Preprints of the 10th IFAC International Symposium on Dynamics and Control of Process Systems
The International Federation of Automatic Control
December 18-20, 2013. Mumbai, India

Robust Plant Friendly Optimal Input Design

Abhishankar Kumar, Sridharakumar Narasimhan*.

*Department of chemical engineering, Indian institute of technology Madras India
(Tel: [+91] (44) 22574177; e-mail: sridharkrn@iitm.ac.in).

Abstract: Optimal experiment design for system identification involves determining an optimal input that is used to perturb the system so that the resulting input-output data is maximally informative. Plant friendly identification requires that constraints on input move sizes, output sizes or variance and experiment time be respected. The solution to the optimum input design problem depends on the unknown parameters to be estimated which is often approximated by an initial estimate. Use of the estimate is likely to result in loss in performance or violation of the constraints. An alternative is to formulate a robust optimization problem with uncertain parameters. The contribution of this work is to use the uncertainty sets originating from a prior identification exercise to solve a robust plant friendly input design problem. The methodology is derived for a general class of systems illustrated using numerical simulations. Simulations validate the expectation that the constraints are probabilistically more likely to be satisfied using the robust design than a nominal design based on uncertain parameters.

Keywords: System identification, input design, convex optimization, robust optimization, semidefinite programming.

1. INTRODUCTION

System identification is the process of identifying dynamic models using a priori system knowledge and input-output data collected during an appropriate experiment. The quality of the identified model depends on the choice of the perturbation (input) signal. Hence, it is mandatory to design an informative input signal that results in maximum information about the system using minimal resources. The quality of the information can be quantified in terms of some scalar norm of the information matrix (Mehra, 1974; Goodwin et al., 1977; Zarrop, 1979; Ljung, 1999). Recently, a cost or a control relevant criterion is used to design the input (Hildebrand et al., 2003; Antoulas et al., 1999). An alternate is the least costly framework (Bombois et al., 2004) where the objective is to minimize the cost of the experiment quantified in terms of excessive input usage and output excursion while ensuring that the identified model is sufficiently accurate. For a wide variety of cost functions, it has been shown that the optimization problems can be reformulated as convex problems and in particular, semidefinite programs (SDPs) or generalized eigenvalue problems (GEVPs). Such problems can be formulated and solved using software such as CVX (Grant et al., 2011) or YALMIP (Lofberg, 2004).

A practical requirement is that experiments are carried out under "plant-friendly" conditions, viz., conditions that cause minimum disruption to normal operation (Rivera et al., 2009). This translates to requirements on input usage, input move sizes (to prevent actuator wear and tear), output excursions and experiment time. These constraints are non-linear and possibly non-convex. Hence, these constraints are relaxed to obtain a convex reformulation (Narasimhan & Rengasawamy, 2011; Narasimhan et al., 2011).

A very intriguing property of several optimal experiment design problems is that the optimization formulation, viz., the objective function and/or constraints are system parameter dependent. However, the parameters are unknown and the purpose of the experiment design is to estimate the unknown parameters. This apparent paradox is recognized in literature (Antoulas & Anderson, 1999). One suggested solution is to perform an initial (sub-optimal) experiment using standard inputs (e.g., PRBS, multi-sine etc.) and obtain an estimate of the parameters. These estimates are then used in the optimization problem and an optimal input is designed which is then used for perturbing the system. The parameters are re-identified and the procedure is iterated, if necessary. This approach is often referred to as the sequential approach. However, since only estimates of the parameters are used at any stage, there is no guarantee of optimality or even constraint feasibility. This is especially relevant in plant friendly identification, where constraints on the output are important.

Alternate methods include Bayesian approach (Chaloner & Larntz, 1989; Atkinson et al., 1993; Sebastiani & Wynn, 2000) or robust approach (Rojas et al., 2007). In the Bayseian approach, the expected value of the cost is minimized, given a prior distribution for the parameters. The robust approach is a min-max or worst case approach, where the objective function is minimized over an uncertain set of parameters, given a characterization of the uncertainty set. This can be shown to be equivalent to a semi-infinite optimization problem with the feature that constraints be enforced at every point in this set.

Currently, we have two strong reasons to solve input design problem in robust framework. First reason is to account parameter uncertainty in problem formulation and second reason is to ensure constraint feasibility with higher probability level. We address this problem using a combination of the sequential and robust approaches. The sequential approach is used to generate an initial estimate of the parameters along with an uncertainty set. PEM methods result in asymptotically normal estimates of the parameters and hence, it can be safely concluded that given a confidence level, the true parameters lie within a confidence ellipsoid (Ljung, 1999). Equivalently, this results in an ellipsoidal uncertainty set for the parameters which are then used in a robust experiment design procedure. The constraints are then imposed for all points in the uncertainty set. Techniques from convex and robust optimization literature Boyd et al., (2004) Ben-Tel et al., (2009); are employed to arrive at robust and computable versions of the constraints.

2. OPTIMAL INPUT DESIGN

We consider the following SISO system with input u_k and output y_k.

$$y_k = \frac{B(q^{-1})}{A(q^{-1})} u_{k-d} + \frac{D(q^{-1})}{C(q^{-1})} e_k \tag{1}$$

Where:

$$A(q^{-1}) = 1 + \sum_{j=1}^{n} a_j q^{-j} \qquad B(q^{-1}) = \sum_{j=0}^{m} b q^{-j}$$

$$C(q^{-1}) = 1 + \sum_{j=1}^{s} c_j q^{-j} \qquad D(q^{-1}) = \sum_{j=0}^{r} d q^{-j} \tag{2}$$

Where q^{-1} is the backshift operator, e_k is a zero mean, finite variance Gaussian white noise sequence. We make the following assumptions:

- A(q) and D(q) have no zero on the closed unit disk and are co-prime.
- There are no pole-zero cancellations.
- The experiment time N is large.
- The true system is in the model set.
- The input is a stationary process with power spectrum (two-sided) $\overline{\Phi}_u(\omega)$ defined on [-π π]. $r_u(\tau)$ is the auto-correlation of $u(t)$ at lag τ and form a Fourier transform pair with $\overline{\Phi}_u(\omega)$. Corresponding to the two-sided power spectrum, we defied an equivalent one-sided power spectrum $\Phi_u(\omega)$ on [0 π]. The relationship between the two is as follow: Hildebrand et al., (2003) given $\overline{\Phi}_u(\omega)$ defined on [-π, π] , $\Phi_u(\omega)$ is defined such that

$$\int_{-\pi}^{\pi} \overline{\Phi}u\,(\omega)\,\varphi(\omega)\,d\omega = \int_{0}^{\pi} \Phi_u(\omega) \frac{(\varphi(\omega) + \varphi(-\omega))}{2} \tag{3}$$

Where $\varphi(\omega)$ is any C^{∞} function on [-π π].

- The class of input is further constrained to those having unit power

$$\int_{-\pi}^{\pi} \overline{\Phi}u\,(\omega) d\omega = 1 \tag{4}$$

or in term of the one -sided power spectrum:

$$\int_{0}^{\pi} \Phi_u(\omega)\, d\omega = 1 \tag{5}$$

Let $P = [a_1 \dots a_n, b_o \dots . b_n]'$ $R = [c_1 \dots c_s,\ d_0 \dots . d_s]'$ then $\theta = [P'R']'$ is the overall vector of parameter to be estimated. The system is perturbed with an input sequence $u_1, u_2 \dots . u_n$ resulting in the output $y_1, y_2 \dots . y_n$ and the resulting data is used to estimate the unknown parameter θ.The quality of the estimated parameter $\hat{\theta}$ can be described in the term of bias and covariance of $\hat{\theta}$. Given an asymptotically unbiased and efficient estimator, such as the Prediction Error method (Ljung , 1999) the covariance of $\hat{\theta}$ is given by the following:

$$cov(\hat{\theta}) = M_{\theta}^{-1} \tag{6}$$

where M_{θ} is Fisher information matrix. A typical problem in experiment design is to minimize a scalar function of M_{θ} subject to certain constraints. For above dynamic system it has been shown that M_{θ} can be partitioned as Zarrop, (1979):

$$\begin{bmatrix} M_P & 0 \\ 0 & M_R \end{bmatrix} \tag{7}$$

where $M_p \in \mathbb{R}^{p \times p}$ is related to the m+n+1 parameters in P and dependent upon input. M_R is related to noise parameters and importantly, is independent of the input and so, it is sufficient to consider cov($\hat{\theta}$) or equivalently the inverse of M_p in input design problem. The objective in a plant friendly input design problem is to maximize some quality of the parameter estimates subject to plant friendly constraints. Typical quality functions include $logdet\,(cov(\hat{\theta}))$, $\lambda_{max} cov(\hat{\theta})$ or $trace(cov(\hat{\theta}))$. Plant friendly considerations include minimizing move size $|u_k - u_{k-1}|$ and output magnitude $|y_k|$. However; it is not possible to translate these instantaneous constraints to the frequency domain and hence it is customary to relax them by constraining $\frac{1}{N}\sum_{i=1}^{N}(u_k - u_{k-1})^2$ and $\frac{1}{N}\sum_{i=1}^{N} y_i^2$.

The frequency domain cov($\hat{\theta}$) is approximately

$$\text{cov}(\hat{\theta}) \cong \sum_{i=1}^{m+n+s+1} L_i x_i \tag{8}$$

where L_i is constant (m+n+1)×(m+n+1) matrix and x_i is trigonometric moment (Narasimhan et al., 2011).

The input power constraint, input move size constraint and the output power constraint are all expressed in frequency domain using Parseval's theorem. Thus for a fixed and large N, the relaxed D-optimal plant friendly design problem is:

$$\min_{\Phi_u} -log\ det\big((\textstyle\sum L_i x_i)\big)$$

$$\text{s.t.}\begin{cases} \int_0^{\pi} \Phi_u d\omega = 1 \\ 2-2\int_0^{\pi} cos(\omega)\Phi_u d\omega \le c_{in} \\ \int_0^{\pi} \left|\frac{B^2(e^{j\omega})}{A^2(e^{j\omega})}\right| \Phi_u(\omega)d\omega \le c_{op} \end{cases} \quad (9)$$

where the constraints are on the input power, input move size and output power respectively and c_{in} and c_{op} are user specified limits on the move size variance and output power.

The above problem is convex but the decision variable $\Phi_u(\omega)$ is infinite dimensional. Two methods have been proposed to convert it into finite dimensional problem (Hjalmarsson., 2005). The first involves parameterizing the input spectrum in term of a finite number of basis function e.g.,

$$\Phi_u = \sum c_j exp(j\omega), \qquad j=1\ldots,M. \quad (10)$$

The other approach is known as the partial correlation approach, where the problem is re-parameterized in terms of trigonometric moment x_i (Narasimhan et al., 2011).

$$x_i = \int_0^{\pi} \left|\frac{1}{D(e^{j\omega})A^2(e^{j\omega})}\right|^2 cos^{i-1}(\omega)\,\Phi_u(\omega)d\omega \quad (11)$$

However, parameterization simply in terms of x_i is not sufficient as it is necessary to ensure that x_i are feasible, or in other word we can say that they satisfy a necessary and sufficient condition to become a valid moment points. Define p=max (2n+r+2, m+s+n+1, m+r+s+1) if p is odd i.e., p=2l+1

$$\underline{\Delta}(x_1, x_2, \ldots, x_{2l+1}) \equiv \begin{bmatrix} x_1 & x_2 & \ldots & x_{l+1} \\ x_2 & x_3 & \ldots & x_{l+2} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ x_{l+1} & x_{l+2} & \ldots & x_{2l+1} \end{bmatrix}$$

$$\overline{\Delta}(x_1, x_2, \ldots, x_{2l+1}) \equiv \begin{bmatrix} x_1 - x_3 & \ldots & x_{l+1} \\ x_2 - x_4 & \ldots & x_{l+2} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ x_l - x_{l+2} & \ldots & x_{2l-1} - x_{2l+1} \end{bmatrix} \quad (12)$$

The obtained matrices $\overline{\Delta}$ and $\underline{\Delta}$ should be positive definite (Narasimhan et al., 2011).

A similar condition can be derived for even p, which is omitted in the interest of brevity and can be found in Narasimhan et al., (2011). The advantage of this re parameterization is that the final optimization problem is a semidefinite program (SDP). The input and output constraints in (9) can be expressed as linear functions of x_i. E.g., $\left|D(e^{j\omega})A^2(e^{j\omega})\right|^2$ as defined in Narasimhan et al., (2011) is a polynomial of degree 2n+r+s in $cos(\omega)$and we denote it by $\sum_{i=1}^{2n+r+1} \alpha_i\, cos^i(\omega)$. Hence the constraint $\int_0^{\pi} \Phi_u d\omega = 1$ can be replaced by a linear equality as follows:

$$\int_0^{\pi} \Phi_u d\omega = \int_0^{\pi} \frac{f(\omega)}{f(\omega)} \Phi_u(\omega)d\omega = \sum_{i=1}^{2n+r+1} \alpha_i\, x_i = 1 \quad (13)$$

The other constraints (input move size and output power) can similarly be expressed as linear functions of x_i (Details are omitted in the interest of brevity). Let $\beta_i, i = 1, \ldots m+n+s+1$ be the coefficient of $\left|A(e^{j\omega})B(e^{j\omega})D(e^{j\omega})\right|^2$. The final optimization problem can be formulated as a SDP:

$$\min_{x_i} -\log det(\textstyle\sum L_i x_i)\text{s.t.}\begin{cases} \sum_{i=1}^{2n+r+1} \alpha_i x_i = 1 \\ 2-2\sum_{i=1}^{2n+r+1} \alpha_i x_{i+1} \le c_{in} \\ \sum_{i=1}^{m+s+n+1} \beta_i\, x_i \le c_{op} \\ \overline{\Delta}(x_1, \ldots\ldots x_p) \succcurlyeq 0 \\ \underline{\Delta}(x_1, \ldots\ldots x_p) \succcurlyeq 0 \end{cases} \quad (14)$$

Other re-parameterization is also possible, e.g., in term of $cos(\omega), cos(2\omega), \ldots$ (Hildebrand et al., 2003) which also yields an LMI. Both approaches are theoretically equivalent.

Inspection of (14) reveals that the optimization formulation depends on the system parameters as they appear directly in the objective function and the output power constraint. The most common solution suggested in literature is to carry out a preliminary identification exercise using a sub-optimal input and obtain an estimate of the parameters, $\hat{\theta}$. These parameters are then used in (14) to obtain an input and the system is perturbed with this computed input and the parameters re-identified. This procedure is repeated, if necessary.

One important issue with this procedure is that the solution obtained may not be optimal or even feasible, i.e., the output power constraint may not be satisfied by the true system if an estimate is used in (14). In the following, we use techniques from robust optimization and characterization of the uncertainty sets to obtain a robust version of (14).

3. ROBUST EXPERIMENT DESIGN

3.1 Motivation

From (14), it is clear that the optimization problem directly depends on the system parameter vector θ. The plant friendly constraints on output power, i.e., $\sum_{i=1}^{m+s+n+1} \beta_i\, x_i \le c_{op}$ directly involves the parameters of the system to be identified $\hat{\beta} = J(\hat{\theta})$. In the sequential approach, an estimate $\hat{\theta}$ is generated using a prior identification experiment and the corresponding $\hat{\beta}$ used to solve (14). Note that $\hat{\beta}$ is only an estimate and hence, when an input corresponding to this solution is actually used to perturb the system subsequently, the output power need not satisfy the above constraint.

Since $\hat{\beta}$ is an estimate and therefore uncertain, if we are able to characterize the corresponding uncertainty sets, we can then formulate a robust optimization problem. Rather than impose the constraints using the estimated values of the system parameters, we require that the constraints be satisfied for all points in the uncertainty set.

3.2 Characterization of uncertainty sets

When θ is estimated using a PEM method, $\widehat{\theta}$ is asymptotically normal with mean θ and covariance given by the Cramer-Rao inequality. This can be interpreted to imply modulo a probability level, that the true θ lies within a confidence ellipsoid. The larger the probability level, the larger the ellipsoid. Equivalently, we can describe the following uncertainty region cantered around $\widehat{\theta}$:

$$\mathcal{U}:\left\{\theta\middle|\left(\theta-\hat{\theta}\right)^{T}cov\left(\hat{\theta}\right)^{-1}\left(\theta-\hat{\theta}\right)\leq\alpha^{2}\right\} \tag{15}$$

where α is indicative of the probability level and $\text{cov}(\hat{\theta})$ is evaluated from (6). Rather than solve (14) using the nominally value, viz., the estimated $\widehat{\theta}$,we allow θ to vary in $\mathcal{U}$.

While (15) characterizes the uncertainty set of the parameters, we are interested in characterizing the uncertainty sets of $[\beta_1, \beta_2, \ldots]^T$. This requires the calculation of the covariance of β as follows:

The covariance of nonlinear parameter can be computed by linearizing it around mean using Taylor series expansion Ljung (1999). Let $\hat{\theta}$ is $\vartheta \times 1$-dimensional vector of estimated parameter with mean θ_0 and covariance Y. Our main interest is to find covariance of $\mu \times 1$ dimensional random variable $\hat{\beta} = J(\hat{\theta})$ which is nonlinear in parameters. We use the following Taylor series expansion as follows:

$$J(\hat{\theta}) \approx J(\theta_0) + J'(\theta_0)(\hat{\theta} - \theta_0)$$

Where J' is the $\mu \times \vartheta$ derivative of J with respect to θ.

$$Cov\, J(\hat{\theta}) = E\left(J(\hat{\theta}) - E\, J(\hat{\theta})\right)\left(J(\hat{\theta}) - E\, J(\hat{\theta})\right)^T$$

For an asymptotically unbiased estimate $E\, J(\hat{\theta}) = J(\theta_0)$

$$Cov\, J(\hat{\theta}) \approx E\left(J(\hat{\theta}) - J(\theta_0)\right)\left(J(\hat{\theta}) - J(\theta_0)\right)^T$$

$$Cov\, J(\hat{\theta}) \approx J'(\theta_0)E\left((\hat{\theta} - \theta_0)(\hat{\theta} - \theta_0)^T\right)\left(J'(\theta_0)\right)^T$$

$$Cov\, J(\hat{\theta}) \approx J'(\theta_0)Y\left(J'(\theta_0)\right)^T = \Omega\Omega^T \tag{16}$$

Thus, given an estimate $\hat{\theta}$, and the corresponding estimate of the transformed vector β, the uncertainty set of the transformed vector β can be described as :

$$\mathcal{U}:\left\{\hat{\beta} + \Omega z\middle|\|z\|_2 \leq \alpha^2\right\}. \tag{17}$$

3.3 Robust formulation

The robust counterpart of output constrain $\sum_{i=1}^{m+s+n+1} \beta_i\, x_i \leq c_{op}$ can be written as $\sum_{i=1}^{m+s+n+1} \beta_i\, x_i \leq c_{op}\ \ \forall\ \beta \in \mathcal{U}$.This relationship will hold if and only if $\max_{\beta\in\mathcal{U}} \sum_{i=1}^{m+s+n+1} \beta_i\, x_i \leq c_{op}$. Equivalently this relationship can be replaced by vector form as follows:

$$\max_{\beta\in\mathcal{U}}\ \beta^T x \leq c_{op} \tag{18}$$

Using equation 17, equation 18 can be written as:

$$\hat{\beta}^T x + \max_{\|z\|_2}(\Omega z)^T x \leq c_{op} \tag{19}$$

Using Cauchy-Schwarz inequality (Boyd et al., 2004) the simplified robust form of output constraint is:

$$\hat{\beta}^T x + \alpha\, \|\Omega x\|_2 \leq c_{op} \tag{20}$$

Although the above constraint is nonlinear, it is a convex and in particular, a second order cone constraint. The robust form of output constraint depends on the covariance of $\hat{\beta}$ which is generally nonlinear in parameter and must be computed before imposing this constraint into optimization formulation.

$$\min_{x_i} -\log det(\textstyle\sum L_i x_i)\text{s.t.}\begin{cases} \sum_{i=1}^{2n+r+1} \alpha_i x_i = 1 \\ 2 - 2\sum_{i=1}^{2n+r+1} \alpha_i x_{i+1} \leq c_{in} \\ \hat{\beta}^T x + \alpha\, \|\Omega x\|_2 \leq c_{op} \\ \overline{\Delta}(x_{1,\ldots\ldots}x_p) \succcurlyeq 0 \\ \underline{\Delta}(x_{1,\ldots\ldots}x_p) \succcurlyeq 0 \end{cases} \tag{21}$$

The obtained problem is standard convex optimization problem and can be solved using CVX (Grant & Boyd, 2011)

3.4 Input signal design

Given the optimal x_i^*, the next step is to design actual input. The theory of Tchebycheff system (Narasimhan, et al., 2011) allows us to obtain an input containing no more than p/2 distinct frequencies. Let the desired input spectrum Φ_u be represented in the form linear combination of weighted frequency.

$$\Phi_u = \textstyle\sum_i \Gamma_i\, \delta(\omega - \omega_i) \tag{22}$$

Where $\delta(\omega - \omega_i)$ is Dirac delta function, $\omega_i$are points in support of Φ_u and Γ_i is the associated weights or contribution of i^th frequency. The frequency ω_i and its associated weight Γ_i can be computed by solving convex optimization problem proposed in Hildebrand and Gevers, 2003 & Narasimhan et al., 2011. Once the frequencies and the weights are known, the optimal input is a multisine (Zarrop, 1979).

$$`u_t = \textstyle\sum_i \lambda_i\, cos(\omega_i t + \phi_i) \tag{23}$$

Where $\phi_i = 0, \lambda_i = (\Gamma_i)^{0.5}$ if $\omega_i = 0, \pi$ .For all other ω_i ,$\lambda_i = (2\Gamma_i)^{0.5}$ and ϕ_i can be chosen arbitrarily.

4. EXAMPLE

To motivate the idea of robust D-optimal plant friendly input design, we have taken single input single output finite impulse response (FIR) system:

Consider the true system

$$y_k = (1 - 0.5q^{-1} + 0.25q^{-2})u_k + e_k \tag{24}$$

Hear u_k is input and e_k is zero mean Gaussian white noise with variance 0.1. The system is to be identified within FIR model structure.

For the identification purpose output signal is generated according to expression (24) using sub optimal input (PRBS) of power 0.5. The generated data is then used to obtained initial estimate of the parameter $(\hat{\theta})$ and corresponding covariance matrix (Y) by using SYSID toolbox in Matlab.

For the above model (FIR) D-optimal plant friendly formulation is as follow:

$$\min_x -\log det \begin{bmatrix} x_1 & x_2 & 2x_3 - x_1 \\ x_2 & x_1 & x_2 \\ 2x_3 - x_1 & x_2 & x_1 \end{bmatrix}$$

$$\text{s.t.} \begin{cases} x_1 = 1 \\ (b_1^2 + b_2^2 + b_3^2 - 2b_1 b_3)x_1 + (2b_2 b_1 + 2b_2 b_3)x_2 \\ \quad +4b_1 b_3 x_3 \quad \leq c_{op} - c_{in} \\ x_2 \geq 1 - \frac{c_{in}}{2} \\ \begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \end{bmatrix} \succcurlyeq 0 \\ x_1 - x_3 \geq 0 \end{cases} \quad (25)$$

The ideal value of the objective function is -28.59 and it is obtained by solving the above problem using the true values of the system parameters [1 -0.5 0.25]. For this system, the information matrix depends on the moments x_1, x_2, x_3. In this particular formation $x_1 = 1$, and hence it is sufficient to consider the feasible space for x_2 *and* x_3. Let the input consist of single frequency ω_0, i.e., $\Phi_u = \delta(\omega - \omega_0)$ where δ is Dirac-delta function centred at ω_0.Clearly, if $\omega_0 = 0$, we have $x_2 = 1, x_3 = 1$ likewise, when $\omega_0 = \pi/2 \;\; x_2 = 0, x_3 = 0$. In similar fashion we continue with different $\omega_0$between 0 to π and will get different x_2 and x_3. Plot of (x_2, x_3) shown in Figure 1 will give the feasible region which is the convex hull of all single frequency design.

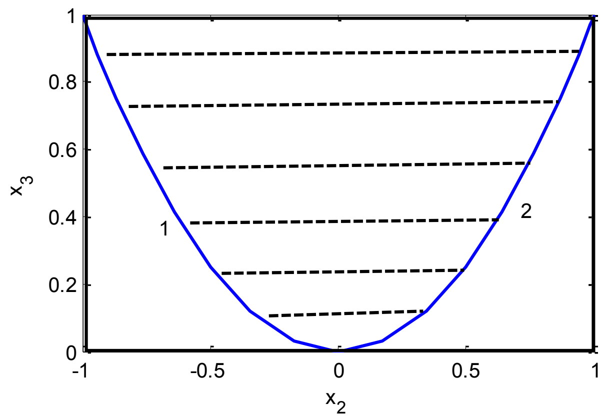

Figure 1. Feasible region

Since the output power constraint and hence, the optimal input design procedure depends on the unknown parameters, an initial estimate of the parameters is obtained by PRBS (Pseudo random binary signal). To simulate this idea we have taken $c_{in} = 1$, *and* $c_{op} = 0.6$. The optimal solution is $[1, 0.4459, 0.3655,]^T$, which lies on output constraint, that is the output constraint is active while input constraint is not. Among other possible optimal input, which includes filtered white noise, autoregressive moving average sequence, we choose the following multi sine because it is easy of generation and subsequent analysis.

$$u_k = 1.2371 cos\left(\frac{\pi}{4}k + \phi_{11}\right) + 0.6225 cos\left(\frac{\pi}{2}k + \phi_{12}\right)$$
$$+0.2026 cos(\pi k) \quad (26)$$

The actual output power corresponding to this input is calculated using the true system parameters. This process is repeated in a Monte Carlo simulation setup with 1000 different noise realization. The parameter estimates using these data sets are used to solve the corresponding D-optimal problem. It is observed that the actual output power constraint is obeyed in only ~ 49.9% of the 1000 realizations. Hence, it can safely surmised that the probability that the output constraint is actually obeyed is approximately 0.495.

In order to address this, we impose a robust version of the output power constraint which inherently accounts for the fact that the parameters are uncertain. This results in a conservative constraint and is shown in Figure 2.

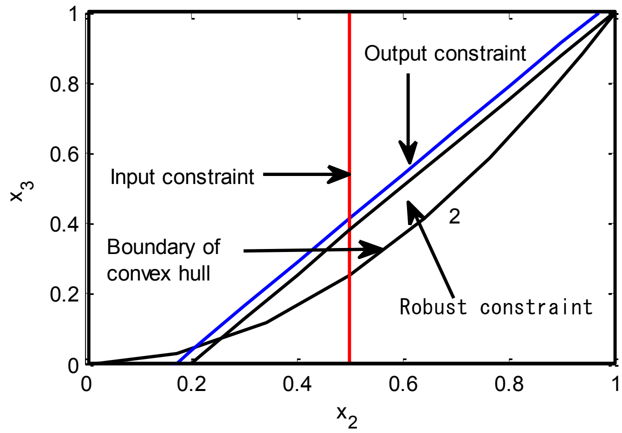

Figure 2. Feasible space with constraints

The robust D-optimal plant friendly formulation is as follows:

$$\min_x -\log det \begin{bmatrix} x_1 & x_2 & 2x_3 - x_1 \\ x_2 & x_1 & x_2 \\ 2x_3 - x_1 & x_2 & x_1 \end{bmatrix}$$

$$\text{s.t.} \begin{cases} x_1 = 1 \\ (\widehat{b_1^2} + \widehat{b_2^2} + \widehat{b_3^2} - 2\widehat{b_1}\widehat{b_3})x_1 + (2\widehat{b_1}\widehat{b_2} + 2\widehat{b_2}\widehat{b_3})x_2 \\ \quad +4\hat{b}_1 \hat{b}_3 x_3 + \alpha\|\Omega x\|_2 \quad \leq c_{op} \\ x_2 \geq 1 - \frac{c_{in}}{2} \\ \begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \end{bmatrix} \succcurlyeq 0 \\ x_1 - x_2 \geq 0 \end{cases} \quad (27)$$

$\beta_1 = b_1^2 + b_2^2 + b_3^2, \;\; \beta_2 = 2b_1 b_2 + 2b_2 b_3, \beta_3 = 4b_1 b_3$

Given an initial estimate of b_i, and the corresponding covariance, the covariance of the nonlinearly transformed β is determined using the Taylor series approximation as described previously and P can be obtained by Cholesky factorisation of this matrix.

To simulate the robust D-optimal formulation we have taken all design parameter same as previous case ($c_{in} = 2$, *and* $c_{op} = 0.6$) *and* $\alpha = 1$. The optimal solution is $[1, 0.4715, 0.3634,]^T$, which lies on output constraint, i.e., the output constraint is active while input constraint is not. The robust input is:

$$u_k = 1.2254cos\left(\frac{\pi}{4}k + \phi_{21}\right) + 0.6607cos\left(\frac{\pi}{2}k + \phi_{22}\right) +0.1758cos(\pi k) \quad (28)$$

As above, the process is repeated with 1000 sets of noise realizations. The parameter estimate using these data sets is used to solve the robust D-optimal problem. It is seen that the actual output power constraint is obeyed in ~84.04% of the 1000 realizations. For sake of illustration the results are also presented for $\alpha = 2.795$ (95 % confidence level). These are tabulated in Table 1. Hence, the robust problem formulation ensures that the probability that the output power constraint is obeyed is substantially higher. This results in a conservative constraint and the trade-off is that the parameter accuracy decreases slightly as compared to the ideal solution.

5. CONCLUSIONS

A robust formulation for plant friendly input design with constraints on input move size and output power is presented. The constraints are shown to be conservative, and hence result in a slight loss in performance. However, the robust formulation ensures that the output constraints are satisfied to a higher degree of confidence.

Table 1

Attribute	$\alpha = 1$	$\alpha = 2.795$
% Output power below than true value in case of (Robust/linear) Output constraint	84.04/49.9	95.4/49.9
True value of objective function	-28.59	-28.59
Objective function range in case of (Robust/linear output Constraint)	-28.5 to -29/ -28.4 to -28.9	-28.5 to -29/-28.4 to -28.85
% of objective function better than true value in case of (Robust/linear) output constraint	15.9/50.2	4.5/50.2

ACKNOWLEDGMENT

This work was partially supported by the Board of Research in Nuclear Sciences under the project "Experiment design on convex optimization".

REFERENCES

Karlin, S. & Studden, W. J. (1966). Tchebycheff Systems: With Applications in Analysis and Statistics *Interscience,*U.S.A.

Mehra, R. (1974). Optimal Input Signals for Parameter Estimation in Dynamic Systems- Survey and New Results *IEEE Transactions on Automatic Control, (19)*, 753-768.

Goodwin, G. & Payne. R. (1977). *Dynamic System: Identification Experiment Design and Data Analysis*, Academic Press, USA.

Zarrop, M. (1979). A Chebyshev System Approach to optimal input design *IEEE Transactions on Automatic Control, (*24), 687-698.

Chaloner, K., Larntz, K., (1989). Optimal Bayesian designs applied to logistic regression experiment. *Journal of Statistical Planning and Inference,* (21), 191-208.

Atkinson, A., Chaloner, K., Juritz, J., Herzberg, A., (1993). Optimum experiment design for properties of a compartmental model. *Biometrica* (29), 325-337.

Ljung, L. (1999). System Identification, Theory for the User, *Prentice Hall*, USA.

Antoulas, A. & Anderson, B. (1999). On the choice of inputs in identification for robust control *Automatica,* **(**35), 1009-1031.

Sebastiani, P., Wynn, H. (2000). Maximum entropy sampling and optimal bayesian experimental design. *Journal of Royal Statistical Society: Series B* 62 (1), 145-157.

Hildebrand, R. and Gevers, M. (2003). Identification for Control: Optimal input design with Respect to A worst-Case ν-gap cost function. *SIAM Journal on Control and Optimization,* (41), 1586-1608.

Bombois, X., Scorletti, G., den Hof, P. V. and Gevers, M. (2004). Least costly identification experiment for control. A solution based on a high-order model approximation. *Proc. American Control Conference, Boston.*

Lofberg, J. (2004).YALMIP: A Toolbox for Modeling and Optimization in MATLAB *Proceedings of the CACSD Conference.*

Gevers, M. (2005). Identification for control: From the early achievements to the revival of experiment design. *European Journal of Control,* **(**11), 1-18.

Hjalmarsson, H. (2005). From experiment design to closed-loop control *Automatica,* , (41*)*, 393 – 438.

Rojas, C.,Welsha, J.,Goodwin, G. & Feuer, A. (2007). Robust optimal experiment design for system identification *Automatica,*(43), 993-1008.

Ben-Tal, A., El Ghaoui, L. & Nemirovski, A. (2009). Robust Optimization, *Princeton University Press.*

Boyd, S. & Vandenberghe, L.(2004). Convex Optimization *Cambridge University Press,* New York.

Rivera, D.E.,Lee, H.,Mittelmann, H.D, & Braun, M.W. (2009). Constrained multi-sine input signals for plant-friendly identification of chemical process systems. *Journal of Process Control,* (19), 623-635.

Narasimhan, S. & Rengaswamy, R. (2011). Plant friendly input design: Convex relaxation and quality, *IEEE Transactions on Automatic Control,* (56), 1467-1472.

Narasimhan, S., Srikanth S. A., Sreeram, J., & Rengaswamy, R. (2011). Optimal Plant Friendly Input Design for System Identification. *I&EC Research*, (50), 13045-13055.

Grant, M. and Boyd, S. (2011). CVX: Matlab Software for Disciplined Convex Programming. http: //Stanford.edu/boyd/ cvx.