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M. Chidambaram and Vivek Sathe  
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# Relay Autotuning for Identification and Control

M. Chidambaram

*Indian Institute of Technology, Chennai*

Vivek Sathe

*Dr. Babasaheb Ambedkar Technological University, Maharashtra*



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## Preface

To design proportional plus integral (PI)/PID controllers, the ultimate values of the controller gain ( $k_u$ ) and frequency of oscillation ( $\omega_u$ ) should be known. The conventional Ziegler and Nichols continuous cycling method requires a large number of experiments to calculate these values. Åström and Hägglund (1984) suggested the use of ideal relay to generate closed loop oscillations. The ultimate gain and ultimate frequency can be found in a single-shot experiment. However, the method is still approximate because of the use of the principal harmonics approximation. Li et al. (1991) reported that for an open loop, stable first order plus time delay (FOPTD) system, an error of -18% to 27% is obtained in the calculation of  $k_u$ . Yu (1999) suggested a saturation feedback test to get better results for the ultimate gain and frequency. However, Yu (1999) did not report any result for large values of delay-to-time constant ratio.

An SOPTD model can incorporate higher order process dynamics better than an FOPTD model. The controller designed on the basis of the SOPTD model gives a better closed loop response than the one designed on an FOPTD model. It is better to have an SOPTD model with equal time constants since only three parameters are to be identified. Li et al. (1991) showed that the conventional analysis of the relay autotune method for an SOPTD model with equal time constants gives -11% to 27% error in the calculation of  $k_u$ . In identifying the transfer function models for unstable systems, it is necessary to estimate 5 parameters to describe such systems. There is no method available to estimate these parameters from a single relay test. Cascade control system is a multi-loop control scheme commonly used in the chemical process industries. It should be noted that there are two ways of relay tuning cascade control systems, namely, simultaneous tuning and sequential tuning procedures. However, there is a need to consider higher order harmonics in the analysis of relay tuning of such systems to obtain improved controllers settings.

The authors have organized the book in the following manner:

Chapter 1 gives an overview of the reported works on relay tuning of single loop control system, cascade controllers, estimation of model parameters and design of PI/PID controllers. Chapter 2 gives a method of considering higher order harmonics in the analysis of symmetric relay tuning method and analytical solutions for model parameters using a single asymmetrical relay test.

Chapter 3 gives a method of considering higher order harmonics in the analysis of series cascade control systems by the sequential tuning method. The method is also extended to

parallel cascade control systems. Chapter 4 analyzes the simultaneous relay tuning of series and parallel cascade control systems.

Chapter 5 gives a simple method to design PID controllers for a series cascade control system. FOPTD models are assumed for both the inner and outer loop sub-systems. The method is based on matching the coefficients of the corresponding powers of  $s$  and  $s^2$  in the numerator to  $\alpha_1$  and  $\alpha_2$  times that in the denominator of the closed loop transfer function model for a servo problem. The method is first applied to design a proportional ( $P$ ) controller for the inner loop and then to design a proportional plus integral ( $PI$ ) controller for the outer loop. The performance of the proposed controllers is evaluated for FOPTD models of the inner loop and the outer loop process transfer functions. This method is also extended to parallel cascade control systems.

Chapter 6 proposes two methods to improve the accuracy of the saturation relay method for FOPTD systems with a large ratio of time delay to time constant. Chapter 7 proposes a method to identify all the three parameters of a first order plus time delay (FOPTD) model using a single symmetrical relay test.

Chapter 8 gives a method to identify all the three parameters of an unstable FOPTD system using a single symmetric relay feedback test. It also proposes a method by incorporating higher order harmonics to explain the error in the calculation of  $k_u$ . Another method is proposed to estimate all the parameters of an unstable FOPTD system.

In Chapter 9, using a single symmetric relay feedback test, a method is proposed to identify all the three parameters of a stable second order plus time delay (SOPTD) model with equal time constants  $[k_p \exp(-Ds)/(\tau s+1)^2]$ . It provides a method to improve the accuracy in the  $k_u$  calculation by incorporating the higher order harmonics.

In Chapter 10, using a single asymmetric relay feedback test, a method is proposed to identify all the four parameters of a stable second order plus time delay (SOPTD) model  $k_p \exp(-Ds)/[(\tau_1 s+1)(\tau_2 s+1)]$ . In Chapter 11, a method is proposed to identify all the 5 parameters of an SOPTD system with a zero,  $G(s) = k_p (\tau_1 s+1) \exp(-Ds)/[(\tau_2 s+1)(\tau_3 s-1)]$ , using an asymmetric relay test.

In Chapter 12, the asymmetrical relay tuning method is extended to multivariable FOPTD systems. In Chapter 13, the asymmetrical relay tuning method is extended to multivariable SOPTD systems. Chapter 14 proposes a simple method of designing centralized multivariable PID controllers for multivariable systems. In Chapter 15, the use of detuned Ziegler–Nichols PID settings as initial guess values is suggested for the design of multivariable controllers by an optimization method using the genetic algorithm to reduce the number of iterations significantly. Chapter 16 summarizes the results and conclusions of the work.

There are three appendices.

Appendix A gives the *Simulink* block diagrams used in the present work.

Appendix B gives the *Matlab* programs used in Chapter 3, Chapter 5 and Chapter 6.

Appendix C gives an improved relay tuning method for integrating plus FOPTD systems.