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Shock stand-off distance for a sphere

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The formula $\Delta/D = 1-\{1-[4/(\gamma+1)^2][1/M^2+(1/2)(\gamma-1)]\}^{1/2}$ is proposed for the stand-off distance of the shock in supersonic or hypersonic flows past a sphere, and is derived from an analogy with the Lorentz transformation.

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In this communication we shall propose a formula for the stand-off distance of the shock in supersonic and hypersonic flows past a sphere. It is given by

$$\frac{\Delta}{D} = 1 - \left[1 - \frac{4}{(\gamma+1)^2}\left(\frac{1}{M^2} + \frac{\gamma-1}{2}\right)\right]^{1/2} \tag{1}$$

where Δ is the stand-off distance of the shock and M is the free-stream Mach number.

As may be seen from the accompanying figure (Fig. 1), the shock detachment distance for a sphere as given by Eq. (1) agrees impressively with the well-known blunt body flow results of Van Dyke [1] Lomax,[2] and others. Equation (1) also clearly brings out the dependence of the shock detachment distance on the ratio of specific heats γ and the free-stream Mach number M. The merging of the detached shock with the body as $M \to \infty$ and $\gamma \to 1$ is also assured in accordance with the hypersonic flow theory. Even though the nose shock detachment distance given by Eq. (1) does not coincide exactly with the numerical values of others for all Mach numbers and various values of γ, we find it interesting to indicate the curious way in which it was obtained.

It is known that there is a shock detachment for a blunt body in supersonic flow. We *assume* that, in the subsonic region between the nose and the shock, Lorentz transformations are applicable. These transformations as given by Kussner[3] are

$$x' = \frac{x + vt}{(1 - v^2/a^2)^{1/2}} \tag{2}$$

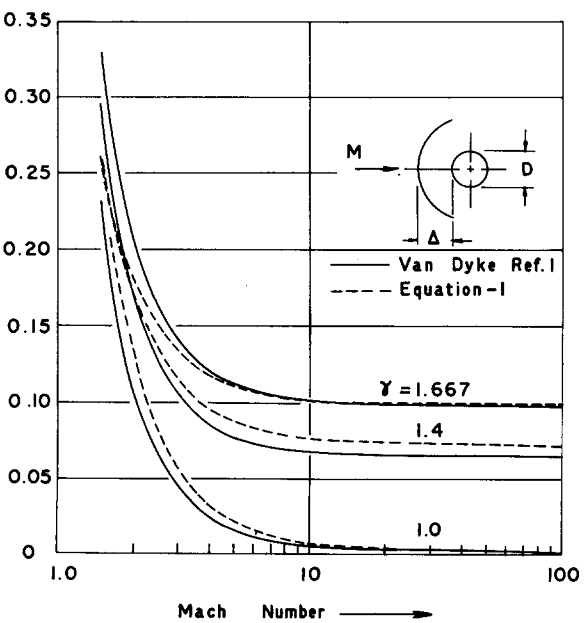

FIG. 1. Shock stand-off distance for sphere.

$$t' = \frac{t + vx/a^2}{(1 - v^2/a^2)^{1/2}}, \tag{3}$$

where t is the time, the primed variables refer to a moving system, v is the subsonic velocity downstream of the shock assumed normal in front of the nose, and a is the speed of sound. Using the Prandtl relation $uv = a^{*2}$ between the upstream and downstream velocities across a normal shock, and substituting for v from this, the transformation relations become

$$x' = \frac{x + a^{*2}t/u}{(1 - a^{*4}/a^2u^2)^{1/2}}, \tag{4}$$

$$t' = \frac{t + a^{*2}x/a^2u}{(1 - a^{*4}/a^2u^2)^{1/2}} \tag{5}$$

where a^* is the critical speed of sound, x is the length, and u is the free-stream supersonic velocity.

It is readily seen that the Lorentz contraction factor in the present case is $(1 - a^{*4}/a^2u^2)^{1/2}$. Thus the diameter D of the sphere changes to D' where

$$D' = D(1 - a^{*4}/a^2u^2)^{1/2} \tag{6}$$

and we attribute $D - D'$ to be the detachment distance. Since we are interested in the shock detachment distance at the nose of the sphere, we now make the assumption that in the immediate neighborhood of the nose the conditions correspond to stagnation conditions. Therefore substituting $a = a_0$ (a_0 is the speed of sound at stagnation conditions), we obtain from Eq. (6)

$$\frac{D-D'}{D} = \frac{\Delta}{D} = 1 - \left(1 - \frac{a^{*4}}{a_0^2u^2}\right)^{1/2}$$

$$= 1 - \left(1 - \frac{a^{*4}}{a_0^4}\frac{a_0^2}{a^2}\frac{a^2}{u^2}\right)^{1/2}$$

$$= 1 - \left(1 - \frac{4}{(\gamma+1)^2}\left[1 + \tfrac{1}{2}(\gamma-1)M^2\right]\frac{1}{M^2}\right)^{1/2}$$

$$= 1 - \left[1 - \frac{4}{(\gamma+1)^2}\left(\frac{1}{M^2} + \tfrac{1}{2}(\gamma-1)\right)\right]^{1/2}$$

for the stand-off distance.

It is surprising that this expression agrees fairly well with the exact numerical values based on a nonlinear theory.

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[1] M. D. Von Dyke and H. D. Gordon, National Aeronautics and Space Administration Report No. NASA TR R-1, 1959 (unpublished).

[2] H. Lomax and M. Inouye, National Aeronautics and Space Administration Report No. NASA TR R-204, 1964 (unpublished).

[3] H. G. Kussner, Luftfahrt-Forsch. 17, 370—8 (1940); [also translated as NACA Tech. Memo. No. 979 (1941)].