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Optimization of Stiffness and Damping for Multi-storey Structures

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Abstract

Efficiency of structural control for dynamic systems is highly dependent on the frequency contents of the excitation and structural frequency. For a given excitation, structural response control can be achieved by optimizing the stiffness and damping of the structure. The structural storey stiffness can be reduced using negative stiffness devices, while damping can be increased by using viscous dampers. A five-storey structure is considered in which stiffness and damping for every storey is optimized for minimum response. It is seen that for the response control, in some cases, storey stiffness is optimized for lesser value than the original storey stiffness. The results indicate that considerable structural control can be achieved for initially soft structures, whereas for very stiff structures, the optimization technique is ineffective.

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1. Introduction

The structural response of multi degree of freedom (MDOF) system depends on internal properties of the system and excitation. The internal properties primarily include mass, stiffness and damping of the structure. Response of the structure can be controlled by several control techniques. Using passive control devices, response of the primary system can be controlled by changing its internal properties.

There is a technique to develop negative stiffness as described in literature [1]. Based on this, stiffness at every floor of the multi-storey structure can be reduced. Structural damping at every storey can be increased by using damping devices. By changing stiffness and damping of the structure, structural parameters, mainly stiffness and damping can be optimized, so that the response of the structure would be minimum.

In this paper, an optimization technique is developed to optimize damping and stiffness of each storey of MDOF system. Damping and stiffness of the system are varied in the vicinity of the original properties of the parent structure (structure with consistent damping and stiffness). For the system with optimized damping and stiffness, the response of the top storey is compared with that of the original system. Various types of structural systems are considered ranging from very soft to very stiff. Finally, the effectiveness of this method is explained using a numerical example.

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2. Theory

2.1. Analysis of MDOF systems

The equation of motion for MDOF system is given by,

$$\mathbf{m\ddot{x}(t)} + \mathbf{c\dot{x}(t)} + \mathbf{kx(t)} = \mathbf{f(t)} \tag{1}$$

where, **m** is the mass matrix, **c** is the structural damping matrix, **k** is the stiffness matrix, **x(t)** is the displacement vector and **f(t)** is the force vector. Eq. (1) can be solved using modal analysis. For MDOF system with N degrees of freedom, response for each DOF can be expressed as,

$$x_j(t) = \sum_{i=1}^{N} q_i(t)\varphi_{ij} \tag{2}$$

where $x_j(t)$ is the response for j^{th} degree of freedom, $q_i(t)$ is the modal response for i^{th} degree of freedom and φ_{ij} is i^{th} component of j^{th} modal vector. The natural frequencies and mode shapes of the system are obtained using eigenvalues and eigenvectors of $\mathbf{m}^{-1}\mathbf{k}$. Using orthogonality of modes, MDOF system can be decoupled. The corresponding modal mass and stiffness matrices are given by,

$$\mathbf{K} = \mathbf{\Phi}^T\mathbf{k\Phi} \text{ and } \mathbf{M} = \mathbf{\Phi}^T\mathbf{m\Phi} \tag{3}$$

In general, only the first few modes govern the overall response of MDOF system. Thus the response of MDOF system can be suitably approximated by considering the first few modes or merely the fundamental mode.

2.2. Evaluation of structural damping using Rayleigh's technique

Generally, damping in the system is not known. It has to be obtained either through experiments or using suitable theoretical model. In this paper, Rayleigh's technique has been implemented to obtain the damping matrix for MDOF system. Rayleigh damping matrix is given by,

$$\mathbf{c} = \alpha\mathbf{m} + \beta\mathbf{k} \tag{4}$$

The coefficients α and β are determined using any two modes i and j by solving the following system of equations [2].

$$\frac{1}{2}\begin{bmatrix} 1/\omega_i & \omega_i \\ 1/\omega_j & \omega_j \end{bmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}\xi_i \\ \xi_j\end{pmatrix} \tag{5}$$

Here ω_i, ω_j are any two modal frequencies and ξ_i, ξ_j are corresponding damping ratios. Usually first two modal frequencies are considered in Eq. (5) since their contribution in the overall response is maximum.

2.3. Spectral analysis of single degree of freedom (SDOF) system

The equation of motion of SDOF system is given by,

$$\begin{aligned} m\ddot{x} + c\dot{x} + kx &= f(t) \\ \ddot{x} + 2\xi\omega_0\dot{x} + \omega_0^2 x &= f(t)/m \end{aligned} \tag{6}$$

where, ξ is the damping ratio and ω_0 is the natural frequency of the system.

$$c/m = 2\xi\omega_0; \quad k = \omega_0^2 m$$

The equation of motion in time domain (Eq. (6)) can be converted to frequency domain using Fourier transform and is given by,

$$\begin{aligned} X(\omega) &= H(\omega)F(\omega) \\ H(\omega) &= 1/(k + i\omega c - \omega^2 m) = 1/\left[m\left(\omega_0^2 + 2i\xi\omega_0\omega - \omega^2\right)\right] \end{aligned} \tag{7}$$

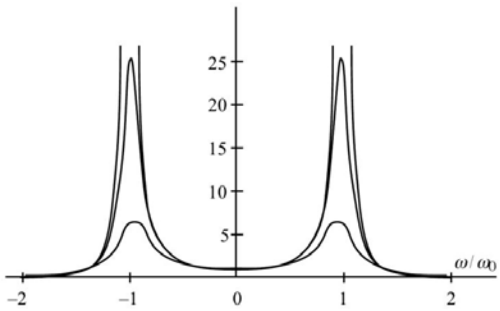


Fig. 1: Plot of $|H(\omega)|^2$ for $\xi = 0.05, 0.10, 0.20$ (Image courtesy: Lutes and Sarkani [3]).

If $f(t)$ is a stationary process, power spectral density function (PSDF) of the response of SDOF system can be written using Eq. (7) as,

$$S_{XX}(\omega) = S_{FF}(\omega)|H(\omega)|^2 = S_{FF}(\omega)/\left[m^2\left\{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\xi\omega_0\omega\right)^2\right\}\right] \tag{8}$$

The response of the system at low frequencies is governed by stiffness and at high frequencies by mass of the system. Near the natural frequency= ω_0, the response is dependent on the amount of damping. As shown in Fig. 1, the SDOF system acts as a bandpass filter for small values of ξ. Only the components of $f(t)$ that are near $\pm\omega_0$ are substantially amplified. Unless $S_{FF}(\omega)$ is much larger for some other non-resonant frequencies, the response PSDF, $S_{XX}(\omega)$, will be dominated by the frequencies near $\pm\omega_0$. This allows the stochastic response of the SDOF system to be considered a narrowband process, thereby simplifying the analysis. On the contrary, the excitation $f(t)$ can be approximated as a broadband process or by an equivalent white noise. The simplest such approximation is the white noise with constant PSDF, $S_0 = S_{FF}(\omega_0)$ as shown in Fig. 2. The most common usage for this is in the computation of the response variance, for which the approximation is given by [3].

$$\sigma_X^2 \approx \frac{\pi S_{FF}(\omega_0)}{2m^2\xi\omega_0^3} \tag{9}$$

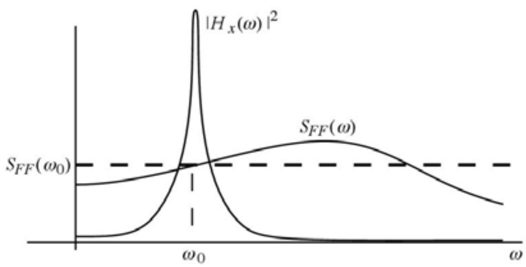


Fig. 2: Equivalent white noise excitation (Image courtesy: Lutes and Sarkani [3]).

3. Methodology

Multi-storey structure is modeled as MDOF system with known mass, stiffness and damping ratio of each storey. The response of top storey is to be minimized by optimizing structural damping and stiffness at each storey. In this paper, numerical problem presented is a five-storey structure. This analysis can be extended for a larger structure following the same methodology. The maximum contribution in the response of SDOF system comes from the fundamental mode. The response of MDOF system is represented by variance of an equivalent SDOF system with mass as fundamental modal mass and frequency as fundamental frequency. The excitation is assumed to be unit uncorrelated white noise with $S_0 = 1$. Hence Eq. (9) can be modified as,

$$\sigma^2 \approx \frac{\pi}{2M_1^2 \xi \omega_1^3} \tag{10}$$

where M_1 is the modal mass of the system corresponding to the fundamental frequency, ω_1, obtained as square root of the highest eigenvalue of $\mathbf{m}^{-1}\mathbf{k}$. The damping ratio, ξ, is approximated as,

$$\xi = C_1/(2M_1\omega_1) \tag{11}$$

where C_1 is the modal damping of the system corresponding to the fundamental frequency.

3.1. Approximation for damping coefficients

For the given multi-storey structure, using mass and stiffness matrices, the damping matrix is estimated using Rayleigh's technique. Its diagonal has contribution from both structural mass and stiffness. Using the diagonal of the damping matrix, damping coefficient for each storey can be approximated as follows. For a five-storey structure with damping coefficient of i^{th} storey as c_i, $\mathbf{c}$ matrix can be written as,

$$\mathbf{c} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & 0 & 0 \\ 0 & -c_3 & c_3 + c_4 & -c_4 & 0 \\ 0 & 0 & -c_4 & c_4 + c_5 & -c_5 \\ 0 & 0 & 0 & -c_5 & c_5 \end{bmatrix} \tag{12}$$

The diagonals of matrices in Eq. (4) and Eq. (12) can be compared to obtain the damping coefficients for each storey. The coefficient c_5 is obtained by equating it with the element c_{55} of $\mathbf{c}$ matrix in Eq. (4). The coefficient c_4 is obtained as $c_{44} - c_5$. This process of equating diagonal terms is continued till the first element c_{11} to obtain all the damping coefficients.

3.2. Optimization Problem

Let the stiffness for the i^{th} storey be k_i. The optimization problem is to minimize

$$\sigma^2(k_i, c_i) \approx \frac{\pi}{2M_1^2 \xi \omega_1^3} \tag{13}$$

Subject to $k_{li} < k_i < k_{ui}$ and $c_{li} < c_i < c_{ui}$
k_{li} and k_{ui} are lower and upper bounds for the structural stiffness of i^{th} storey.
c_{li} and c_{ui} are lower and upper bounds for the structural damping of i^{th} storey.

These bounds are determined based on maximum change in the structural storey damping and stiffness that can be achieved in reality using different devices [1]. The optimization can be performed as a constrained optimization using *fmincon* function in MATLAB [4].

3.3. Comparison of the response at top storey for original and optimized system

Extending Eq. (8) to MDOF system, response PSDF matrix can be expressed as

$$\mathbf{S_{XX}}(\omega) = \mathbf{H}(\omega)\mathbf{S_{FF}}(\omega)\mathbf{H}^*(\omega) \tag{14}$$

$\mathbf{H}^*(\omega)$ is the conjugate transpose of $\mathbf{H}(\omega)$. For MDOF system, $\mathbf{H}(\omega)$ is given by,

$$\mathbf{H}(\omega) = \left[\mathbf{k} - \omega^2\mathbf{m} + i\omega\mathbf{c}\right]^{-1} \tag{15}$$

For n-storey structure subjected to uncorrelated unit excitation, $\mathbf{S_{FF}}(\omega)$ would be an n×n identity matrix. Therefore Eq. (14) reduces to

$$\mathbf{S_{XX}}(\omega) = \mathbf{H}(\omega)\mathbf{H}^*(\omega) \tag{16}$$

$\mathbf{H}(\omega)$ is computed for the original and optimized **c** and **k** matrices using Eq. (15). Then using Eq. (16), response PSDF matrix for both original and optimized structure can be evaluated. The area under auto-PSDF for each DOF gives variance of the response of the corresponding DOF. Mathematically, this can be expressed as,

$$\sigma^2_{X_i} = \int_{-\infty}^{\infty} S_{X_iX_i}(\omega)\mathrm{d}\omega \tag{17}$$

Variance of the top storey response is computed for both original and optimized system. The effectiveness of the proposed optimization method is studied by comparing the top storey response for original and optimized system.

4. Numerical example

In order to illustrate the proposed optimization technique, a five-storey structure is considered, see Fig. 3. In this example, mass of each storey is assumed to be 3000 kg. To study various types of systems ranging from very soft to very stiff, four cases with different storey stiffness are considered. Stiffness for all the storeys in each case is assumed to be equal. Based on maximum possible change in structural properties, stiffness of each storey is varied between 0.8 to 2.0 times that of original value and damping for each storey between 1.0 to 2.0 times that of original value. The damping ratio of the structure is taken as 2%. The structure is subjected to unit uncorrelated white noise excitation.

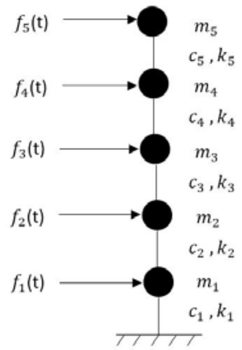


Fig. 3: Five-storey structure modeled as five degrees of freedom system.

4.1. Very soft structure

In this case, stiffness for each storey is chosen so that fundamental frequency of the original structure is ω_1 = 1.16 rad/s (T = 5.4 sec), refer Table 1. Using the top storey response PSDF as shown in Fig. 4(a), standard deviations of the top storey response for the original and optimized structure are obtained as, σ_{X_5} = 1.000 mm and $\sigma_{X_{5opt}}$ = 0.992 mm.

Table 1: Stiffness and damping coefficients of all the storey for original (very soft) and optimized structure.

Storey	Original Structure		Optimized Structure	
	Stiffness (k_i) (kN/m)	Damping Coefficient (c_i) (kg/s)	Stiffness (k_i) (kN/m)	Damping Coefficient (c_i) (kg/s)
1	50	543	100	1086
2	50	439	100	878
3	50	543	100	1086
4	50	439	100	878
5	50	543	40	1086

4.2. Soft structure

In this case, stiffness for each storey is chosen so that fundamental frequency of the original structure is ω_1 = 1.64 rad/s (T = 3.8 sec), refer Table 2. Using the top storey response PSDF as shown in Fig. 4(b), standard deviations of the top storey response for the original and optimized structure are obtained as, σ_{X_5} = 0.603 mm and $\sigma_{X_{5opt}}$ = 0.554 mm.

Table 2: Stiffness and damping coefficients of all the storey for original (soft) and optimized structure.

Storey	Original Structure		Optimized Structure	
	Stiffness (k_i) (kN/m)	Damping Coefficient (c_i) (kg/s)	Stiffness (k_i) (kN/m)	Damping Coefficient (c_i) (kg/s)
1	100	770	200	1540
2	100	620	200	1240
3	100	770	200	1540
4	100	620	200	1240
5	100	770	189	1540

4.3. Stiff structure

In this case, stiffness for each storey is chosen so that fundamental frequency of the original structure is ω_1 = 2.32 rad/s (T = 2.7 sec), refer Table 3. Using the top storey response PSDF as shown in Fig. 4(c), standard deviations of the top storey response for the original and optimized structure are obtained as, σ_{X_5} = 0.358 mm and $\sigma_{X_{5opt}}$ = 0.362 mm.

4.4. Very stiff structure

In this case, stiffness for each storey is chosen so that fundamental frequency of the original structure is ω_1 = 3.29 rad/s (T = 1.9 sec), refer Table 4. Using the top storey response PSDF as shown in Fig. 4(d), standard deviations of the top storey response for the original and optimized structure are obtained as, σ_{X_5} = 0.213 mm and $\sigma_{X_{5opt}}$ = 0.308 mm.

Table 3: Stiffness and damping coefficients of all the storey for original (stiff) and optimized structure.

Storey	Original Structure		Optimized Structure	
	Stiffness (k_i) (kN/m)	Damping Coefficient (c_i) (kg/s)	Stiffness (k_i) (kN/m)	Damping Coefficient (c_i) (kg/s)
1	200	1090	400	2170
2	200	880	400	1760
3	200	1090	321	2170
4	200	880	297	1760
5	200	1090	223	2170

Table 4: Stiffness and damping coefficients of all the storey for original (very stiff) and optimized structure.

Storey	Original Structure		Optimized Structure	
	Stiffness (k_i) (kN/m)	Damping Coefficient (c_i) (kg/s)	Stiffness (k_i) (kN/m)	Damping Coefficient (c_i) (kg/s)
1	400	1540	321	3070
2	400	1240	321	2480
3	400	1540	321	3070
4	400	1240	321	2480
5	400	1540	321	3070

5. Results and discussions

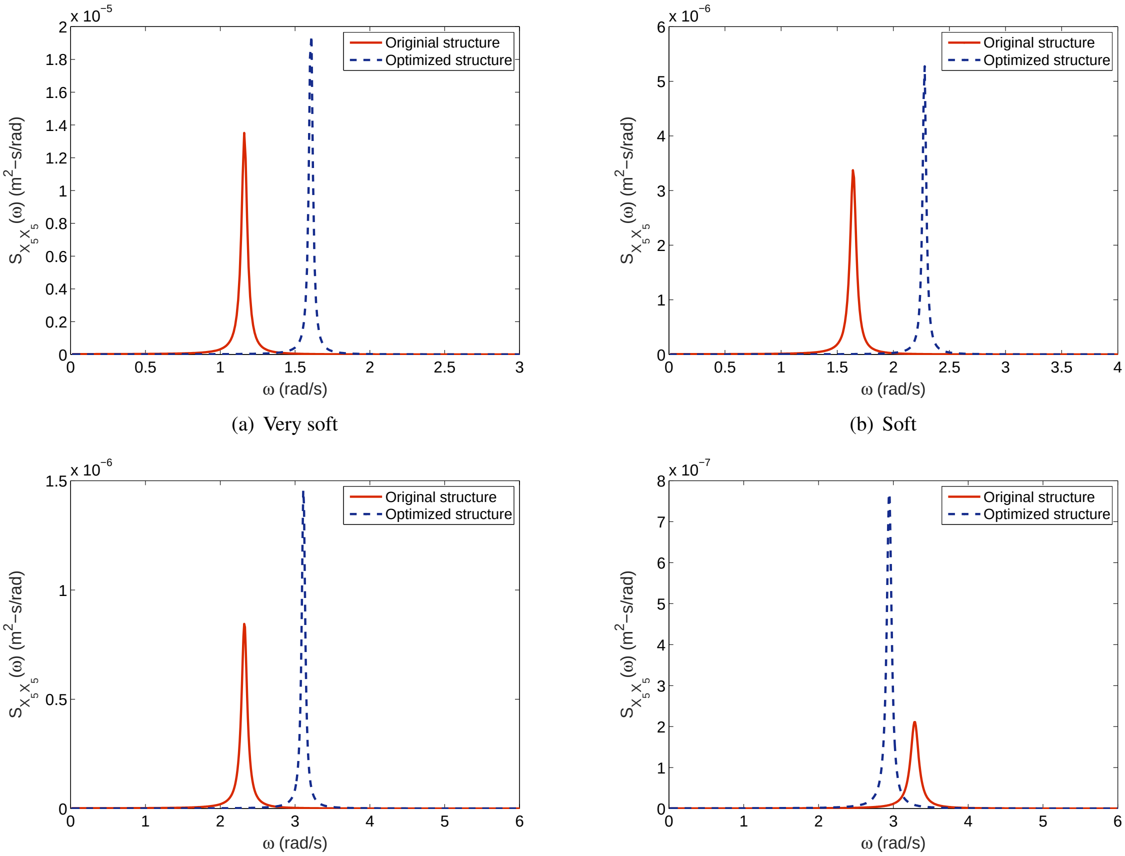
It is observed that, in some cases, the storey stiffness coefficient is optimized for lesser value than its original value. The optimized storey damping coefficients, on the other hand, are twice their original values in almost all the cases. The response at top storey is compared for all the cases as shown in Table 5. The comparison shows that this technique works well for structures which are comparatively softer having natural period close to 3.8 sec. This method is slightly advantageous for very soft structures, whereas it is ineffective for very stiff systems.

Table 5: Comparison of the response at top storey for original and optimized structures.

Type of structure	Fundamental time period (s)	σ_{X_5} (mm)	$\sigma_{X_{5opt}}$ (mm)	Percentage change
Very soft	5.4	1.000	0.992	-0.8
Soft	3.8	0.603	0.554	-8.0
Stiff	2.7	0.358	0.362	+1.0
Very stiff	1.9	0.213	0.308	+44.0

6. Conclusions

Dynamic response of MDOF system is studied for its different internal properties. Structural properties, mainly stiffness and damping are varied within practical range to achieve response which is minimum. The original stiffness coefficient of the system is varied from 0.8 times to 2.0 times while damping coefficient is varied from 1.0 times to 2.0 times. To achieve the reduction in stiffness coefficient, negative stiffness device can be utilized. It is observed that structural internal properties can be optimized for controlling the response to a certain limit. Following observations are made.



(a) Very soft (b) Soft

(c) Stiff (d) Very stiff

Fig. 4: PSDF of the response at top storey for original and optimized structure.

1. Relatively soft structure shows better control in response when optimized stiffness and damping are used.
    (a) Percentage control achieved for soft structure is 8%.
    (b) Percentage control achieved for very soft structure is 0.8%.
2. Relatively stiff structure does not show good control in response when optimized stiffness and damping are used. In case of very stiff structure, 44% increase in response is observed.

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