Reply

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New theoretical formulation for the determination of radiance transmittance at the water-air interface: reply

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Abstract: Gordon and Voss in their comment challenge the recently published paper [Opt. Express **25**, 27086 (2017)] on the unity factor of radiance transmittance of the assumed hypothetical case (i.e., for the albedo equal to 1) and question the dependence of particulate contribution to the refractive index of water. Here, we provide answers to their comments.

OCIS codes: (010.4450) Oceanic optics; (010.4455) Oceanic propagation; (010.5620) Radiative transfer; (010.5630) Radiometry; (010.7340) Water.

References and links

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4. P. J. Dev and P. Shanmugam, "New theoretical formulation for the determination of radiance transmittance at the water-air interface," Opt. Express **25**(22), 27086–27103 (2017).

1. Transmittance in the hypothetical water

The comment presumes that we have termed the well-established n^2 – *law for radiance* Eq. (1) equation to be incorrect. We object to this supposition and we do not ignore or term it as incorrect, which can be evidenced by the use of the same equation in our formulation. The geometry ascribed in n^2 – *law for radiance* is for the radiance interacting with the interface *only once* [1,2], and the multiple interactions are completely omitted. Here, we answer to the comment challenging the transmittance of the assumed hypothetical water is unity. The hypothetical situation assumed by Gordon and Voss is analogous to our hypothetical assumption, in which they used R = 1 and we assumed the water medium as non-absorbing and fully upward scattering. Both assumptions state that no photons are retained within the water body in any form, *i.e.,* fully reflecting medium. Now, we prove that Eq. (2) of the comment itself shows the transmittance of hypothetical waters turns to unity, when multiple surface reflections of the photons at the waterside are considered. For that we provide the proof below. The n^2 – *law for radiance* applied to the air-water medium is given by,

$$L_u^+(\theta_a,\phi_a) = \frac{L_u^-(\theta_w,\phi_w)}{n_w^2} t_f^-(\theta_w,\varphi_w). \quad (1)$$

Upwelling radiance *before* crossing the interface, inclusive of multiple surface interactions at the waterside (same as represented in the comment), is expressed as,

$$L_u^- = L_{u0}^- + L_{u1}^- + L_{u2}^- + \ldots = L_{u0}^- + \overline{r}RL_{u0}^- + (\overline{r}R)^2 L_{u0}^- + \ldots = \frac{L_{u0}^-}{1-\overline{r}R}. \quad (2)$$

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L_{u0}^{-} is the in-water radiance *before* striking the interface and L_{u1}^{-}, L_{u2}^{-}, ... are the successive surface reflected components of L_{u0}^{-}. Radiance *after* crossing the interface [Eq. (2) in Eq. (1)],

$$L_u^+ = \frac{t_f^-}{n_w^2}\frac{L_{u0}^-}{1-\overline{r}R} = \frac{t_f^-}{n_w^2}\left[\frac{1}{1-\overline{r}R}\right]L_{u0}^-. \tag{3}$$

Comparing Eq. (3) and Eq. (1), we get an extra term $\left[\frac{1}{1-\overline{r}R}\right]$. As the L_{u0}^{-} is the in-water radiance *before* striking the interface, the L_{u0}^{-} in Eq. (3) and the $L_u^-(\theta_w,\phi_w)$ in Eq. (1) contain the same number of photons within the solid angle. Therefore, $\left[\frac{1}{1-\overline{r}R}\right]$ cannot be the component of radiance as stated in the comment [Eq. (2)]. It is clearly a component of the interface and water. The term $\left[\frac{1}{1-\overline{r}R}\right]$ varies with the optical properties of water and interface, which additionally contributes to the existing transmittance factor (t_f^- / n_w^2) and has nothing to do with the radiance. For the hypothetical case of Gordon and Voss (in the comment) $R = 1$,

$$L_u^+ = \frac{t_f^-}{n_w^2}\left[\frac{1}{1-\overline{r}}\right]L_{u0}^- = 0.541\times\left[\frac{1}{1-0.475}\right]L_{u0}^- = 1.03\times L_{u0}^-. \tag{4}$$

Clearly, the above equation results in *unity*. The value of $\overline{r}$ given in Gordon *et al*. [3] and the comment is 0.475 (approx.). Practically, there cannot be 103% of L_{u0}^{-}. We understand the constant $\overline{r} \sim 0.475$ is an approximate value, and for a 100% of L_{u0}^{-}, the calculation results in the value of $\overline{r} = 0.459$. The correct value of $\overline{r}$ can be obtained by considering a uniform upwelling light field above and below the interface.

$$E_u^+ = \int_{\Omega_d} L_u^+(\theta_a,\phi_a)\cos\theta_a d\Omega_a = \pi L_u^+. \tag{5}$$

$$E_u^- = \int_{\Omega_d} L_u^-(\theta_w,\phi_w)\cos\theta_w d\Omega_w = \pi L_u^-. \tag{6}$$

For this assumed hypothetical case, if no photons are retained within the medium, then Eqs. (5) and (6) will be equal and the Eq. (3) reduces to $\frac{t_f^-}{n_w^2}\left[\frac{1}{1-\overline{r}}\right]=1$. This leads the $\overline{r}$ value to 0.459 by assuming t_f^- / n_w^2 = 0.541. Now, we consider the present formulation (Eq. (11) in Dev and Shanmugam [4]), where L_{u0}^{-} is the in-water radiance *before* striking the interface. For a non-absorbing and a fully upward scattering medium, $\mu_u = 1$ and $\omega = 1$,

$$L_u^+ = \frac{t_f^-}{n_w^2}\left[\frac{1-\mu_u\omega}{r_f^2}+\frac{\mu_u\omega n_w^2}{t_f^-}\right]\times L_{u0}^- = \frac{t_f^-}{n_w^2}\left[\frac{1-1}{r_f^2}+\frac{n_w^2}{t_f^-}\right]\times L_{u0}^- = 1.00\times L_{u0}^-. \tag{7}$$

Equations (4) and (7) conclude the same and agree with each other that the transmittance in the hypothetical water turns to be unity, after the inclusion of multiple surface reflections of photons with the water-air interface. The only difference between these two formulations is that the comment deals the processes from the standpoint of *internal reflection*, whereas the

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present formulation deals from the standpoint of *transmittance* of photons on every interaction with the interface. If $\left[\frac{1-\mu_u\omega}{r_f^2}+\frac{\mu_u\omega n_w^2}{t_f^-}\right]$ replaces the $\left[\frac{1}{1-\overline{r}R}\right]$ in Eq. (2), all the photons will eventually cross the interface satisfying the Eq. (3) to Eq. (7) (in the comment) as described. Therefore, no violation of physical concepts in both the cases.

Here, we provide an example that allows us to examine the correctness of Gordon and Voss (GV) interpretation [Eq. (2)]. According to GV, L_u^- is the measurable quantity and L_{u0}^- is non-measurable and a theoretical construct.

Now, let us assume radiance at two depths z_1 and z_2 in a homogeneous and a diffuse medium as $L_u^{z_1^-}$ and $L_u^{z_2^-}$. Now, let us extrapolate $L_u^{z_1^-}$ to 'just below-water surface radiance denoted by $L_u^{z_0^-}$. Using Eq. (8), $L_u^{z_1^-}$ can be extrapolated to $L_u^{z_0^-}$.

$$L_u^{z_0^-}=L_u^{z_1^-}e^{K_{Lu}z_1}. \tag{8}$$

where, $K_{Lu}=\frac{1}{(z_2-z_1)}\left[ln\left(L_u^{z_1^-}\right)-ln(L_u^{z_2^-})\right]$. As we know, $L_u^{z_1^-}$ and $L_u^{z_2^-}$ are not involved in any interactions with the interface, and the obtained $L_u^{z_0^-}$ though the extrapolation is also not involved in any interaction with the interface. This $L_u^{z_0^-}$ and GV's L_{u0}^- are analogous to each other because both radiances have not yet interacted with the interface. On propagating both the quantities through the interface, $L_u^{z_0^-}$ can be successfully translated to L_w, but GV's L_{u0}^- requires $1-\overline{r}R$ (according to the comment) which is not possible to translate though the interface. Therefore, two things we can understand from this:

1) GV's interpretation failed to explain Eq. (8) and the propagation process through the interface.
2) L_{u0}^- is not a theoretical construct as claimed in the comment, since we have simulated it using Eq. (8) ($L_u^{z_0^-}$ is same as their L_{u0}^-).

Now, it became clear that $1-\overline{r}R$ cannot be represented along with the radiance, as it is the property of the interface and the water. For this reason, we interpreted GV's L_{u0}^- as the radiance *before* interacting with the interface. The only way to avoid the failure of GV's interpretation is to consider L_{u0}^- as L_{u0}^- as we have shown above. It means that the usage of the term L_{u0}^- is incorrect, and hence, Eq. (2) becomes invalid.

2. Dependence of particulate contribution to the refractive index of water and the transmittance

We initially suspected that there would be significant differences in the Snell cones of the pure water and the turbid water (especially high scattering water) because of large difference in the particle concentrations between these waters. To find the Snell cones of these two water regimes, we performed an experiment in our laboratory to measure the critical angles for the pure and the turbid water by tracing the incident ray. Pure water was filled inside the glass tank (1 ft x 1 ft x 1 ft) and red laser source was used. Tracing the incident ray for the critical angle measurement, it provided the critical angle of 48 degrees. Then the particles were introduced to it slowly until it becomes very turbid. During this measurement, the light became too diffuse, hindering the observable point, however critical angle hovered around 47 degrees. We were only able to observe one 1 degree of difference between the pure and the turbid water which gives the difference in refractive index about (1.3628-1.34 = 0.0228). Our

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experiment may contain measurement errors, as the experiment was not performed with sophisticated instruments.

Since the particulates contribute significantly less to the refractive index of waters, $n(\lambda)$ can be replaced with the $n_w(\lambda)$ in the Eq. (8) of the published paper [4]. The corrected equations are given below. The transmittance equation Eq. (10) in [4] becomes,

$$\tau_{w,a} = \frac{(1-\rho_{w,a})}{n_w^2}\left[(1-\mu_u\omega)+\frac{\mu_u\omega n_w^2}{(1-\rho_{w,a})}\right]. \tag{9}$$

Similarly, the water-leaving radiance (Eq. (11) in [4] and translation of below to above-water remote sensing signal (Eq. (18) in [4]) become,

$$L_w(air,\lambda) = \frac{(1-\rho_{w,a})}{n_w^2}\left[(1-\mu_u\omega)+\frac{\mu_u\omega n_w^2}{(1-\rho_{w,a})}\right]L_u(0^-,\lambda). \tag{10}$$

$$R_{rs}(0^+,\lambda) = \frac{(1-\rho_{a,w})(1-\rho_{w,a})}{n_w^2}\left[(1-\mu_u\omega)+\frac{\mu_u\omega n_w^2}{(1-\rho_{w,a})}\right]r_{rs}(0^-,\lambda). \tag{11}$$

The term representing the particulate contribution to the refractive index of the water, r_f^2, has been removed, and it is the only change between this reply and the original manuscript [4]. The results do not vary significantly in the case of clear oceanic waters, but show some variations (approximately up to 15%) in the case of high scattering waters. However, the multiple scattering effect is significantly high in turbid waters as confirmed by this study.

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