Response dynamics of a freely oscillating cylinder under the effect of noise

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Abstract

The present work revisits free vibration of a circular cylinder undergoing Vortex Induced Vibrations situations in terms of a two dimensional CFD framework, under the effect of a random input flow. Randomness in the oncoming fluid and its effect on the overall dynamical behaviour of the system is a likely scenario that has not received its due attention in the community. The analytical models, prove to be good enough in capturing the qualitative behaviour of the responses, with appropriate tuning of parameters. The effect of stochastic fluctuations in the oncoming flow is analyzed on the structural amplitude. Noise seems to have major effects on the structural amplitude, the onset and range of lock-in. A stability analysis of the structure for both deterministic and stochastic cases is complete only when the spatial and temporal evolution of the wake pattern is also considered. In this context, a Navier Stokes high fidelity solver is used, to determine the bifurcation behavior of the system for deterministic and stochastic cases. Effects of a few relevant system parameters such as Reynolds number, structural stiffness have been studied to resolve the qualitative and quantitative changes in the response dynamics. The intensity of noise seems to have major effects in determining the quality and quantity of the response.

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1. Introduction

Vortex induced vibrations [VIV] and their study is an important area of research in many practical situations, especially in heat exchangers, riser tubes, marine engineering structures, towing pipes, power transmission lines etc. Flow around circular cylinders stands as the first step towards a proper understanding of bluff body flows as in these applications. In the present study, we make efforts to capture the dynamics of an elastically mounted rigid cylinder undergoing transverse oscillations for high-mass to damping ratio cases. Govardhan \textit{et al.} \cite{1} and Feng \cite{2} presented interesting experimental results for low-mass and high-mass to damping cases respectively. Analytical formulations characterizing VIV are effective in giving an understanding about the physics of the problem and to understand the overall dynamics with lesser computational efforts. Analytical formulations which have marked themselves as phenomenological are those based on wake oscillators in the form of Van der pol or Rayleigh equations such as that

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presented by Bishop and Hassan [3], Hartlen and Curie [4]. Fachinetti et al.[5] model was a major breakthrough since they brought an effective coupling model to characterize VIV due to circular cylinders. It is important to forego the assumptions involved in featuring the flow field in these formulations such as inviscid approximations and use a high fidelity Navier Stokes solver to capture the exact flow and structural dynamics. Singh and Mittal [6] used a stabilized finite element method to study VIV at subcritical Reynolds number. They report self-excited oscillations at Reynolds number as low as 20. Prasanth and Mittal [7]. An intermittent mode of switching is reported during the transition between the two branches of response obtained. Li et al. [8] investigates the nonlinear characteristics of VIV around a circular cylinder. With the help of Poincare maps and phase spaces, they report the bifurcation of lift coefficients and displacements.

Though extensive numerical studies have been made to study VIV phenomenon, all these have been carried out considering deterministic flow environments. But encountering realistic flow situations, the flow might be characterised with noise or randomness. The incoming flow velocity can have fluctuations which can be either be inherent or due to environmental disturbances. Since we are solving the coupled FSI [Fluid Structure Interaction] problem, where the flow and structural responses being coupled affect each other mutually at every time, even a small perturbation to the incoming flow largely affects the structural response. It is important that we take into consideration this randomness and study the effects that a stochastic environment of the flow field brings in to the response. In the present work, we incorporate a noise in the incoming flow at every time step, by modelling it through a uniform distribution.

The organization of the paper as follows: Section 2 provides the governing equations and boundary conditions along with the numerical solver details. The validation of the flow solver considering a fixed and oscillating cylinder along with the grid independence study is done in section 3. Section 4 describes the results and discussions followed by conclusion in Section 5.

2. Problem Formulation and Numerical implementation

2.1. Computational Domain

Numerical simulations are carried out in two dimensions for an infinitely long circular cylinder. The cylinder radius [R] is taken as 1 m. We are investigating a laminar regime and a Reynolds number of 100 has been chosen. The computational domain which has been used is shown in Figure 1. The dimensions of the computational domain are 63R x 50R. The two dimensional mesh used has a total of 66000 cells, discretized using structured grids. An O-type grid has been used around the cylinder which is further extended to the far field by means of structured grids. The meshed domain and a closeup view of the mesh around the cylinder are shown in Figures 2 and 3. The boundary conditions are as follows: zero gradient pressure and uniform velocity at inlet, zero gradient velocity and atmospheric pressure at outlet, slip boundary conditions and zero gradient pressure at the walls, no slip boundary conditions and zero gradient pressure in the cylinder walls. The spatial and temporal discretization schemes used are second order accurate. A maximal Courant number of 1 is set with adaptive time steppings. Pressure Implicit with Splitting of Operator [PISO] algorithm is used for the pressure velocity coupling. Simulations are done in OpenFOAM, a finite volume based CFD solver.
2.2. Governing equations

2.2.1. Structural solver

We consider an infinitely long cylinder oscillating in the transverse direction under the influence of a Uniform flow as shown in Figure 1. The equation of motion of the structure is given by (1)

\[ m\ddot{y} + c\dot{y} + ky = L(t) \]  

In this equation, \( m \), \( c \) and \( k \) are the mass, damping and stiffness of the cylinder and \( L(t) \) is the unsteady lift forces occurring due to the flow which is changing at every time step. The structural equation is integrated using a fourth order Runge-Kutta method with the time step of integration same as that of the flow solver.

2.2.2. Flow solver

The flow equations are the incompressible Navier Stokes equation (2) and (3). An ALE (Arbitrary Langrangian Eulerian) formulation has been used to solve these equations in a moving grid. ALE method offers higher mesh quality near the boundary layer. In this method, mesh on the boundaries and interfaces move along with the structure by which it optimises the shapes of the elements. Due to this motion of mesh, an additional equation of motion (4) is also solved while computing the values of the internal points of the deforming mesh. This equation makes sure that the cell volume changed after each time step is the cell volume changed by the boundary. The mesh deformation method used is the Radial Basis Function [9] due to the superior mesh quality it provides even during large deformations. The governing equations of the flow are as follows:

\[ \nabla \cdot \overrightarrow{u} = 0 \]  

\[ \frac{\partial \overrightarrow{u}}{\partial t} + (\overrightarrow{u} - \overrightarrow{u}^m) \cdot \nabla \overrightarrow{u} = -\frac{\partial p}{\partial \rho} + \nu \nabla^2 \overrightarrow{u} \]  

\[ \frac{\partial \overrightarrow{u}}{\partial t} + \nabla \cdot \overrightarrow{u}^m = 0 \]

Here \( \overrightarrow{u} \) is the velocity of the flow, \( \overrightarrow{u}^m \) is the grid point velocity, \( p \) is the pressure, \( \rho \) is the density of the fluid, \( \nu \) is the dynamic viscosity, \( \Gamma \) is the cell volume.

3. Grid independence study and validation

3.1. Flow around a fixed cylinder

In order to validate the solver, the numerical results are obtained by simulating the flow around a cylinder fixed in a flow field, encountering a uniform flow of 1 m/s perpendicular to the cylinder’s axis. The two dimensional
mesh used has 280 points on the surface of the cylinder. This choice of grid was confirmed after doing a grid independence study, by varying the number of points on the surface of the cylinder. The comparison with two different grid resolutions has been done in Figure 4 where the lift coefficient (Cl) has been plotted with time. Four Reynolds number values were chosen: Re 60, 80, 100, 120. The mean drag coefficient was validated against the values got by Placzek et al.[10] and is shown in Figure 5.

3.2. Flow round a cylinder forced to oscillate in the transverse direction

In order to make sure the accuracy of our moving mesh algorithm, we simulate and validate the flow field of a cylinder which is forced to oscillate in the transverse direction. All the simulations were carried out at a Reynolds number of 100, with maximum amplitude of oscillations set as 0.5R and by varying the forcing frequency. The results, shown in Figure 6, are validated against the numerical results of Anagnostopoulos [11] and Nobari and Nareman [12].

4. Results and Discussions

4.1. Deterministic responses

Before studying the stochastic cases, it is important to first analyse the system for the deterministic case. This is important to gain a physical insight of the VIV system and to identify the various states through which the deterministic system passes. The bifurcation parameter chosen is the structural stiffness, k. The values of the structural parameters are as follows: \( m = 100 \) kg, \( c = 10 \) Ns/m. The structural stiffness is varied on a range from 10 N/m to 100 N/m to
examine the responses and results are presented for three different typical k values indicating three types of qualitatively different states. For the deterministic case, lock-in region has been found to occur between stiffness values of 30 N/m and 70 N/m. As the bifurcation parameter is changed, the system passes through the pre-lock in, lock in and post lock-in regions. The time histories when the system is in these three different regimes are given below in figure 7 corresponding to (a) k=15 (b) k=30 (c) k=80 respectively. In figure 7(a), at k=15 N/m, it is seen that the structure has very low amplitude oscillations indicating that the system is still in the pre-lock in regime. Figure 7(b) shows high amplitude oscillations indicating that the system has reached lock-in. For the above given parameters, it was seen that the lock-in regime extended till k=70 N/m. After this, the oscillations again started becoming low amplitudes indicating the transition to the post lock-in state. Such a post lock-in state has been shown in Figure 7(c) for k=80 N/m.

4.2. Effect of noise on the response dynamics

Now the response of the system is examined under the effect of noise. In the present work, we have modelled noise mathematically through a uniform distribution at every one second. With this formulation, the incoming velocity \( u \) effectively becomes

\[
u = U_m + \sigma \cdot c\] (5)

where \( U_m \) is the mean incoming velocity and \( \sigma \) is the standard deviation or the intensity of the noise, \( c \) is a random value from a uniform distribution which changes at every time step. The values of \( c \) can go between 0 and 1. In the present case, value of \( U_m \) has been chosen as 1 m/s. As in the deterministic case, we gradually vary our bifurcation parameter to examine the role of noise on the system. The evolution of the structural displacement with time for noise intensity \( \sigma = 0.1 \), \( U_m = 1 \) m/s, for two different values of stiffness corresponding to two different states of the system are shown in Figure 8 and Figure 9. Figure 8 corresponds to the structural response at k=5 N/m. As is seen from the figure, the system is having high amplitude oscillations. From our earlier discussions on the deterministic system, we know that lock-in in the deterministic case occurs at k=30 N/m only. For k=5 N/m, the deterministic system exhibits extremely low amplitude oscillations. But, in the stochastic situation, for the same stiffness value of 5 N/m, the structure is exhibiting high amplitude oscillations indicating that the system has already gone to lock-in for this parameter. In other words, we don’t see a pre-lock in region at all for this value of noise intensity. The maximum amplitudes attained in the lock-in regime are also much higher compared to the deterministic cases. With

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Fig. 7. Deterministic responses at (a) k=15 [pre lock-in] (b) k=30 [lock-in] (c) k=80 [post lock-in]

Fig. 8. Structural response under effect of noise at k=5 N/m

Fig. 9. Structural response under effect of noise at k=100 N/m
further increase of stiffness, the system slowly moves out of the lock-in state and exhibit low amplitude oscillations. Such a low amplitude oscillation for $\sigma = 0.1$ is given for $k=100$ N/m in Figure 9.

A similar study has been conducted for $\sigma = 0.01, 0.05$ etc. These have been not reported in the paper now, in order to prevail brevity. In these cases also, we have seen that noise can alter the bifurcation behaviour of the system. It was seen that the bifurcation point at which lock-in happens was shifted forefront with addition of noise. Also, the amplitude of oscillations were higher. As discussed above, with higher values of noise intensities, these effects started dominating more and more and could even completely take off the pre lock-in regime as in the $\sigma = 0.1$ case.

It is evident that the addition of even less than a ten percent noise has brought out noticeable changes in the response output. These changes are reflected in quantitative and qualitative ways. In realistic situations where circular cylinders undergo free vibrations, these facts count substantially in determining the quality and quantity of responses. Lock-in is considered to be dangerous in majority of cases involving fluid structure interactions. Calculation of structural loads and aerodynamic loads would often be under the assumptions of deterministic flow environments. The above discussed results show that fluid structure interaction problems have to be assessed and analysed considering the uncertainties and studies based on just deterministic assumptions are not sufficient.

5. Conclusion

In this study, we observe that stochastic fluctuations play a major role in affecting the response of a freely vibrating structure undergoing fluid structure interactions. We have modelled a fluctuating flow field, adhering to real life conditions, and studied how this affects the quality and quantity of the structural response. It is seen that a slight perturbation to the flow field brings in noticeable changes in the oscillations of the body. Some of these changes include the amplification of amplitudes of the response, advancement of the lock-in regime. This work emphasises on the importance of modelling stochastic uncertainties of fluid structure interaction problems due to the alarming changes they bring in the structural and aerodynamic loads. It has been also observed in a preliminary study that additional new frequencies occur in the response output under the effect of noise. A study on these new frequencies which are reflected in the response and the effect of other types of noise such as Gaussian, are further being investigated by the authors currently.

References