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Protocol for measuring permeability and form coefficient of porous media

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The experimental determination of permeability K and form coefficient C, two hydraulic properties necessary to characterize a porous medium, is beset with undesired secondary effects, which augment the uncertainties in their determination. This study sets forth a new measuring protocol, with derived model equations, to guide the design of experiments for accurate determination of K and C, using Darcy's law of flow through a porous medium and Newton's law of flow around a bluff body as constitutive equations defining K and C, respectively. The analysis shows that the model equation for measuring C requires the separation between the viscous-drag effect imposed by the porous medium and the viscous effect of the boundary walls on the measured pressure drop when defining K. Furthermore, the model equations suggest large aspect ratio channels and laminar flow with maximum Re as the best choice for measuring K and C (contrary to prevailing belief). The protocol is applicable to either individual or concurrent determination of K and C. *© 2005 American Institute of Physics*. [DOI: 10.1063/1.1979307]

Permeability $K(\mathrm{m}^2)$ is one of the two hydraulic properties necessary to characterize the flow of a fluid through a porous medium, the other one being the form coefficient $C(\mathrm{m}^{-1})$. During the years since their original propositions, by Darcy[1] and Dupuit,[2] respectively, the two equations defining K and C have progressively lost their distinction of being constitutive equations. Instead, they are now commonly presented as momentum balance equations[3,4] for predicting the fluid velocity in porous medium flows. A constitutive equation is understood here as an equation necessary to reduce the number of unknowns in a balance equation by defining a material property.

To determine the property defined by a constitutive equation, such as K and C, one has to design specific experiments to isolate the effect of the particular property from other effects. The resulting experiment allows considerable simplifications to the balance equation. Sometimes the simplification is such that the resulting balance equation becomes identical to the constitutive equation. This aspect induces the mischaracterization of the constitutive equations as the balance (of momentum) equations.

This study aims at clarifying the limitations of the original constitutive equations for K and C in light of the practical aspects of their experimental determination. We begin by analyzing the determination of permeability.

When presented as originally intended, i.e., as a constitutive equation,[1] the permeability equation[5]

$$K = \frac{\mu}{(\Delta P_\nu / L)} u \tag{1}$$

requires the measurement of the cross-section-averaged fluid speed $u(\mathrm{m\,s^{-1}})$, and of the pressure drop $\Delta P_\nu(\mathrm{Pa})$, in the flow of a Newtonian fluid, along a length $L(\mathrm{m})$ of the channel occupied by a porous medium.

Let us now recall the differential form of the volume-averaged momentum equation for the flow of a Newtonian fluid through a porous medium, with uniform, isotropic, and constant properties,[5] namely,

$$\frac{\rho}{\phi}\left[\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\phi}(\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla p + \mu_{\mathrm{eff}}\nabla^2\mathbf{v} + \frac{\mu}{K}\mathbf{v} + \rho C|\mathbf{v}|\mathbf{v}, \tag{2}$$

where $\rho(\mathrm{kg\,m^{-3}})$ and $\mu_{\mathrm{eff}}(\mathrm{kg\,m^{-1}\,s^{-1}})$ are the fluid density and effective dynamic viscosity, ϕ is the porosity of the porous medium (defined as the ratio of fluid-occupied volume to the total volume) necessary to correct the increased speed of the fluid through the pores as compared to the clear (of porous medium) channel case, $\mathbf{v}(\mathrm{m\,s^{-1}})$ is the local fluid velocity, and $p(\mathrm{Pa})$ is the local pressure. The terms to the left side of the equal sign of Eq. (2) represent the fluid acceleration (local and convective). To the right of the equal sign we have pressure gradient, viscous diffusion, viscous drag, and form drag, respectively.

For steady, fully developed (unidirectional) flow and negligible viscous-diffusion and form-drag effects, the momentum balance equation [Eq. (2)] reduces to

$$\frac{\Delta P}{L} = \frac{\mu}{K} u, \tag{3}$$

which is almost identical to Eq. (1). The difference between the constitutive equation defining K [Eq. (1)] and the momentum balance equation [Eq. (3)] is on the pressure drop. The single value K originating from the constitutive equation [Eq. (1)] represents a material property of the porous medium. Hence, the implicit assumption behind Eq. (1) is that, once determined, the value of K would not change with

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changes in the porous medium length, channel geometry, or flow rate, but only with changes in the internal structure of the porous medium. To satisfy this assumption, the pressure drop ΔP_ν of Eq. (1) must be a measure of the pressure drop caused by the viscous drag only, induced by the internal structure of the solid porous matrix. The momentum balance equation version of Darcy's law [Eq. (3)] need not have this restriction. Consequently, the applicability of Eq. (1) for determining K is more restricted than that of the momentum balance equation [Eq. (3)].

To guarantee that only the viscous-drag effect is accounted for in using Eq. (1) (i.e., for the measured pressure drop ΔP_m to be identical to ΔP_ν), the testing channel must be straight and of uniform cross section. Moreover, the effect imposed on the fluid pressure drop by the bounding walls of the channel would have to be made negligible as well (boundary viscous-diffusion effects). Obviously, this last requirement might be difficult to satisfy in practice. Two alternatives exist to resolve this dilemma. The first alternative is to relax the requirement for ΔP being affected only by the solid matrix viscous-drag effect. Accordingly, Eq. (1) would be rewritten as

$$K = \frac{\mu}{(\Delta P_{c\nu}/L)} u, \tag{4}$$

where $\Delta P_{c\nu}$ is the measured pressure drop, which includes the viscous-drag (ΔP_ν) and the channel boundary (ΔP_c) contributions. Two important consequences of this alternative are as follows: (a) K determined from Eq. (4) would be dependent on the channel geometry, i.e., K is no longer a material property of the porous medium alone, and (b) Eq. (4) would yield a nonzero permeability value even in the absence of a solid porous matrix inside the flow channel.

Exploring this alternative a little further, we can consider the pressure-drop versus average fluid-speed relationship for a clear channel flow, for which $\Delta P_m = \Delta P_c$. In general, an analytical expression for the channel pressure-drop ΔP_c versus fluid-speed relationship for steady, fully developed laminar flow in a straight channel with a uniform cross-section can be written as

$$\frac{\Delta P_c}{L} = \frac{\mu}{b} u, \tag{5}$$

where b is a coefficient dependent only on the geometry of the channel. Comparing Eqs. (4) and (5), for $\Delta P_{c\nu} = \Delta P_c$, we conclude that b plays the role of permeability for a clear channel. In general,

$$b = \frac{D_h^2}{2\text{Po}}, \tag{6}$$

where Po is the Poiseuille number, $\text{Po} = 2\tau_w D_h / \mu u$, $\tau_w(\text{N m}^{-2})$ is the wall shear stress, and $D_h(\text{m})$ is the hydraulic diameter of the channel. For a parallel-plate channel, for instance, Po=24 and $D_h = 2h$, so

$$b = \frac{D_h^2}{48} = \frac{h^2}{12}, \tag{7}$$

where $h(\text{m})$ is the distance between the plates. For a circular tube channel, of diameter $d(\text{m})$, Po=16 and $D_h = d$, so

$$b = \frac{D_h^2}{32} = \frac{d^2}{32}. \tag{8}$$

For a square cross-section channel, $b = D_h^2/28.46$, and for an equilateral triangular cross section, $b = D_h^2/26.66$ (see Cengel and Cimbala[6] for a detailed corroboration). Consequently, there is a relationship between cross-sectional geometry and b, so we surmise that there exists a geometry which leads to a maximum b value, or, equivalently, to a maximum permeability among all possible wall-bounded channel flows.

The second alternative to resolve the channel boundary viscous effect dilemma is to model the contributions by the boundary viscous-diffusion and viscous-drag effects to the total pressure drop as additive. In this case, the pressure-drop contribution by the viscous-drag effect alone can be estimated as

$$\frac{\Delta P_\nu}{L} \equiv \frac{\Delta P_{c\nu}}{L} - \frac{\Delta P_c}{L}. \tag{9}$$

Following this alternative, a different conclusion is reached. In general the channel boundary effect on $\Delta P_{c\nu}$ is not known, but conservatively one can use the smallest possible boundary effect, which is the pressure drop of steady, fully developed laminar flow as indicated previously. So, using Eqs. (1) and (5), Eq. (9) can be rewritten as

$$K = \left(\frac{\Delta P_{c\nu}}{L\mu u} - \frac{2\text{Po}}{\phi D_h^2} \right)^{-1}. \tag{10}$$

As defined in Eq. (10), K is no longer limited to a maximum value, but instead it tends to infinity when the channel tends to a clear channel because, in this case, $\phi \to 1$ and $\Delta P_{c\nu}/L \to \mu u 2\text{Po}/D_h^2$, according to Eqs. (4)–(6).

Consider now the K definition given by Eq. (4). Setting $K = b$ and using Eq. (6), the Darcy number, defined as $\text{Da} = K/D_h^2$, becomes $\text{Da} = 1/(2\text{Po})$. This Darcy number value, dependent on the channel cross-section geometry, represents the maximum possible Darcy number for the wall-bounded channel flow (without porous medium). However, when defined by Eq. (10), the value of K (and by consequence that of Da) has no upper bound. So, any parametric (numerical) study involving wall-bounded channel flow through a porous medium using $\text{Da} > 1/(2\text{Po})$ is valid only provided K is defined by, and measured according to, Eq. (10). The striking difference in the maximum attainable permeability values when K is defined by Eq. (4) and (10) highlights the importance of defining K rigorously.

The second hydraulic parameter characterizing a porous medium is the form coefficient C. Although C can be related to K in some specific cases,[4,5] the form coefficient in general should be regarded as an individual coefficient dependent on the form (shape) of the porous obstruction.[7,8] In line with the model proposed by Dupuit,[2]

$$C = \frac{(\Delta P_f/L)}{\rho u^2}, \tag{11}$$

where ΔP_f is the pressure drop due to form drag alone. Observe that Eq. (11) can be obtained from Eq. (2) when acceleration, viscous-drag, or boundary viscous effects are negligible in uniform flow across a porous medium of length L. As explained by Lage,[5] model Eq. (11) is better interpreted as a simple extension to flow in a porous medium of Newton's law of fluid flow around a bluff body.

Similar to Eq. (1), Eq. (11) is also a constitutive equation, now defining C. In practice, the determination of C also involves the flow of the fluid through a walled channel bounding the porous medium and the measurement of ΔP_m along this channel at a certain fluid speed u. The applicability of Eq. (11) for the accurate determination of C requires ΔP_m to match ΔP_f. Again, and according to Eq. (2), the testing channel must be straight and of uniform cross section so as to hold acceleration effects negligible. Likewise, the viscous-diffusion effect imposed by the bounding walls of the channel and the viscous-drag effect of the porous medium would have to be made negligible as well.

Analogous to the permeability issue, the requirements of negligible viscous-drag and channel wall effects might be very difficult to satisfy in practice. The option of incorporating these effects on the form coefficient itself, similarly to what was considered in the case of K, is problematic because the nature of the pressure-drop dependency on the fluid speed for viscous drag and the overall viscous-diffusion channel wall effect differs from the nature of the fluid-speed dependency for form drag (linear versus quadratic).

Hence, the only alternative in the present case is to model the measured pressure drop ΔP_m as the sum of a component due to the form drag, ΔP_f, and a component due to the other viscous effects. Let us consider first the case in which the other viscous effects (channel boundary viscous effect and viscous-drag effect) are lumped together and represented as $\Delta P_{c\nu}$, or equivalently, assuming K is defined by Eq. (4). In this case,

$$\frac{\Delta P_f}{L} = \frac{\Delta P_m}{L} - \frac{\Delta P_{c\nu}}{L}. \tag{12}$$

Substituting Eqs. (4) and (11) into Eq. (12) and rearranging the terms, we obtain

$$C = \frac{1}{\rho u^2}\left(\frac{\Delta P_m}{L} - \frac{\Delta P_{c\nu}}{L}\right) = \frac{1}{\rho u^2}\left(\frac{\Delta P_m}{L} - \frac{1}{K}\mu u\right). \tag{13}$$

As defined by Eq. (13), the form coefficient C would be zero for laminar fully developed flow through a straight, uniform cross-section, clear channel, for which, from Eq. (5), $\Delta P_m/L = \Delta P_c/L = \mu u/b = \mu u/K$. This result is physically consistent with the expectation of a straight channel imposing zero form drag.

A difficulty hindering the use of Eq. (13) is the possibility of turbulence in the channel flow. Recall that turbulence can become important in the porous channel flow before or after the form-drag effect becomes relevant (see discussion by Lage[5] and experiments by Wilson, Narasimhan, and Venkateshan[9]). If one attempts to use Eq. (4) for determining K using measurements for a turbulent channel flow, the result will be inconsistent because the turbulence effect on ΔP_m is likely to be quadratic in the fluid speed, not linear as assumed in Eq. (4) (where $\Delta P_m \equiv \Delta P_{c\nu}$). Therefore, to allow for the possibility of having channel wall effects caused by turbulence setting in prior to the form-drag effects, we are compelled to use Eq. (10) instead of Eq. (4). In this case, Eq. (12) becomes

$$\frac{\Delta P_f}{L} = \frac{\Delta P_m}{L} - \left(\frac{\Delta P_c}{L} + \frac{\Delta P_\nu}{L}\right). \tag{14}$$

In consideration of possible turbulence effect, it is more convenient to represent the channel wall viscous effect in terms of Darcy's friction factor f, namely,

$$\frac{\Delta P_c}{L} = \frac{1}{2D_h}\rho u^2 f. \tag{15}$$

In this case, an equivalent equation to Eq. (10) for determining K when form drag is negligible would be

$$K = \left(\frac{\Delta P_m}{L\mu u} - \frac{1}{2\phi^2 D_h \mu}\rho u f\right)^{-1}. \tag{16}$$

Combining Eqs. (1), (11), (14), and (15),

$$C = \frac{\Delta P_m/L}{\rho u^2} - \frac{f}{2\phi^2 D_h} - \left(\frac{1}{K}\right)\frac{\mu}{\rho u}. \tag{17}$$

Equation (17) in dimensionless form becomes

$$\mathrm{Du} = \zeta\ \mathrm{Eu} - \frac{f}{2\phi^2} - \frac{1}{\mathrm{Da\ Re}}, \tag{18}$$

where Dupuit (Du), aspect-ratio (ζ), Euler (Eu), Darcy (Da), and Reynolds (Re) numbers are defined, respectively, as

$$\mathrm{Du} = D_h C, \quad \zeta = \frac{D_h}{L}, \quad \mathrm{Eu} = \frac{\Delta P_m}{\rho u^2}, \quad \mathrm{Da} = \frac{K}{D_h^2}, \quad \mathrm{Re} = \frac{\rho D_h}{\mu}u. \tag{19}$$

Similarly, Eq. (16) in nondimensional form becomes

$$\mathrm{Da} = \left(\zeta\ \mathrm{Eu\ Re} - \mathrm{Re}\frac{f}{2\phi^2}\right)^{-1}. \tag{20}$$

Equations (16) and (17), or the equivalent Eqs. (18) and (20), are model equations for K and C in terms of directly measurable quantities. These equations are in line with the original constitutive equations for K and C, respectively, Eqs. (1) and (11), for subtracting from the measured pressure drop the other flow effects known to exist but not related to the property of interest.

The resulting models for K and C are also consistent with known results for particular flow configurations. For instance, the limit of no porous medium in the channel (i.e., clear channel) with $\phi \to 1$ would lead to $\Delta P_m/L \to \rho u^2 f/(2D_h)$, which, from the definitions listed in Eq. (19), can be written as $(\zeta\,\mathrm{Eu}) \to f/2$ or as $(\zeta\,\mathrm{Eu\,Re}) \to \mathrm{Re}\,f/2$. Consequently, from Eq. (20) $\mathrm{Da} \to \infty$ and hence, $\mathrm{Du} \to 0$ from Eq. (18). These results are consistent for a clear channel flow configuration. Notice also that the constitutive equation defining the friction factor for a wall-bounded clear channel flow is recovered from Eq. (18) when no porous medium is present in the channel. This result is only possible when the representation of K excludes boundary wall effects [as in Eq. (16)], in line with Eq. (18).

In the limit of a very impermeable porous medium, we have $K \to 0$ and, consequently, $\mathrm{Da} \to 0$. Observe that the term $(f/2\phi^2)$ in Eq. (18) remains finite, as long as Re is finite. In this case, for a finite Du, the term $1/(\mathrm{Da\,Re})$ can only be balanced by the $(\zeta\,\mathrm{Eu})$ term. Consequently, Eq. (18) approaches the limiting behavior

$$\mathrm{Da} \to \frac{1}{\zeta \mathrm{Eu\,Re}}. \tag{21}$$

Equation (21), with $\Delta P_m \to \Delta P_\nu$, tends to a result identical to Eq. (1). That is, Darcy's constitutive equation for K is recovered, independent of the values f, Re, and Du might have when the porous medium permeability tends to zero, as one would expect.

Finally, observe that Eq. (20) results from setting $\mathrm{Du} = 0$ in Eq. (18), a consequence of assuming negligible form drag when deriving Eq. (20). It is important to consider when this assumption is valid. Physically we know the viscous-drag and form-drag concepts are useful in modeling a single drag phenomenon. In reality, these two drags are always present in any practical (finite Re) flow. Therefore, the best we can do is to compare the two. Using Eq. (18), we find that the viscous-drag and form-drag effects are comparable when $\mathrm{Du} \sim 1/(\mathrm{Re\,Da})$, which in dimensional form translates into $(\rho u C K/\mu) \sim 1$. This criterion for transition from viscous-drag-dominated to form-drag-dominated regime is identical to the criterion suggested by Lage.[5]

In practice, K and C should be measured together, following specific recommendations.[8] When designing an experiment to measure K and C, the experimentalist should seek to minimize the viscous effect of the channel wall boundary on the measured pressure drop, from Eq. (18)

$$\zeta > \frac{f}{2\phi^2 \mathrm{Eu}}. \tag{22}$$

Clearly from Eq. (22), having small f is beneficial. Hence, maximizing Re but maintaining the flow laminar seems to be the best approach to determine K and C with better accuracy. In cases in which K is already known, C may be determined using Eq. (18). For accuracy, especially when we anticipate C to be small, it is optimal for the bounding channel wall and porous medium viscous-drag effects to be kept minimal, which, according to Eq. (18), requires

$$\zeta > \frac{f}{2\phi^2\,\mathrm{Eu}} + \frac{1}{\mathrm{Eu\,Da\,Re}}. \tag{23}$$

Observe that Eq. (23), in comparison to Eq. (22), has an additional term dependent on $(1/\mathrm{Re})$. There might be situations under the laminar flow requirement for which Eq. (22) demands particular values of Eu, which, combined to particular values of Da, make it difficult to satisfy Eq. (23). In this case, it might be necessary to increase Re even further to neutralize the effect of the last term of Eq. (23). Aiding this requirement, it is worth recalling that for flow in regular channels f remains constant under turbulent flow (assuming a rough wall).

To summarize, the measurement of permeability K and form coefficient C, the two defining hydraulic properties of a porous medium, are usually affected by secondary (acceleration and viscous diffusion) effects. This note treats Darcy's law of flow through a porous medium and Newton's law of flow around a bluff body as constitutive equations defining K and C, respectively. A study of the behavior of these constitutive equations [Eqs. (1) and (11)] leads to more general model equations involving relevant dimensionless numbers, i.e., Eqs. (18) and (20). These model equations are shown to yield consistent results when limiting situations with known results are considered.

Finally, the resulting equations provide firm guidelines—minimizing secondary effects—in the design of experiments for accurate determination of K and C. If a choice is made to determine K and C separately, then use of Eq. (20), along with Eq. (22) and maximizing Re while maintaining laminar flow to minimize f, is proposed for the accurate determination of permeability K for any porous medium. With a known K, the accurate determination of C can be done using Eq. (18), together with Eq. (23), and if necessary with higher Re (since f is constant for a fully turbulent channel flow). If K and C are to be determined concurrently, via interpolation of experimental data as recommended by Antohe, Lage, Price, and Weber,[8] then Eq. (18) is suggested. Because the criterion requires values for K and C, to be determined by the interpolation of the experimental data, an iterative procedure would have to be followed in this case.

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