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Prediction of mechanical response of geomaterials using an advanced elasto-plastic constitutive model

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Abstract

Phenomenological models using plasticity theory are constantly evolving with the increased computational advancements. This evolution is inevitable especially for modeling heterogeneous, anisotropic materials systems like granular or cemented granular materials as newer aspects of their physics come to light. In this study, we present a brief compendium of the evolution of constitutive models for granular materials. The essence of this research study lies in selecting an appropriate advanced third generation elasto-plastic constitutive model and checking its efficacy in predicting the various aspects of granular and cemented granular response. The Lade's isotropic single hardening elasto-plastic constitutive model is selected for this study. The model description is laid out for granular system and the modifications that are made to incorporate the effect of cohesion are also clearly established. Further, the material/model parameters are obtained from appropriate experimental procedures and a comparison between the single point integrated model predictions and experimental results are made.

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Keywords: cemented granular materials; ductile-brittle transition; constitutive modelling

1. Introduction and history of constitutive models for soils

The goal of providing a competent infrastructure requires an extensive understanding of underlying components. With increasing demand on space to build structures, the number of high rise buildings are increasing day by day. Due to these structures a great amount of load is transferred to the soil through substructure which introduces a possibility of failure of soil. Therefore a complete understanding of the mechanics of soil, structure, and its interaction is inevitable. With the advent of high speed computing it is possible to ensure safety and serviceability requirement of a structure by analyzing and designing it through numerical modeling. These numerical models should be validated, using a laboratory experiment or *in-situ* test results, before using them for analysis and design purpose.

Soil is a complex material (non-homogeneous, anisotropic, multi-phase) to predict its response mechanically due to varied composition (unknown structure, non uniform grain shape and size), loading history, drainage condition, etc. A satisfactory design of geotechnical structure should take care of safety and serviceability criteria. Safety requirement is related to failure or bearing capacity while serviceability requirement is related to settlement (mostly elastic). Prior to

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advent of high speed computing tools, settlement and bearing capacity were obtained using analytical solutions using linear elasticity and limit equilibrium or limit analysis, respectively. Hooke's law, with material parameters - elastic modulus (E) and Poison's ratio (ν), was used to calculate settlement assuming soil as linear elastic and isotropic. Limit equilibrium methods assumed soil as perfectly plastic with failure identified with Mohr-Coulomb criteria. Such simplifying assumptions often fails to capture the important characteristics of the soil behavior. The next stage of advancement came through utilizing a non-linear elastic hyperbolic model [1–3] with two parameters. Even though such a non-linear model was able to capture the stress strain relations for specific loading paths in small strain domain, it was incapable of predicting the real soil behavior under general conditions (large strain, varied stress paths, drainage conditions, cyclic loading, etc). This hyperbolic model had no strong theoretical basis like linear elastic isotropic model. These two aspects of constitutive models (elastic and plastic) were clubbed into elastic perfectly plastic models with elastic stress-strain relation, failure criteria, associated flow rule. In elastic-perfectly plastic models, history effect as observed in compression was not taken into account. Therefore it failed to differentiate between loading, unloading and reloading response. With associated flow rule, model predicted excessive dilation during plastic shearing. Additionally, model predicted infinite elastic volumetric strain under isotropic compression since there was no cap along hydrostatic axis.

To overcome drawbacks of elastic-perfectly plastic models, [4] introduced a movable cap along with fixed yied cone, the movement of cap was due to the volumetric plastic strain hardening of the cap. These models were called cone-cap models which assumed associated flow rule for the yield cap. The models were capable of capturing volumetric (ϵ_v^p) and deviatoric (ϵ_q^p) plastic strains. Perhaps one of the important developments in soil constitutive models came in 1958 when [5] introduced a framework called "critical state soil mechanics". A modified cam clay model given by [6] based on these critical state concepts assuming an associated flow rule and isotropic hardening/ softening has become one of the most frequently used constitutive models in prediction and design. The important feature that distinguishes this model from other plasticity models is the dependency on history of the material which evolves as a function of accumulated plastic strains and is capable of simulating both consolidation and shearing of soils. These models were able to reproduce a number of facets of real soil behavior including stress-strain response, volumetric or pore pressure behaviour under drained and undrained conditions. However predictions of super-critical region, large deformations and cyclic response were not captured accurately. To refine this model further [7] introduced Hvorslev's failure surface in place of Cam clay surface in super-critical region. This generation models used associative plasticity which satisfies stability and uniqueness postulates but predicts excessive dilation.

The next generation of constitutive models were marked with the introduction of true triaxial and hollow cylinder experiments. These apparatus were equipped to provide a mapping of yield surface, failure surface along with plastic strain increments and its direction with stress increments. Results from these experiments enabled to introduce combined hardening of yield cap and cone with ϵ_v^p and ϵ_q^p [8–12] whereas previous generation models only considered hardening of yield cap with ϵ_v^p along with a fixed yield cone [4,7]. These hardening concepts were extended from combined isotropic hardening to kinematic hardening. The models of this generation incorporates non-associated flow rule by introducing plastic potential function to avoid excessive dilatancy. The isotropic hardening models have the capability to predict the material response under the monotonic load history with irreversible plastic deformation.

The material response is elastic as predicted by previously discussed models with stress state lying within the current yield surface, which implies models inability to predict the response under radial loading or cyclic loading history for soils. Significant amount of plastic deformation can be observed in case of cyclic loading even under elastic limits (of monotonic stress history) which causes the evolution of center along with changes in the shape and size of the yield surface. Such a response can be modeled using anisotropic hardening i.e. mix hardening models. To incorporate this, in the recent past the ideas of bounding surface and multi-surface plasticity were introduced [13,14]. The yield surface of previous models is identified as bounding surface evolving with the internal variable according to isotropic hardening rules. These models allowed irreversible deformation inside the bounding surface to simulate the response of soils under cyclic loading. To do so, an inner yield surface was conceived inside bounding surface which evolves with the mix hardening rule. This inner yield surface demarcates the region of elastic and plastic deformations (of slightly lesser magnitude in comparison to plastic deformations due to bounding surface). To evaluate the model response a translation rule and interpolation rules are employed for yield surface and hardening variables, respectively. Further development in this field came with the introduction of bubble models [15]. In regular loading unloading experiments it can be observed that the point of unload does not match with the point of onset of plastic deformation during reloading. The bubble model tries to address this issue. To this end it has been realised that addressing one issue affects the other and complexity of the model also increases.

The above discussed models belong to elasto-plastic framework, in which, underlying components are obtained from curve fitting or phenomenological in nature. Several researchers have tried other frameworks such as rational continuum mechanics and thermodynamics, etc. The models from these belongs to hypo-plastic models [16,17], endochronic theory [18], multi-laminate [19], disturbed state model [20] and breakage mechanics models [21].

In this study we examine one of the advanced elasto-plastic constitutive models, Lade's model [12,22,23] to predict mechanical response of weakly cemented granular material. Model was established phenomenologically for the frictional granular materials (such as sand) and extended later to incorporate effect of cohesion or bond between the cohesionless grains. This model is equipped with failure criteria, yield criteria, non-associated flow rule and hardening/softening rule. Model has good prediction ability with purely frictional granular materials [22]. The model surfaces are translated in stress space to adapt bond strength between the grains. We provide a brief description of model components and model material used along with experiments performed to calibrate and validate the model. A detail discussion on the performance of model for predicting the response of cemented granular material is provided.

2. Lade's constitutive model

A single hardening elasto-plastic constitutive model given by [12,22,24,25] is utilized in this study. Components of the model are described below.

2.1. Elastic stress-strain relation

A non-linear elastic stress-strain relation is employed in this model. Structure of elastic stress strain relation is similar to linear elastic isotropic solid

$$\boldsymbol{d\sigma} = \mathbb{C}^e\, \boldsymbol{d\epsilon} \tag{1}$$

Where $\mathbb{C}^e$ is fourth order elasticity tensor. The young modulus as derived by Lade and Nelson (1988) is given below

$$E(I_1, J_2) = M\, P_a \left[\left(\frac{I_1}{P_a} \right)^2 + 6 \frac{(1+\nu)}{(1-2\,\nu)}\, \frac{J_2}{P_a} \right]^{\lambda} \tag{2}$$

Where I_1 is the first invariant of stress tensor and J_2 is the second invariant of deviatoric stress tensor. P_a is the atmospheric pressure and ν is the Poisson's ratio. Parameters M and λ are elastic model parameters.

2.2. Failure criteria

In this criteria, peak stress state is used as failure point which is also used to differentiate between hardening and softening. The locus of these points (failure criteria $F(I_1, I_3) = 0$) is given below

$$F(\boldsymbol{\sigma}) = f_n(I_1, I_3) - \eta = \left(\frac{{I_1}^3}{I_3} - 27 \right) \left(\frac{I_1}{P_a} \right)^m = 0 \tag{3}$$

Where I_3 is the third invariant of stress tensor and m, η are the failure parameters which controls geometry of the failure surface.

2.3. Flow rule

Flow used in this model is non-associative with the plastic potential function given by

$$g_p(I_1, I_2, I_3) = \left(\psi_1 \frac{{I_1}^3}{I_3} - \frac{{I_1}^2}{I_2} + \psi_2 \right) \left(\frac{I_1}{P_a} \right)^{\mu} \tag{4}$$

Where ψ_1, ψ_2, and μ are the plastic potential parameters.

2.4. Yield criteria and work hardening/softening function

Yield criteria is used to address hardening and softening with state variable as plastic work (W_p).

$$f(\boldsymbol{\sigma}, W_p) = f_1(\boldsymbol{\sigma}) - f_2(W_p) = \left(\psi_1 \frac{I_1{}^3}{I_3} - \frac{I_1{}^2}{I_2}\right)\left(\frac{I_1}{P_a}\right)^h \exp(q) - f_2(W_p) \tag{5}$$

Where $q = \frac{\alpha S}{1-(1-\alpha) S}$, $q \in (0, 1)$ and S is the stress level ($S = \dfrac{f_n}{\eta}$). h, α are the yield parameters.
The function $f_2(W_p)$ is defined as follows

$$f_2(W_p) = \left(\frac{W_p}{D\,P_a}\right)^{\dfrac{1}{\rho}} \qquad \text{hardening regime} \tag{6}$$

$$f_2(W_p) = A\,\exp\left(-B\,\frac{W_p}{P_a}\right) \qquad \text{softening regime} \tag{7}$$

Where $D = \frac{C}{(27\psi_1+3)^{\rho}}$, $\rho = \dfrac{p}{h}$ and C, p are hardening parameters.

The model was originally coined for cohesionless soils which was modified later to accommodate cohesion between the grains. To allow the cohesion, original stress space is translated by bond strength ($\sigma_t = a\,P_a$) along the hydrostatic axis since cohesion will act as extra confinement. This exercise is fairly common for models of elasto-plastic framework where cohesion between the grains is treated as extra confinement. Current study focuses on critically assessing the efficacy of such a treatment.

3. Model material and experimental details

Natural soils are susceptible to additional cohesion due to moisture, silicates, carbonates and presence of organic matter. The cohesion imparted by these can change the mechanical behaviour of cohesionless soils significantly. Many a times to provide additional strength and stiffness, cohesion is artificially added to cohesionless grains. The effect of this cohesion is studied experimentally by choosing an appropriate model material whose characteristics are known a priori and a uniformity can be maintained among the experiments performed throughout the study. For the current study, a weakly cemented sand with angular sand and ordinary Portland cement (53 grade) is artificially reconstituted in the laboratory. Mean grain size of the angular sand (specific gravity = 2.65) is 0.45 mm with optimum moisture content and maximum dry density of 18% and 1.6 g/cc, respectively. A hollow cylinder apparatus was employed to perform tests on specimen with 200 mm height, 20 mm thickness and 100 mm outer diameter. This apparatus is capable of performing test along different radial stress path unlike triaxial apparatus which can only perform triaxial compression and extension. In this study, isotropic compression test, unload-reload test, triaxial compression tests are performed to calibrate the Lade's constitutive model. Then model is validated using test performed at constant mean effective stress of 300 kPa and with varying intermediate principal stress ratio ($b = \frac{\sigma_2-\sigma_3}{\sigma_1-\sigma_3}$). Table 1 presents the model parameter obtained by calibration exercise. An extensive description of extraction of material parameter is given in [22,24,25].

Table 1. Model parameters determined from triaxial compression test

Elastic parameters			Failure parameters			Plastic potential			Hardening		Yield	
ν	M	λ	a	m	η	ψ_1	ψ_2	μ	C	p	h	α
0.230	456.886	0.265	1.125	0.105	27.92	0.027	-3.619	2.552	0.000352	1.6	1.0562	0.065

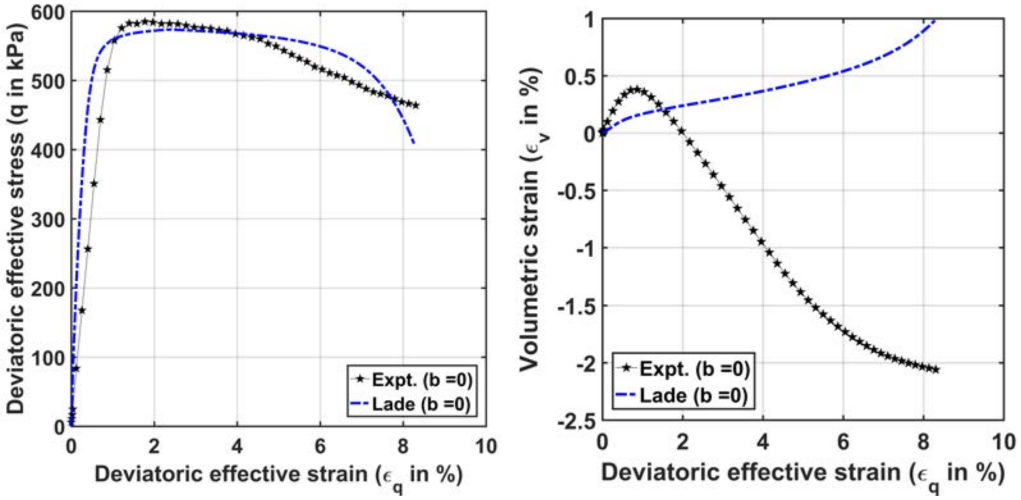

Fig. 1: a: Comparison of predicted and experimentally obtained response of stress strain behaviour at $b = 0$ and mean effective stress (p') of 300 kPa b: Comparison of predicted and experimentally obtained response of volumetric behaviour at $b = 0$ and mean effective stress (p') of 300 kPa

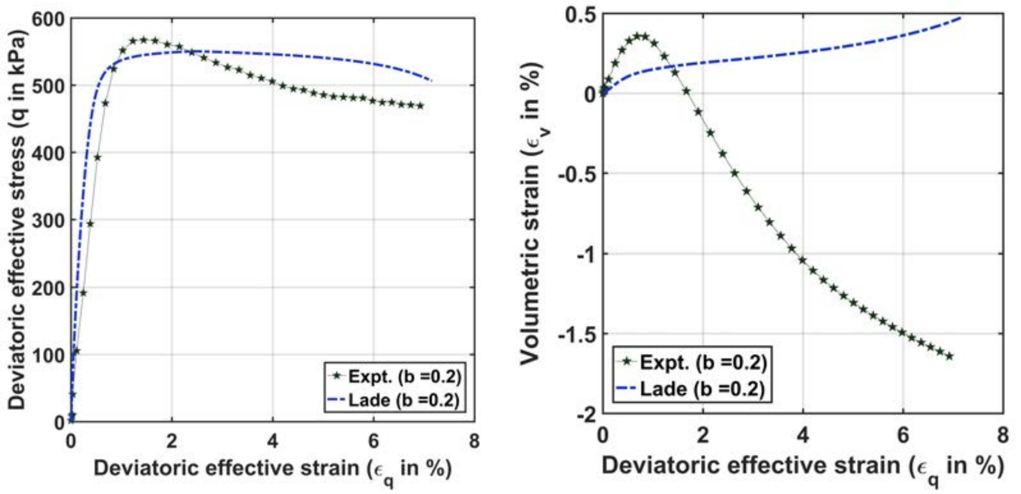

Fig. 2: a: Comparison of predicted and experimentally obtained response of stress strain behaviour at $b = 0.2$ and mean effective stress (p') of 300 kPa b: Comparison of predicted and experimentally obtained response of volumetric behaviour at b = 0.2 and mean effective stress (p') of 300 kPa

4. Result and discussion

Current study focuses on constitutive modeling of weakly cemented granular materials and its prediction ability. In the calibration exercise, we have utilised experiments on the triaxial plane ($\sigma_2 = \sigma_3$) for extraction of model parameters. Validation of the model is performed by comparison of experimentally obtained results on octahedral plane with predicted response. The results from this comparison exercise indicates the accuracy of failure surface, plastic potential function, yield surface and its evolution in stress space. Results are plotted between deviatoric effective strain (ϵ_q *in* %) vs deviatoric effective stress (q *in kPa*), deviatoric effective strain vs volumetric strain (ϵ_v *in* %) as shown in fig 1, fig 2 and fig 3 for b values 0.0, 0.2 and 0.4.

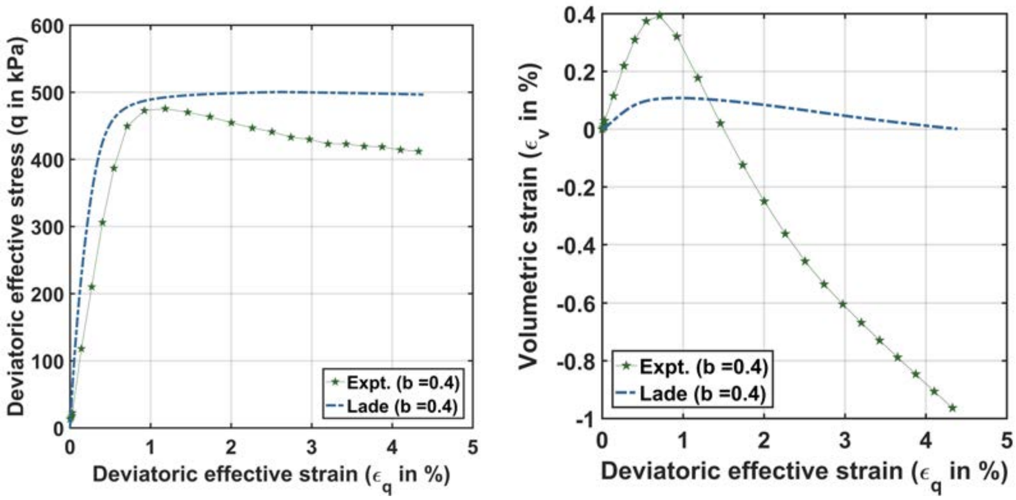

Fig. 3: a: Comparison of predicted and experimentally obtained response of stress strain behaviour at b = 0.4 and mean effective stress (p') of 300 kPa b: Comparison of predicted and experimentally obtained response of volumetric behaviour at b = 0.4 and mean effective stress (p') of 300 kPa

Parameter b or intermediate principal stress ratio takes values from 0 to 1 where 0 reflects compression test and 1 is for tension test. Figure 1a shows that the stress strain behaviour is predictd quite well but as we move from compression to tension regime the degree of mismatch increases particularly during softening (fig 2a, fig 3a). Prediction of volumetric response is poor since the predicted response shows only contraction whereas material shows dilation after contraction (fig 1b, fig 2b, fig 3b). And other contradiction between predicted and experimentally obtained response arises from observing that with increasing b material shows more contractive behaviour but the predicted response becomes more dilative.

This exercise implies that failure criteria is fairly accurate whereas softening rule and plastic potential function do not stand correct for cemented granular materials. Other cause of mismatch can be the sensitivity of model material parameters to the extraction process and choice of experiments. In a good constitutive model, material parameters should be independent of choice of experiments to calibrate the model.

5. Conclusion

Constitutive models are essential components for analysis and design of a structure subjected to body forces, Neumann and Dirichlet boundary condition. Due to complexity of accurate physical and mechanical characterization of granular materials in geomechanics, constitutive models are keep on evolving to give better prediction. This study presents a brief description on evolution of soil models from elastic-perfectly plastic to multisurface plasticity or bubble models. An advance third genration constitutive model (Lade's model) with 13 material parameters is chosen for calibration and validation exercises for cemented granular materials. Model was originally devised for granular materials without cohesion which has been extended to cemented granular materials by accounting cohesion or bond strength as additional confinement and translating the stress space along hydrostatic axis. This study comprises of several triaxial compression tests and constant mean effective stress tests. Model is calibrated using data obtained from triaxial compression tests, isotropic compression test and unload reload tests. Then stress-strain behaviour and volumetric response, as obtained from prediction exercise, is validated using experimental results for constant mean effective test result with b values of 0.0, 0.2, 0.4. From the validation exercise, we conclude that predicted stress-strain response is satisfactory in comparison to volumetric behaviour which is not properly replicated through model integration.

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