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New capacitive micromanometer

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The theory, design, and preliminary test results of a new capacitive micromanometer for differential pressure measurement are presented. A U-tube manometer and two cylindrical cross capacitors are used to transduct pressure variation to capacitance variation linearly. The change in capacitance is measured with the help of a 2-winding transformer ratio bridge employing an 8-decade Inductive Voltage Divider (IVD). The value of differential pressure is obtained with a resolution of 60 m Pa in terms of the IVD ratio readings. The unit constructed can measure pressures from very low values to 10 k Pa. It is considered that the proposed system can be developed as a primary standard for the measurement of low range differential pressures.

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INTRODUCTION

The precise and accurate measurement of differential pressure in the low and medium ranges is required in connection with a number of scientific and technical applications. Most of the resistive, inductive, and capacitive transducers used for this purpose[1,2] have accuracies limited to about 0.1% and linearity limited to 1% of full scale, due to the characteristics of the primary sensor and/or the final measuring circuit. The micromanometer is considered to be the most sensitive among the purely mechanical instruments. Different methods have been employed to sense the difference in fluid levels in the limbs of the manometer. A highly accurate and sensitive ultrasonic micromanometer with a resolution of 1.4 m Pa and an accuracy of 0.01% has been developed by Heydemann *et al.*[3] Another interferometric oil manometer has been developed at NPL(UK) with an accuracy of 0.5% and a range of 0.3–6 Pa.[4]

The system presented here uses a capacitive secondary transducer in conjunction with a U-tube mercury manometer as the primary sensor. Two cylindrical cross capacitors are placed at the same level and the two columns of the U tube are placed coaxially in their central spaces as shown in Fig. 1. The effective lengths of the two capacitors are fixed by the levels of the grounded mercury columns in the two limbs of the U-tube manometer. On the application of pressure, one capacitance increases while the other decreases. The differential change in capacitance is measured on a transformer bridge which comprises the two capacitors, an IVD, and a detector connected as shown in Fig. 2. Ideally, the change in pressure and the change in IVD ratio required for bridge balance are linearly related. As the ratio of a good IVD can be read with as high an accuracy and resolution as 1 ppm, the same advantages with some limitations imposed by the pressure–capacitance transduction are carried over to the measurement of differential pressure. As a result of the differential system used, some of the effects due to temperature and humidity tend to cancel out. The theory and experimental results on the constructed model indicate that the proposed system lends itself to development as a primary instrument for the precise and accurate measurement of pressures in the low range.

Another scheme using the same transducer, described elsewhere,[5] requires a considerably longer time for measurement during which the pressure should be held steady. In the present method the differential pressure is obtained after a single balancing adjustment after pressure is applied.

I. THEORY

In the cylindrical cross capacitor of the shape suggested by Thompson,[6] four uniform circular cylinders are arranged with their axes parallel and situated in free space at the corners of a square, with the gaps between the adjacent cylinders kept very small. Let C_1 and C_2 be the capacitances between the opposite pairs of cylinders. If $C_1 = C_2 = C_0$, then it has been shown[6] that $C_0 = (\epsilon_0/\pi)(\ln 2)(L)$ farads, where L is the length of the cylinder in meters. It has also been shown that even if C_1 deviates slightly from C_2, their mean value very closely approaches the above expression for C_0. For instance, the mean value differs from the above expression for C_0 by only 0.01 ppm for 340 ppm difference between C_1 and C_2. When a conducting grounded rod is inserted into the central space of the cylindrical cross capacitor, the capacitance of the latter decreases due to the shielding action of the former.[7,8] Over an appreciable range, the change in the position of the conducting rod is linearly related to the change in capacitance, unaffected by the fringing effect present at the tip of the conducting rod,[6] provided the flux distribution is reproducible in both positions. This effect is utilized in the transducer system to be described.

In the present scheme, two cylindrical cross capacitors are fabricated and kept vertical with their bottom ends at the same level. A U-tube manometer is positioned such that its two mercury columns are symmetrically and coaxially situated in the central spaces of the two cross

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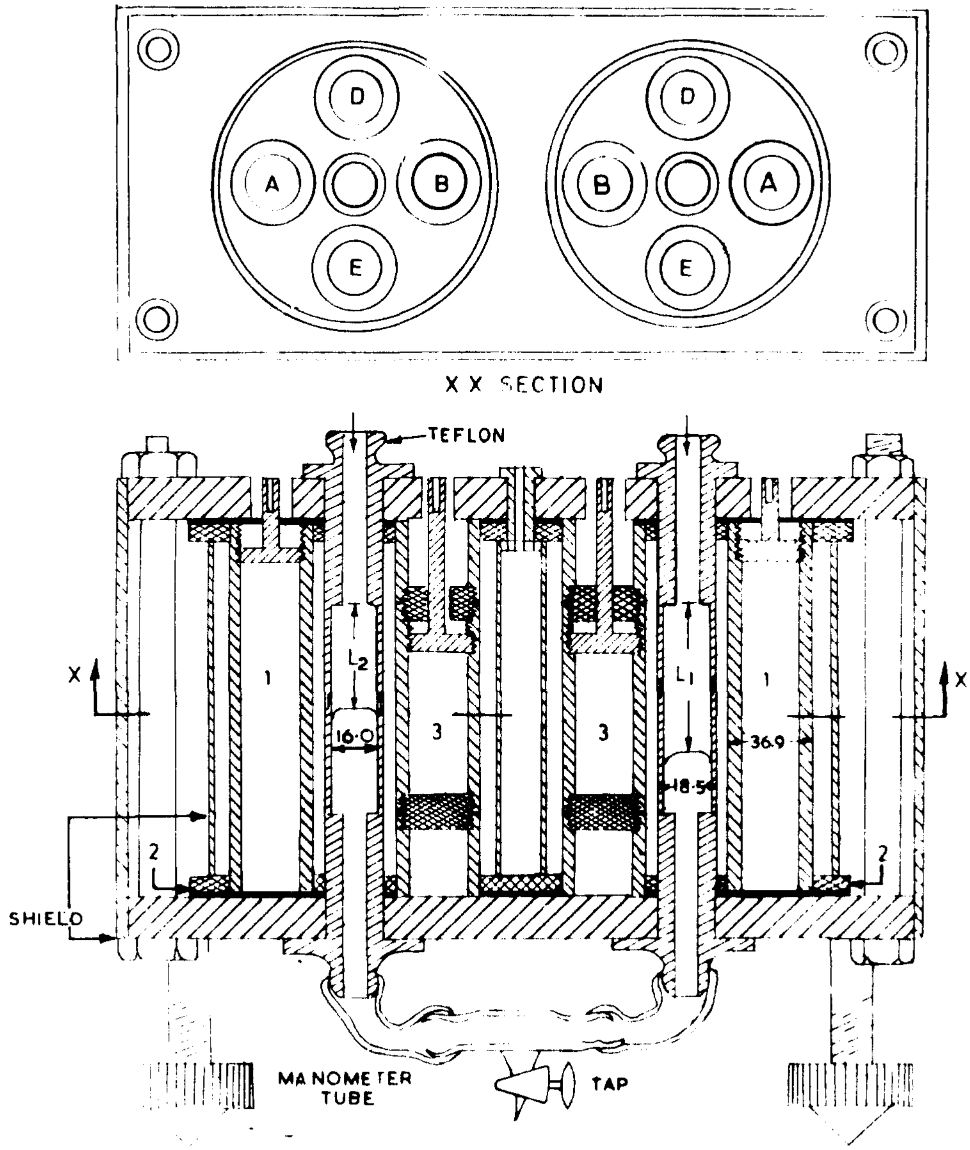

FIG. 1. Constructional details of the capacitive micromanometer: (1) higher potential electrode, (2) sheet with holes, (3) guarded electrode. Dimensions are in millimeters.

capacitors. The mercury is kept at ground potential. The effective lengths of the capacitors are now decided by the levels of the mercury in the two columns of the U tube as shown in Fig. 1. The capacitors are connected in series and their common terminal is connected to the detector while their higher potential terminals are connected to an 8-decade IVD as shown in Fig. 2.

Let L_1 and L_2 be the effective lengths of the two capacitors at zero-differential pressure and let

$$L_1 = L \text{ meters, and } L_2 = (L - \delta L_0) \text{ meters.} \quad (1)$$

The capacitances of the first and second capacitors are given by the following expressions:

$$C_{01} = (\epsilon_0/\pi)\,(\ln 2)(K)(L) \text{ farads} \quad (2)$$

$$C_{02} = (\epsilon_0/\pi)\,(\ln 2)(K)(L - \delta L_0) \text{ farads,} \quad (3)$$

where K is a factor which accounts for the presence of the manometer tube material inside the capacitors and assumed to be equal for both the tubes.

Suppose $x_0 = (0.5 + \delta x_0)$ is the ratio of the IVD at bridge balance under this condition. Then,

$$\frac{1 - x_0}{x_0} = \frac{C_{02}}{C_{01}} = \frac{L - \delta L_0}{L}. \quad (4)$$

Neglecting the product of second order quantities, we can deduce that

$$L_0 = 4L\delta x_0. \quad (5)$$

Now pressure 'p' is applied. As a consequence, let L_1 increase by δL_1 and L_2 decrease by δL_2. Let the bridge be again balanced with an IVD ratio of $(0.5 + \delta x)$. Then the ratio of the new values of capacitances is given by

$$\frac{(C_{02})_p}{(C_{01})_p} = \frac{L - \delta L_0 - \delta L_2}{L + \delta L_1} = \frac{0.5 - \delta x}{0.5 + \delta x}. \quad (6)$$

From (5) and (6)

$$\delta L_1 + \delta L_2 = 4L(\delta x - \delta x_0) - 8L\delta x_0 \delta x + 2\delta x(\delta L_1 - \delta L_2). \quad (7)$$

Assuming a high degree of symmetry, δx_0 and $(\delta L_1 - \delta L_2)$ tend to be vanishingly small. Hence to a good order of approximation,

$$\delta L_1 + \delta L_2 = 4L(\delta x - \delta x_0). \quad (8)$$

The above derivation is made under the assumption that the two individual cross capacitances of each capacitor system are equal. For a more accurate treatment, their average value may be measured and taken into consideration. It is possible to find the ratio of the two average cross capacitances by measuring an appropriate set of three ratios of the four cross capacitances taken two at a time, with the help of the circuit shown in Fig. 2.

In a U-tube manometer, differential pressure is expressed in terms of the density of liquid, gravity, and the changes in liquid level in the two limbs, i.e., $p = \rho g(\delta L_1 + \delta L_2)$. Hence

$$p = 4\rho g L(\delta x - \delta x_0). \quad (9)$$

Thus the differential pressure can be measured in terms of the readings of the IVD with and without the applied pressure and in terms of the constants g, ρ, and L. The value of L is initially electrically determined by the following procedure. (a) The average cross capacitance of the first capacitor without the manometer tube is measured with the bridge circuit of Fig. 2 using a reference capacitor (which itself may be a cross capacitor). (b) The manometer tube without mercury is now inserted in the cross capacitor and the new value of average cross capacitance is measured in the same manner as above. (c) The ratio of the two average cross capacitances in (b) and (a) yields the value of K in Eq. (2). (d) After filling up the manometer tube with mercury, C_{01} is measured with the circuit of Fig. 2, replacing the second capacitor by a reference capacitor. L is then computed from Eq. (2) using the value of K determined as above.

II. CONSTRUCTIONAL DETAILS

The cross-sectional view of the prototype model constructed is shown in Fig. 1.

Precisely machined brass cylinders (36.9 mm o.d.) are mounted parallel to each other with their axes passing through the corners of a square. The gap between two adjacent cylinders is taken to be 3 mm (less than one tenth of the diameter as suggested by Thompson).[6] Two adjacent cylinders in each of the cross capacitors are provided with guard electrodes to enable measurement of the two cross capacitances. The design value of the guarded length of each capacitor (including half the insulating gap thickness at each guard) is 179.70 mm.

The cylinders are kept in position with the help of two thick top and bottom perspex sheets in which matching holes are bored with a jig boring machine with an accuracy of $\pm 5\ \mu$m. The whole system is made to rest between two thick stainless steel plates with appropriate insulation for the high potential electrodes. The angle between the plates and cylinders is checked to be 90°.

The vertical portions of the U-tube manometer are made of teflon tubes having an internal diameter of 16 mm and an external diameter of 18.5 mm. Teflon is chosen in preference to materials like glass and perspex owing to its low coefficient of friction, low dielectric constant,

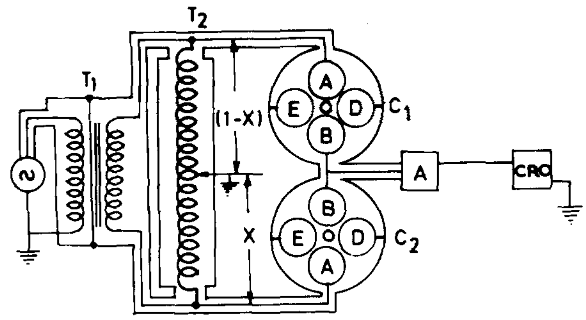

FIG. 2. Circuit arrangement; T_1—isolating transformer, T_2—8-decade inductive voltage divider, A—amplifier, CRO—oscilloscope detector, C_{12}, C_2—right hand and left hand cross capacitors.

low absorption of moisture, high temperature stability, and easy machinability.[9] The tubes are made out of teflon rods taking extreme care in fabrication to have a uniform cross section throughout the length. A tap is provided in the U bend made of polyethylene tube to take out mercury.

III. ANALYSIS OF ERRORS

The differential pressure is measured in terms of the fixed constants and measured values in accordance with Eq. (9). The contribution of these quantities to the uncertainty of the measured differential pressure and other sources of error is now discussed.

The errors may be broadly divided into those proportional to the measured differential pressure, for example, those contributed by the uncertainties in the values of g, ρ, L, and those independent of pressure like varying fringing and capillary effects, varying temperature gradients, etc.

A. Gravity (g)

The ambient value of the acceleration due to gravity at any location can be calculated with an uncertainty of 100 ppm or less. If necessary it can be measured with special arrangements with an absolute accuracy of 1 ppm or less.

B. Density (ρ)

Since mercury does not dissolve gases and is easily purifiable, the density is quite reproducible and its value at any temperature is available with an accuracy of 1 ppm.[3]

C. Length (L)

Contributions to the error in the determined value of the length L arise from the uncertainty in the value of the reference capacitor, the inequality of the two cross capacitances, constancy of the value of K, and fringing effect at the mercury surface.

In our experiments a General Radio reference capacitor having an accuracy of 5 ppm has been used. The two values of the cross capacitance of the capacitor system built were found to differ by less than 0.1%, thereby

TABLE I. Monitoring of zero reading in a working day. Constant of the unit $(\partial p/\partial x)$ = 56704.7 Pa.

Time of day	Temperature	I.V.D. Ratio at zero differential pressure
9 AM	27.0°C	0.500520
11 AM	26.5°C	0.500526
1 PM	26.0°C	0.500523
3 PM	25.9°C	0.500525
5 PM	26.0°C	0.500522

enabling the mean capacitance value to be related to length with an error of less than 0.1 ppm.

Nonuniformity of the value of K along the length of the capacitor arises due to variation in dimensions, dielectric constant, and positioning of the manometer tube. By employing a manometer tube made of a material having a low dielectric constant with a large diameter and small wall thickness, the value of K can be made to approach unity, a value which it would take in the absence of the manometer tube. Thereby the percentage variation in K due to variations in the above mentioned parameters will be greatly restricted. In the system built, the experimentally determined values of K for the two limbs are 1.0582 and 1.0581. To check indirectly the uniformity of K along the length of the capacitor, an experiment was performed, as described in the next section, to evaluate the variation in the ratio of capacitances of the two cross capacitors for equal lengths. It is observed that this ratio does not vary by more than 12 ppm over an appreciable range.

The fringing effect at the mercury surface at one end of the capacitor causes the value of L calculated from Eq. (2) by the procedure described in Sec. II to be slightly smaller than the actual length of the capacitor. However, as long as there exists a reproducible fringing effect in both the tubes with and without the pressure, it is the same modified value of L which enters Eq. (9) and hence the error gets cancelled.

An error is introduced if the cylinders are not kept vertical on the horizontal stainless steel plate. If the angle between the vertical line and the axis of the cylinders is θ rad, then the vertical component of the level difference in the manometer is Cosθ times $(\delta L_1 + \delta L_2)$, leading to a relative error of $\theta^2/2$ for small values of θ. This cosine error is made negligible by careful grinding of the base plate and lapping of the bottom ends of the cylinders. Fine threads provided on the base permit the horizontal alignment of the latter. To limit the cosine error to 10 ppm, the tilt angle needs to be kept below 0.25°.

Capillary depressions in the two manometer tubes and variations in the meniscus shape are minimized by having a fairly large manometer tube diameter (≥15 mm) as recommended by standards[10] and using mercury free from impurities. Errors are caused in our system only due to the *variations* in the capillary depressions and fringing effects due to varying meniscus shapes. As the contact angle of mercury with teflon is considered fairly reproducible,[9] the errors on this account can be expected to be small. The results of an experiment described in the

TABLE II. Repeatability of zero reading after pressure excursions.

Sl. no.	Pressure pascals	I.V.D. Ratio
1.	0 100	0.500513
2.	0 −100	0.500515
3.	0 100	0.500512
4.	0 200	0.500513
5.	0 500	0.500515
6.	0 1000	0.500513
7.	0	0.500517

TABLE III. Results of the test for checking the uniformity of the value of K (mercury is dropped slowly).

Sl. no.	Reduction in height of mercury column	I.V.D. Ratio at zero diff. pressure
1.	0.0	0.500514
2.	0.5 cm	0.500514
3.	1.0 cm	0.500513
4.	2.0 cm	0.500516
5.	3.0 cm	0.500515
6.	5.0 cm	0.500516
7.	7.0 cm	0.500516

TABLE IV. Comparison of readings with a mechanical micromanometer.

Sl. no.	Reading of mechanical micromanometer, mm methanol (pascals)	I.V.D. Ratio at bridge balance χ	$\delta x - \delta x_0$	Calculated pressure, pascals
1.	0	0.500523		
2.	5.000 (39.132)	0.501228	0.000705	39.977
3.	10.000 (78.265)	0.501880	0.001357	76.948
4.	15.000 (117.397)	0.502603	0.002080	117.946
5.	20.000 (156.529)	0.503287	0.002764	156.732
6.	25.000 (195.66)	0.503943	0.003420	193.930

Reference Capacitor used: General Radio type 1404, 10 pF value.
Value of K (determined experimentally) = 1.0582.
Initial length L (determined experimentally) = 0.10709 m.
Density of mercury at 26°C = 13531.14 Kg/m^3.
Value of acceleration due to gravity = 9.78309 m/s^2.
Constant of the unit $(\delta p/\delta x)$ = 56704.7 Pa.
IVD ratio at zero diff. pressure x_0 = 0.500523, δx_0 = 0.000523.
Difference in length δL_0 = 0.000224 m.
Density of methanol (as given by the manufacturer of mechanical micromanometer) = 800 Kg/m^3.

next section show that the balance condition at zero differential pressure after repeated pressure excursions from this value is reproducible to within 10 ppm, lending confirmatory evidence to the above conclusion.

IV. EXPERIMENTAL RESULTS

Experiments[11] were conducted on a prototype model to study the performance of the proposed system.

The variation of the zero reading of the instrument (IVD ratio at zero differential pressure) is monitored over a working day. The results are given in Table I. The variation in the reading is primarily due to changing temperature gradients, tilts, and other random effects.

The zero reading of the instrument is taken repeatedly after applying and removing different values of differential pressure. The scatter in these readings is expected to be due to varying capillary depressions, varying fringing pattern, and motion-induced electrostatic charge generation. The results are given in Table II.

The extent of uniformity of the value of K along the length of the tubes is indirectly checked by monitoring the balance conditions at zero differential pressure when mercury is slowly withdrawn, altering the level of mercury in both tubes by the same amount. The readings provide a measure of nonlinearities due to dimensional or dielectric variations. The results are tabulated in Table III.

The constant of the instrument viz., $\delta p/\delta x = 4\rho g L$ is 56705 Pa as noted in Tables I and IV. Even though the instrument is designed for pressure measurement without reference to any other standard, it was felt desirable to compare its readings with those of a readily available mechanical type micromanometer to provide a rough check. The readings are tabulated in Table IV.

V. DISCUSSION

The prototype instrument has a resolution of about 57 mPa, corresponding to the sixth decimal position of the IVD ratio. In principle the resolution can be increased by avoiding all kinds of vibrations at the test site and having a better detection scheme.

Experiments conducted on the prototype show that there is uncertainty in the IVD ratio beyond the fifth decimal place for the experimental conditions met with. On this basis the uncertainty in the measured value of the differential pressure is estimated to be 0.6 Pa + 0.02% of the indicated pressure on a conservative basis. The instrument therefore yields good accuracy at medium pressures. If the zero value is reset before a reading is taken, the percentage error even at low pressures would be quite low. It is believed that the errors can be further reduced by employing a larger manometer tube and better fabrication methods, and that the proposed scheme can be developed further to serve as a primary standard for the measurement of low and medium differential pressures.

1 Wasim Ahmad and M. Khan, IEEE Trans. Vol. IECI-25, 26–29 (1978).
2 A. E. Fribance, *Industrial Instrumentation Fundamentals* (McGraw–Hill, New York, 1962), pp. 247–286.
3 P. L. M. Heydemann, C. R. Tilford, and R. W. Hyland, J. Vac. Sci. Technol. **14**, 597–605 (1977).
4 K. F. Poulter and P. J. Nash, J. Phys. E **12**, 931–936 (1979).
5 M. Rehman and V. G. K. Murti, J. Phys. E (to be published).
6 A. M. Thompson, Proc. IEE **106B**, 307–310 (1959).
7 W. H. Clothier, Metrologia **1**, 35–56 (1965).
8 G. H. Raynor, IEEE Trans. Vol. IM-21, 361–365 (1972).
9 Kirk–Othmer Encyclopedia of Chemical Technology, Vol. 9, p. 708 (1967).
10 M. T. Yothers, editor, *Standards and Practices for Instrumentation*, 5th ed., Manometer Tables (Instrument Society of America, Pittsburgh, 1977), p. 6.

11 The authors are thankful to the reviewer who suggested these experiments for an evaluation of the instrument.