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Stability analysis of a gravity driven miscible two-fluid flow: Role of wall slip

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Abstract

A linear stability analysis of a gravity-driven miscible two-fluid flow with matched density and different viscosities, containing a thin mixed layer between the two fluids, down a slippery inclined plane is considered. Results reveal the stabilizing effects on the surface (S), the overlap (O_1 and O_2) and the TS (not shown) modes that exist for a configuration with more viscous fluid adjacent to the free surface under overlap conditions. There is a possibility of destabilizing the overlap mode (O_1) when the mixed layer is very close to the slippery wall, indicating the dual role played by the wall slip. The stabilizing characteristics of slip can be favourably used to suppress the nonlinear breakdown that may happen due to the coexistence of the unstable modes in a flow over a substrate with no slip.

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Keywords: miscible two-fluid flow; Velocity slip; Viscosity stratification; Instability.

1. Introduction

Motivated by its relevance in several applications[1], Ghosh *et al.*[2] have addressed a linear stability of pressure driven miscible two-fluid flow with same density and varying viscosity in a channel with velocity slip at the walls. Their results suggest an effective strategy to control the flow system, in a channel with slippery walls. This conclusion is a consequence of the dual role displayed by velocity slip at the wall in either promoting or suppressing the instabilities that occur for the flow in a rigid channel[3]. This sparks the natural curiosity to examine the stability characteristics of miscible two-fluid free-surface flow (MTF) with varying viscosity and matched density down an inclined substrate with velocity slip. In addition, a single layer flow down an inclined slippery substrate is destabilized as compared to that over a substrate with no slip at the onset of instability by lowering the critical Reynolds number; however, beyond the instability threshold, slip has a stabilizing effect[4]. As the MTF over a rigid substrate[5] exhibits stability characteristics different from immiscible fluids (sudden jump in viscosity) and infinite miscible fluids (finite viscosity gradient throughout the film), in the sense that, apart from the usual surface mode, the TS mode and a new type of

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overlap mode (the O-mode, similar to that observed by Craik[6] in Couett flow) exist due to the overlap of the mixed layer with the critical layer, it is of interest to see the role of slip on MTF and this is analyzed in the present study.

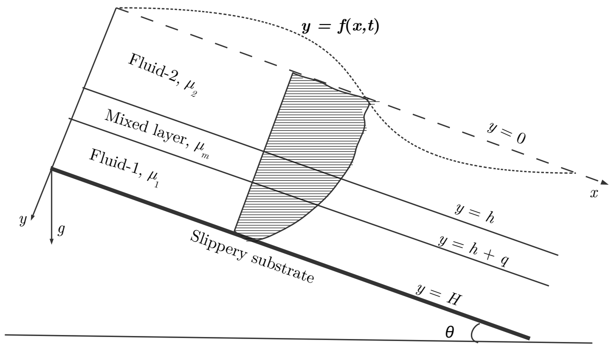

Fig. 1. Schematic of the flow system considered over a slippery inclined plane

2. Formulation and Discussion

We consider a gravity-driven two-dimensional free-surface flow of two miscible, incompressible, Newtonian fluids of matched density (ρ) and different viscosities down an inclined slippery substrate. The angle of inclination to the horizontal is θ. Fig. 1 presents the geometric structure of the flow system and the location of fluid-i, $i = 1, 2$ with respect to the Cartesian coordinate system whose x-axis ($y = 0$) is at the unperturbed free surface. $y = f(x, t)$ is the equation of the free surface at time t and μ_i ($i = 1, 2$) is the viscosity of fluid-i. A local viscosity stratification is created by a thin layer (mixed layer) of thickness q between the two fluids and it occupies the region $h \le y \le h + q$ (h is the thickness of fluid layer-2). The Peclet number is assumed to be high and this allows us to neglect the downstream growth of the mixed layer thickness.

2.1. Governing equations

The flow dynamics is governed by the continuity, the Navier-Stokes and a scalar-transport (for viscosity) equations. The boundary conditions are the Navier-slip condition[7] at the inclined plane; normal, shear stress balance and the kinematic conditions at the free surface. The set of dimensional equations and boundary conditions are given below.

$$u_x + v_y = 0, \tag{1}$$

$$\rho\left[u_t + uu_x + vu_y\right] = \frac{\partial}{\partial x}\left[-p + 2\mu u_x\right] + \frac{\partial}{\partial y}\left[\mu(u_y + v_x)\right] + \rho g \sin(\theta), \tag{2}$$

$$\rho\left[v_t + uv_x + vv_y\right] = \frac{\partial}{\partial x}\left[\mu(u_y + v_x)\right] + \frac{\partial}{\partial y}\left[-p + 2\mu u_y\right] + \rho g \cos(\theta), \tag{3}$$

$$\mu_t + u\mu_x + v\mu_y = \chi\left[\mu_{xx} + \mu_{yy}\right]. \tag{4}$$

On the free surface $y = f(x, t)$,

$$p = \frac{2\mu}{(1 + {f_x}^2)}\left[v_y - v_x f_x + u_x {f_x}^2 - u_y f_x\right] + \frac{\sigma f_{xx}}{(1 + {f_x}^2)^{3/2}}, \tag{5}$$

$$\mu(1 - f_x^{\,2})(u_y + v_x) - 4\mu u_x f_x = 0, \tag{6}$$

$$v = f_t + u f_x \tag{7}$$

and on the slippery substrate $y = H$,

$$u = l_s u_y,\ v = 0. \tag{8}$$

Here u, v are velocity components along the x and y directions respectively; p, σ and g correspond to the pressure, the surface tension coefficient and the gravity; l_s is the slip length. The base state solution can be derived from the above equations after non-dimensionlization by using H as the length scale, μ_1 as the viscosity scale and U as the velocity scale which is the average velocity over the film. The temporal stability characteristics of the base flow are examined using a linear stability analysis by giving an infinitesimal perturbation to the flow variables. Applying the two-dimensional disturbances in normal mode form as,

$$(u, v, p, s) = (U_B(y), 0, P_B(x), \mu_B(y)) + (\hat{u}, \hat{v}, \hat{p}, \hat{\mu})(y)\mathrm{e}^{\mathrm{i}\alpha(x-ct)}, \tag{9}$$

one can derive the following modified Orr-Sommerfeld[5,8] system (after suppressing hat (ˆ)):

$$\begin{aligned}\mathrm{i}\alpha\, Re\left[\phi''(U_B - c) - \alpha^2\phi(U_B - c) - U_B''\phi\right] = \mu_B\phi'''' + 2\mu_B'\phi''' + (\mu_B'' - 2\alpha^2\mu_B)\phi'' - \\ 2\alpha^2\mu_B'\phi' + (\alpha^2\mu_B'' + \alpha^4\mu_B)\phi + U_B'\mu'' + 2U_B''\mu' + (U_B''' + \alpha^2 U_B')\mu,\end{aligned} \tag{10}$$

$$\mathrm{i}\alpha\, Pe\left[(U_B - c)\mu - \mu_B'\phi\right] = (\mu'' - \alpha^2\mu), \tag{11}$$

$$\phi' = -\beta\phi'', \quad \phi = \mu = 0 \quad \text{at} \quad y = 1, \tag{12}$$

$$\phi'' + \alpha^2\phi + U_B''\eta = 0 \quad \text{at} \quad y = 0, \tag{13}$$

$$\alpha\, Re(U_B - c)\phi' + \mathrm{i}\mu_B(\phi''' - 3\alpha^2\phi') + \mathrm{i}\mu_B'(\phi'' + \alpha^2\phi) + 2\mathrm{i}U_B''\mu - \alpha(Gcot\theta + \sigma\alpha^2)\eta = 0 \quad \text{at} \quad y = 0, \tag{14}$$

$$\phi + (U_B - c)\eta = 0 \quad \text{at} \quad y = 0, \tag{15}$$

where prime ($'$) denotes differentiation with respect to y; $\mathrm{i} \equiv \sqrt{-1}$; ϕ, μ and η are respectively the amplitudes of the disturbances of the stream function, viscosity and free surface. $\beta = \frac{l_s}{H}$ is the dimensionless slip parameter and $Pe = \frac{UH}{\chi}$ (χ is the mass diffusivity), $Re = \frac{\rho UH}{\mu_1}$ (U is the velocity scale) and $Sc = Pe/Re$ are the Peclet, the Reynolds and the Schmidt number respectively. U_B, μ_B are the base state velocity and viscosity profiles with

$$\mu_B(y) = \begin{cases} m & \text{if} \quad 0 \le y \le h, \\ \mu_m(y) & \text{if} \quad h \le y \le h + q, \\ 1 & \text{if} \quad h + q \le y \le 1, \end{cases} \tag{16}$$

where $m = \mu_2/\mu_1$ and $\mu_m(y)$ is given by a tangent hyperbolic profile. The flow is linearly unstable if $Im(c) > 0$, where c is the wave speed and α (real and positive) is the wave number. In the absence of slip, the above system reduces to those by Usha *et al.*[5] and when Pe is set to infinity, to those by Craik and Smith[9].

2.2. Results

The numerical solutions of the system (10)−(15) are computed using Chebyshev spectral collocation method[10] with suitable grid stretching[3]. The open source software LAPACK is used for obtaining eigenvalues. The code recovers exactly the results of Samanta *et al.*[4] for a film over a slippery substrate ($m = 1$; Fig. 2(a), curves with symbols) after taking into account the velocity scale used by them. Slip destabilizes the surface mode (S) at the onset of instability and beyond this, at higher Reynolds numbers (Re), it stabilizes the flow by decreasing the bandwidth of unstable wave numbers (Fig. 2(b)). The computations for the present study show the existence of surface mode when a more viscous fluid is adjacent to the free surface ($m = 1.5$). In contrast to the single layer case, the surface mode is stabilized at the onset as well as beyond, by the presence of velocity slip. Also, the miscible two-fluid flow system ($m = 1.5$, $Sc = Pe/Re = 20$) over a slippery substrate is more stable than the single layer flow ($m = 1.0$, $Sc = 0$)

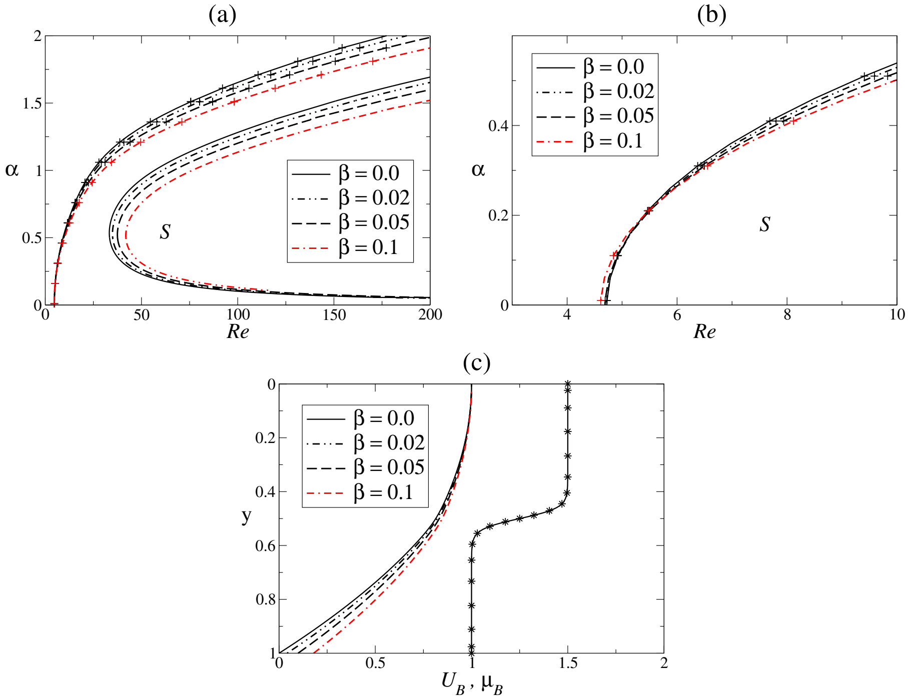

Fig. 2. (a) Neutral stability curves (Surface mode) for $h = 0.4$, $q = 0.2$, $m = 1.5$, $Sc = Pe/Re = 20$ and $\theta = 10$; (b) zoom of Fig. 2(a); (c) base velocity (U_B) and viscosity profile (μ_B; curve with star symbols) . The curves with plus symbols correspond to single fluid flow ($m = 1$) over a slippery substrate. The solid curve ($\beta = 0$) corresponds to the results for MTF over a rigid substrate[5].

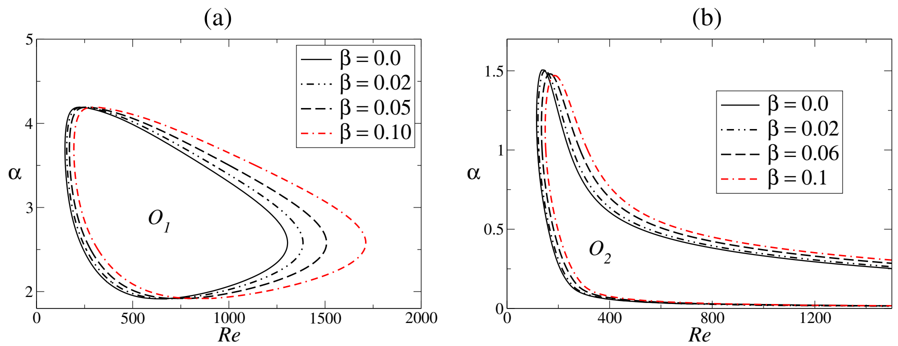

Fig. 3. Effect of velocity slip (β) on the neutral stability boundaries for $h = 0.4$, $q = 0.2$, $m = 1.5$, $Sc = 20$ and $\theta = 10$: (a) O_1 mode; (b) O_2 mode.

down a slippery substrate. The corresponding base velocity (μ_B) and base viscosity (U_B) profiles are given in Fig. 2(c). There is no change in base viscosity (curve with star symbols) with slip parameter β but base velocity is affected by β. Fig. 2(c) clearly indicate that the base velocity near the wall increases with β, as a result wall shear is less for the case of slippery wall ($\beta \neq 0$) than that for rigid wall ($\beta = 0$).

For the same configuration ($m = 1.5$, $h = 0.4$, $q = 0.2$), when the phase speed is smaller than the free surface velocity, the simultaneous existence of two other modes (O_1 and O_2) are also observed due to the overlap of the critical layer with the mixed layer (Fig. 3). Fig. 3(a) shows the effects of slip on the O_1 mode. O_1 mode is also stabilized by the slip at the onset; however, the unstable region becomes larger with an increase in β. The unstable region extends to higher Re and the range of unstable wave number increases in this region, with slip. The same trend is exhibited by the O_2 mode also, with respect to slip (Fig. 3(b)). The difference is that, O_2 mode exists for smaller wave numbers and the unstable region has no upper bound for Re. For this configuration, the surface mode is the dominant mode and it is stabilized by slip.

Fig. 4 presents the variation of growth rate (ω_i) as a function of wave number (α) for different Re values. The other parameters are the same as in Fig. 2. It is to be noted that the range of α values considered in this figure corresponds to the growth rate of O_1 mode. In Fig. 4(a) (for $Re = 200$) growth rate (ω_i) decreases with an increase in β; as a result for this Re, wall slip (β) has a damping effect on the flow system. On the other hand, Fig. 4(b) indicates the destabilizing effects of wall slip for $Re = 1300$ as ω_i becomes positive and takes higher values with an increase in β.

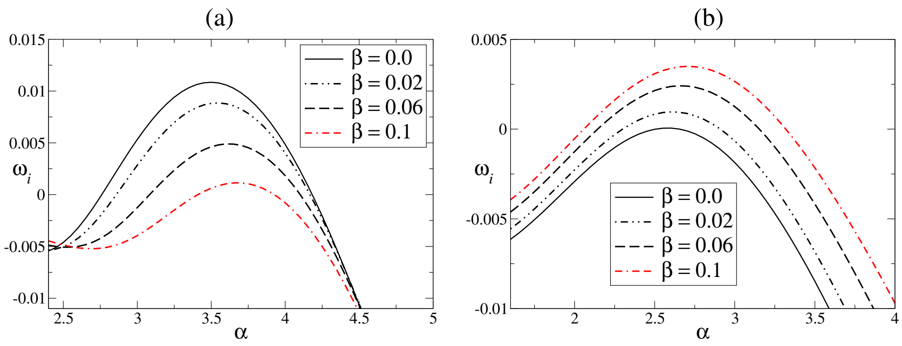

Fig. 4. Growth rate ($\omega_i = \alpha c$) curves for different β (slip) for $h = 0.4$, $q = 0.2$, $m = 1.5$, $Sc = 20$ and $\theta = 10$: (a) $Re = 200$ and (b) $Re = 1300$.

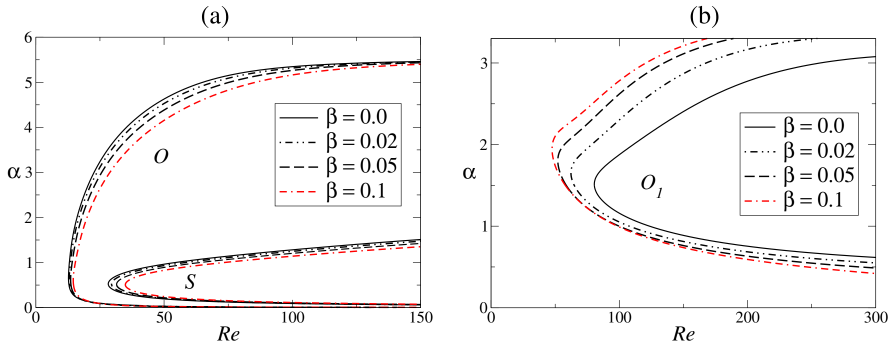

Fig. 5. Effect of β on the neutral stability boundaries for $q = 0.2$, $m = 1.5$ and $\theta = 10$: (a) O and Surface (S) modes, $h = 0.4$, $Sc = 100$; (b) O_1 mode, $h = 0.75$, $Sc = 20$.

If however, when Sc is increased to $Sc = 100$, the coalescence of the O_1 and the O_2 modes occur and it is observed to be the most dangerous mode (Fig. 5(a)). Still, slip stabilizes this overlap mode as well as the surface mode. Fig. 5(b) presents the O_1 mode when the mixed layer is located very close to the slippery substrate ($h = 0.75$). It is interesting to observe the destabilizing role of β for this configuration (Fig. 5(b), $h = 0.75$, $Sc = 20$).

3. Conclusions

Velocity slip on the substrate has a dual role on the stability of miscible two fluid flow (MTF) down an inclined slippery substrate and the behaviour is influenced by the location of mixed layer (h) and the diffusivity of two fluids (Sc). The study is exploring whether the stabilization that we obtain under some situations can be enhanced to the point of becoming useful as a passive control option for MTF, by designing the substrate as a hydrophobic[11] or a rough[12] or a porous substrate with small permeability[13]. The mechanism for the dual role of slip in this study can be examined via the energy budget analysis.

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