Eq. (24) in the form

$$K^2\int_{y_1}^{y_2} \rho_0(1+f)|G|^2 dy = \int_{y_1}^{y_2} \rho_0(1+f)\left(\frac{U'^2}{4} - \frac{g\beta}{(1+f)}\right)\left|\frac{G}{W}\right|^2 dy - \int_{y_1}^{y_2} \rho_0(1+f)|G'|^2 dy ,$$

and noting that $|W|^{-2} \le c_i^{-2}$, we get

$$K^2 \int_{y_1}^{y_2} \rho_0(1+f)|G|^2 dy \le \frac{1}{c_i^2} \max\left(\frac{U'^2}{4} - \frac{g\beta}{(1+f)}\right) \times \int_{y_1}^{y_2} \rho_0(1+f)|G|^2 dy ,$$

which gives a bound for growth rate

$$K^2 c_i^2 \le \max|U'^2/4 - g\beta/(1+f)| .$$

VI. RAYLEIGH-SYNGE CRITERION

As in Sec. III, with $c_i > 0$ for an unstable mode, one can introduce $\eta = W^{-n}H$, and select a definite branch in case n is not an integer. Substitution for η in terms of H in (21) results in the following equation for H

$$[\rho_0(1+f)W^{2(1-n)}H']' - \{K^2\rho_0(1+f)W^{2(1-n)} + nW^{1-2n}[\rho_0(1+f)U']' + \rho_0(1+f)W^{-2n} \times[n(1-n)U'^2 - g\beta/(1+f)]\}H = 0 . \tag{33}$$

Multiplying (33) by H^*, the complex conjugate of H, and integrating over (y_1, y_2) we get

$$\int_{y_1}^{y_2} \rho_0(1+f)W^{2(1-n)}(|H'|^2 + K^2|H|^2)dy + n\int_{y_1}^{y_2} W^{(1-2n)}[\rho_0(1+f)U']'|H|^2 dy + \int_{y_1}^{y_2} \rho_0(1+f)W^{-2n}[n(1-n)U'^2 - g\beta/(1+f)]|H|^2 dy = 0 . \tag{34}$$

From this, one can obtain the stability criterion derived in Sec. III and the semi-circle theorem derived in Sec. IV by taking $n = \frac{1}{2}$ and $n = 0$, respectively. For $n = 1$, the imaginary part of Eq. (34) gives the Rayleigh–Synge criterion, namely, for instability a necessary condition is that the expression

$$[\rho_0(1+f)U']' - 2\rho_0 g\beta|W|^{-2}(U - c_r)]$$

should change sign in (y_1, y_2).

VII. SEMI-ELLIPSE THEOREM

In this section following Kochar and Jain,[17] we have obtained a semi-elliptic region for unstable modes which lies wholly within the semi-circular region given in Sec. III thus improving the region for stable modes.

We observe that the transformation

$$G = (U - c)^{1/2}\eta , \tag{35}$$

implies

$$|G'|^2 \ge |U - c||\eta'|^2 + (U'^2/4|U - c|)|\eta|^2 - |U'||\eta'||\eta| . \tag{36}$$

Multiplying Eq. (36) by $\rho_0(1+f)$, integrating over (y_1, y_2), and using Eqs. (35) and (24) one can easily obtain the inequality

$$(1 - 4J_0)B^2 \ge E^2 + B^2 - \int_{y_1}^{y_2} \rho_0(1+f)|U'||\eta'||\eta|dy , \tag{37}$$

where

$$B^2 = \int_{y_1}^{y_2} \frac{\rho_0(1+f)U'^2|\eta|^2 dy}{4|U - c|} ,$$

$$E^2 = \int_{y_1}^{y_2} |U - c|Q\, dy , \tag{38}$$

and J_0 denotes the minimum value, over the interval (y_1, y_2), of the modified local Richardson number $g\beta/(1+f)U'^2$. Using Schwarz' inequality, we have

$$\int_{y_1}^{y_2} \rho_0(1+f)|U'||\eta'||\eta|dy \le \left[\int_{y_1}^{y_2} \frac{\rho_0(1+f)|U'|^2|\eta|^2}{|U - c|} dy \times \int_{y_1}^{y_2} \rho_0(1+f)|U - c||\eta'|^2\right]^{1/2} dy ,$$

$$\int_{y_1}^{y_2} \rho_0(1+f)|U'||\eta'||\eta|dy \le \left[\int_{y_1}^{y_2} \frac{\rho_0(1+f)|U'|^2|\eta|^2}{|U - c|} dy \times \int_{y_1}^{y_2} \rho_0(1+f)|U - c|(|\eta'|^2 + K^2|\eta|^2)\right]^{1/2} dy ,$$

$$\int_{y_1}^{y_2} \rho_0(1+f)|U'||\eta'||\eta|dy \le (4B^2E^2)^{1/2} . \tag{39}$$

Inequalities (37) and (39) lead to the inequalities

$$1 - (1 - 4J_0)^{1/2} \le E/B \le 1 + (1 - 4J_0)^{1/2} . \tag{40}$$

Since $|U - c| \ge c_i$, Eq. (38) and inequality (40) give

$$c_i \int_{y_1}^{y_2} Q\, dy \le \frac{1}{4c_i}[1 + (1 - 4J_0)^{1/2}]^2 \times \int_{y_1}^{y_2} \rho_0(1+f)U'^2|\eta|^2 dy ,$$

from which one can deduce

$$\int_{y_1}^{y_2} \rho_0 g\beta|\eta|^2 dy \ge \frac{4J_0 c_i^2}{[1 + (1 - 4J_0)^{1/2}]^2} \int_{y_1}^{y_2} Q\, dy . \tag{41}$$

Using (41) in (30), for an unstable mode we get

$$[c_r - \tfrac{1}{2}(U_{max} + U_{min})]^2 + 2c_i^2/[1 + (1 - 4J_0)^{1/2}] \le \tfrac{1}{4}(U_{max} - U_{min})^2 \tag{42}$$

implying that the complex wave speed for an unstable mode lies inside the semi-ellipse in the upper half-plane with the range of $U(y)$ for the length of the major axis and $\{\frac{1}{2}[1 + (1 - 4J_0)^{1/2}]\}^{1/2}$ times the range of $U(y)$ for the length of the minor axis.

Since $f > 0$, it is easily seen that

$$\min[g\beta/(1+f)U'^2] \le \min(g\beta/U'^2)$$

so that in the case of a dusty gas the minor axis is greater than that in the case of a clean gas implying that the region of instability is enlarged by the addition of fine dust as observed by Saffman.[1]

VIII. SUMMARY AND DISCUSSION

We have examined the effect of fine dust particles on the stability of parallel shear flows of a stably stratified liquid based upon Saffman's model, in which a two-phase continuum is assumed, under the assumptions that the mass concentration is very small, that the relaxation time for the dust particle is very much less compared with the time scale characterizing the basic flow, and that the sedimentation effects can be neglected. We have found that the sufficient conditions for stability will remain the same as those originally conjectured by Taylor,[18] later proved by Miles, and independently by Howard, namely, that $U'(y) \neq 0$, $U(y)$ being the basic shear flow, and the local Richardson number $J(y) \geq \frac{1}{4}$, provided that the Richardson number is redefined by

$$J(y) = g\beta/(1+f)U'^2 .$$

Howard's semi-circle theorem that the complex wave velocity for any unstable mode lies inside the semicircle in the upper half-plane which has the range of $U(y)$ for the diameter also holds. We have further shown that an improved estimation of the region for unstable modes is a semi-ellipse enclosed by the semicircle.

We have tacitly assumed in the basic equations given in Sec. II that the buoyancy forces and the pressure gradient forces on the dust particles are not significant enough to be included in Eq. (2) compared with the inertial and Stokes' drag forces. The terms corresponding to these forces may contribute $O(f)$ correction terms. Further investigation will be necessary to see the effect of the terms on the stability criterion established here.

We have further to add that several modes which may presumably be due to the dust motions themselves and not merely due to the property changes in the fluid get lost in the act of approximating Eq. (20) by Eq. (21).

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