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New branches of non-supersymmetric attractors in $N = 2$ supergravity

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ABSTRACT

In this paper we analyse non-supersymmetric single centred extremal black hole solutions in $N = 2$ supergravity theory coupled to n vector multiplets with purely cubic pre-potential in four dimensions. We consider the algebraic attractor equations in their most general form at the black hole horizon. We explicitly construct a new class of solutions for these attractor equations. These solutions are characterised by a set of involutory matrices. These involutions are obtained from a constraint involving the parameters in the pre-potential and generate new attractor points in the moduli space.

1. Introduction

The attractor mechanism plays a central role in understanding the macroscopic origin of black hole entropy in gravity theories coupled to scalar fields [1]. The mechanism shows that the scalar fields must run into a fixed point at the horizon irrespective of the value they take at the asymptotic infinity, with their values at the fixed point being entirely determined by the black hole charges. This explains why the black hole entropy is only a property of its horizon and must be independent of the asymptotic data involving the scalar fields.

The attractor mechanism has first been realised for supersymmetry preserving black holes in the context of four dimensional $N = 2$ supergravity coupled with arbitrary number of vector multiplets [2]. Soon it has been generalised for dyonic black holes [3]. Various aspects of the mechanism have been studied subsequently. One issue of great importance pertaining to the attractor mechanism is the existence of non-supersymmetric attractors [4,5]. Subsequently, it has been shown that the attractor mechanism is really a consequence of extremality of the black holes. An extremal black hole may or may not be supersymmetric, however it always exhibits the attractor behaviour.

One of the reasons the attractor mechanism plays an important role in understanding black hole entropy is the uniqueness of these attractors [6]. Though a given single centre charge configuration appears to admit a unique supersymmetric attractor, it is not always the case. For example, the five dimensional supergravity admits multiple basin of supersymmetric attractors [7]. This, of course, depends on the topology of the moduli space of scalar fields coupled to the gravity multiplet [8]. For supersymmetric attractors in four dimensions, a classification of charged orbits has been carried out when the moduli space is a symmetric space [9]. More recently the uniqueness issue of supersymmetric attractors carrying $D0-D4-D6$ charges has been investigated in detail [10]. A classification of all the supersymmetric solutions has been carried out for the above charge configuration. For this class of black holes it has been shown that there exist domains in the charge lattice such that the attractor solution is unique in a given domain. The moduli space metric becomes degenerate at the boundaries of these domains and hence single centre black hole ceases to exist at these boundaries. However the black hole undergoes a kind of phase transition as one changes the values of the charges from one domain to the other. The functional form of the attractor point as well as the entropy changes as well.

The non-supersymmetric attractors are very similar to their supersymmetric counterparts in a number of aspects [11]. For example, the functional forms of the respective entropies are identical for a given charge configuration. In the case of axion free attractors, there is a set of first order flow equations determining the exact behaviour of the black hole in space–time [12,13]. These equations are obtained upon the extermization of a fake superpotential which is analogous to the central charge for supersymmetric black holes. Thus it is worth asking if there exist analogous results in the case of non-supersymmetric attractors.

The goal of the paper is to explore these new branches in non-supersymmetric extremal black holes. In the next section we will review the required background to study these solutions. In §3 we will briefly outline the previously known extremal configurations. Subsequently in §4 we will analyse the attractor equations and will solve them with a specific ansatz. Finally, we will summarise our results in §5.

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2. The model

In the present work we will focus $N=2$ supergravity theory in four dimensions coupled to n abelian vector multiplets. The bosonic part of the Lagrangian density is given by

$$\mathcal{L} = -\frac{R}{2} + g_{a\bar{b}}\partial_\mu x^a \partial_\nu \bar{x}^{\bar{b}} h^{\mu\nu} - \mu_{\Lambda\Sigma}\mathcal{F}^{\Lambda}_{\mu\nu}\mathcal{F}^{\Sigma}_{\lambda\rho}h^{\mu\lambda}h^{\nu\rho} - \nu_{\Lambda\Sigma}\mathcal{F}^{\Lambda}_{\mu\nu} * \mathcal{F}^{\Sigma}_{\lambda\rho}h^{\mu\lambda}h^{\nu\rho} . \quad (2.1)$$

We use the standard notations and conventions as in [4] to describe the system. In particular, we use $h_{\mu\nu}$ to denote the four dimensional space–time metric with R being the corresponding Ricci scalar. The complex scalars x^a parametrise the moduli space for n scalar fields in the vector multiplet and $g_{a\bar{b}}$ is the metric on it. $\mathcal{F}^{\Sigma}_{\lambda\rho}$ is the field strength for the gauge fields $\mathcal{A}^{\Sigma}_{\mu}$. The indices Λ, Σ take $n+1$ values due to the presence of an additional gauge field coming from the gravity multiplet. The gauge couplings $\mu_{\Lambda\Sigma}$ and $\nu_{\Lambda\Sigma}$ are derived from the $N=2$ pre-potential F. In this paper we will entirely focus on the purely cubic pre-potential.

For static, spherically symmetric configurations carrying dyonic charges (p^Λ, q_Σ) the system reduces to an effective one dimensional theory with the Lagrangian density:

$$\mathcal{L}(U, x^a(\tau), \bar{x}^a(\tau)) = \left(\frac{dU}{d\tau}\right)^2 + g_{a\bar{b}}\frac{dx^a}{d\tau}\frac{d\bar{x}^b}{d\tau} + e^{2U}V_{\text{eff}} , \quad (2.2)$$

with the corresponding Hamiltonian density being constrained to vanish [4]. Here U is the warp factor appearing in the space–time metric:

$$ds^2 = e^{2U(\tau)}dt^2 - e^{-2U(\tau)}(d\vec{x})^2 , \quad (2.3)$$

and τ is the inverse of the radial separation $\tau = 1/r$. The effective black hole potential V_{eff} is determined in terms of the Kähler potential K and the superpotential W which in turn are derived from the pre-potential F by:

$$K = -\ln\left(i\sum_{\Lambda=0}^{n}\left[\bar{X}^\Lambda \partial_\Lambda F(X) - X^\Lambda \bar{\partial}_\Lambda \bar{F}(X)\right]\right) , \quad (2.4)$$

and

$$W = \sum_{\Lambda=0}^{n}(q_\Lambda X^\Lambda - p^\Lambda \partial_\Lambda F). \quad (2.5)$$

The superpotential W is related to the central charge Z by $Z = e^{K/2}W$. Note that the physical scalar fields x^a $(a = 1, \ldots, n)$ appearing in the effective one dimensional Lagrangian (2.2) as well as in the supergravity Lagrangian (2.1) are given in terms of the symplectic sections X^Λ as $x^a = X^a/X^0$. The effective potential V_{eff} has the expression [4]:

$$V_{\text{eff}} = e^K\left[g^{a\bar{b}}\nabla_a W (\nabla_b W)^* + |W|^2\right], \quad (2.6)$$

where the action of the Kähler covariant derivative on W is given by $\nabla_a W \equiv \partial_a W + \partial_a K W$.

The supersymmetric attractors are obtained by extremising the central charge. The condition can be expressed in terms of the superpotential W as

$$\nabla_a W = 0 . \quad (2.7)$$

The supergravity theory however admits more general black hole configurations. For extremal black holes, existence of a regular horizon requires that the effective potential V_{eff} is extremized on it. For the effective potential (2.6) this condition can explicitly be stated as [11]:

$$g^{b\bar{c}}\nabla_a\nabla_b W\overline{\nabla_c W} + 2\nabla_a W\overline{W} + \partial_a g^{b\bar{c}}\nabla_b W\overline{\nabla_c W} = 0 . \quad (2.8)$$

Clearly, the supersymmetric configurations do satisfy the above equation. However there is a possibility more general configurations exist, which solve (2.8) and for which $\nabla_a W \neq 0$. Such non-supersymmetric extremal black hole attractors have been explored extensively during the past decade and their properties have been studied in detail. In the remaining part of this paper we will examine the equations of motion (2.8) more carefully and find some new solutions which were previously unknown.

3. Extremal solutions

In the present work we will entirely focus on $N=2$ supergravity theories with the purely cubic pre-potential:

$$F = D_{abc}\frac{X^a X^b X^c}{X^0} \quad (3.1)$$

The parameters D_{abc} are totally symmetric and take arbitrary values in general. This pre-potential takes a prominent role because of its appearance in large volume compactification of type IIA string theory on a Calabi–Yau manifold $\mathcal{M}$. In this case the parameters D_{abc} are no longer arbitrary and are given in terms of the triple intersection numbers of $\mathcal{M}$:

$$D_{abc} = \frac{1}{6}\int_{\mathcal{M}} \alpha_a \wedge \alpha_b \wedge \alpha_c , \quad (3.2)$$

with $\{\alpha_a\}$ denoting a basis of the integral cohomology group $H^2(\mathcal{M}, \mathbb{Z})$.

We will focus on configurations carrying $\{q_0, p^a, p^0\}$ charges. From the string theory point of view these will correspond to $D0-D4-D6$ configurations carrying q_0 number of $D0$-branes, p^a number of $D4$-branes wrapping four cycles dual to α_a and p^0 number of $D6$ branes wrapping $\mathcal{M}$.

For convenience we will set $x^a = X^a/X^0$ and choose the gauge $X^0 = 1$. With this choice of the gauge, the Kähler potential K and the superpotential W, for the above configuration, respectively, have the expressions

$$K = -\ln\left(-iD_{abc}(x^a - \bar{x}^a)(x^b - \bar{x}^b)(x^c - \bar{x}^c)\right) , \quad (3.3)$$

and

$$W = (q_0 - 3D_{ab}x^a x^b + p^0 D_{abc}x^a x^b x^c) . \quad (3.4)$$

This configuration admits the well known supersymmetry preserving solution [14],

$$x^a = p^a t$$

with

$$t = \frac{1}{2D}\left(-p^0 q_0 \pm i\sqrt{q_0(4D - (p^0)^2 q_0)}\right) . \quad (3.5)$$

Here we use the notation $D = D_{abc}p^a p^b p^c$. For the attractor solution to be non-singular, we require $q_0(4D - (p^0)^2 q_0) > 0$.

The attractor equation (2.8) however admits a more general extremal solution. The existence of such non-supersymmetric attractors was first investigated in [11] with the ansatz $x^a = p^a t$. The real and imaginary parts of t are given respectively by

$$t_1 = \begin{cases} \dfrac{2}{s}\dfrac{\left(1+\frac{p^0}{s}\right)^{1/3} - \left(1-\frac{p^0}{s}\right)^{1/3}}{\left(1+\frac{p^0}{s}\right)^{4/3} + \left(1-\frac{p^0}{s}\right)^{4/3}} & |\frac{s}{p^0}| > 1 \\[2ex] \dfrac{2}{p^0}\dfrac{\left(1-\frac{s}{p^0}\right)^{1/3} + \left(1+\frac{s}{p^0}\right)^{1/3}}{\left(1-\frac{s}{p^0}\right)^{4/3} + \left(1+\frac{s}{p^0}\right)^{4/3}} & |\frac{s}{p^0}| < 1 , \end{cases} \quad (3.6)$$

and

$$t_2 = \begin{cases} \frac{4s}{(s^2-(p^0)^2)^{1/3}\left((s+p^0)^{4/3}+(s-p^0)^{4/3}\right)} & |\frac{s}{p^0}| > 1 \\ \frac{4s}{((p^0)^2-s^2)^{1/3}\left((|p^0|+s)^{4/3}+(|p^0|-s)^{4/3}\right)} & |\frac{s}{p^0}| < 1 \,. \end{cases} \quad (3.7)$$

Here we introduced the variable $s = \sqrt{(p^0)^2 - \frac{4D}{q_0}}$ for convenience. Note that the above non-supersymmetric solution is non-singular provided $q_0(4D - (p^0)^2 q_0) < 0$.

4. New branches

More recently the supersymmetric conditions for black holes carrying $D0 - D4 - D6$ charges were analysed in more detail [10]. It was realised that the configuration described by (3.5) is not the most general solution for supersymmetric attractor carrying these charges. There exists a family of solutions determined by involutory matrices $I^a{}_b$ satisfying

$$D_{abc} I^b{}_e I^c{}_f = D_{aef} \,. \quad (4.1)$$

The most general solution for (2.7) is given by $x^a = x_1^a + i x_2^a$ with

$$x_1^a = \frac{1}{p^0}\left(p^a - \frac{D - \frac{1}{2} q_0 {p^0}^2}{D_c I^c{}_d p^d} I^a{}_b p^b\right), \quad (4.2)$$

$$x_2^a = \frac{1}{p^0}\left(1 - \left(\frac{D - \frac{1}{2} q_0 {p^0}^2}{D_c I^c{}_d p^d}\right)^2\right)^{1/2} I^a{}_b p^b \,. \quad (4.3)$$

For all these solutions the charges must satisfy $q_0(4D - (p^0)^2 q_0) > 0$. However, this is not the only criteria for the existence of a smooth solution. A more fundamental requirement is the positive definiteness of the moduli space metric at the attractor point. This requirement divides the charge lattice into several domains and different involutions give rise to unique attractor configurations in each such domain of the charge lattice [10].

In the following we will derive analogous solutions for the non-supersymmetric attractors. We will first obtain the equations of motion in its general form for the pre-potential (3.1) and then analyse them to obtain specific solutions. Let us now compute various terms in (2.8). We write, $x^a = x_1^a + i x_2^a$, and introduce the notation $D_{ab} = D_{abc} p^c$, $D_a = D_{ab} p^b$, $\nu_{ab} = D_{abc} x_2^c$, $\nu_a = \nu_{ab} x_2^b$, $\nu = \nu_a x_2^a$ for convenience. We further introduce the variable $\omega^a = (p^a/p^0) - x_1^a$ and define $\mu_{ab} = D_{abc}\omega^c$, $\mu_a = \mu_{ab}\omega^b$, $\mu = \mu_a \omega^a$ for easy reading of the equations. The superpotential W can now be expressed as

$$W = X_1 + i Y_1 \quad (4.4)$$

with

$$X_1 = q_0 - \frac{2D}{(p^0)^2} + 3\frac{D_a \omega^a}{p^0} + 3p^0 \nu_a \omega^a - p^0 \mu$$
$$Y_1 = -p^0 \nu - \frac{3 D_a x_2^a}{p^0} + 3p^0 \mu_a x_2^a \quad (4.5)$$

The covariant derivative of the superpotential $\nabla_a W$ is given by

$$\nabla_a W = \frac{3}{2}\left(\left(-\frac{2D_a}{p^0} + 2p^0 \mu_a - 2p^0 \nu_a - \frac{\nu_a}{\nu} Y_1\right) + i\left(-4p^0 \nu_{ab}\omega^b + \frac{\nu_a}{\nu} X_1\right)\right) \quad (4.6)$$

The supersymmetric solutions are obtained by setting the real and imaginary parts of $\nabla_a W$ to zero. The most general solution to these equations is given by eqs. (4.2) and (4.3). For non-supersymmetric configurations we also need to compute $\nabla_a \nabla_b W$. The real and imaginary parts of the above quantity are given respectively by

$$\frac{3}{2\nu}\left(\nu_{ab} - 3\frac{\nu_a \nu_b}{\nu}\right) X_1 + \frac{9p^0}{\nu}(\nu_a \nu_{bc} + \nu_b \nu_{ac})\omega^c - 6p^0 \mu_{ab}$$

and

$$6p^0 \nu_{ab} + \frac{3}{2\nu}\left(\nu_{ab} - 3\frac{\nu_a \nu_b}{\nu}\right) Y_1 - \frac{9}{2\nu}\left(\frac{1}{p^0}(\nu_a D_b + \nu_b D_a) + 2p^0 \nu_a \nu_b - p^0(\mu_a \nu_b + \mu_b \nu_a)\right)$$

We also need the inverse of the moduli space metric and its derivative:

$$g^{b\bar{c}} = -\frac{2\nu}{3}\left(\nu^{bc} - \frac{3}{\nu} x_2^b x_2^c\right)$$
$$\partial_a g^{b\bar{c}} = i\left(\nu_a \nu^{bc} \frac{\nu}{3} D_{ade} \nu^{bd} \nu^{ce} - \delta_a^b x_2^c - \delta_a^c x_2^b\right)$$

We now substitute the above expressions in the equations of motion (2.8). After a straightforward, but tedious computation we obtain:

$$3(p^0)^2 \nu_a \nu_b \omega^b + \nu_{ab}\omega^b p^0 (Y_1 - p^0 \nu) + \left(X_1 - 3p^0 \nu_b \omega^b\right)\left(D_a/p^0 - p^0 \mu_a\right) + p^0 \nu \nu^{bc} \mu_{ab}\left(D_c/p^0 - p^0 \mu_c\right) = 0 \,, \quad (4.7)$$

for the real part of the equations of motion, and

$$\nu_a\big(2X_1^2 - 12p^0 X_1 \nu_c \omega^c + 36(p^0)^2(\nu_c\omega^c)^2 + Y_1^2 - (p^0)^2\nu^2 + 2p^0 Y_1 \nu\big) + 2Y_1\nu\left(D_a/p^0 - p^0\mu_a\right) - \nu^2 \nu^{bd}\nu^{ce} D_{ade}\left(D_b/p^0 - p^0 \mu_b\right)\left(D_c/p^0 - p^0\mu_c\right) - 24(p^0)^2 \nu \nu_{ab}\omega^b \nu_c \omega^c + 2\nu^2\big(D_a + (p^0)^2 \mu_a\big) = 0. \quad (4.8)$$

for the imaginary part. This is the most general form of the non-supersymmetric equation of motion. Because of the complicated structure it is extremely hard to obtain the most general solution for the above equations. However, taking the clue from the existing solutions for their supersymmetric counter part we can look for appropriate ansatz to construct a class of new non-supersymmetric solutions for the above equations. We set

$$x_2^a = I^a{}_b p^b x \text{ and } \omega^a = I^a{}_b p^b \omega \,, \quad (4.9)$$

where the involution $I^a{}_b$ is assumed to satisfy (4.1).[1] Substituting the above in eqs. (4.7) and (4.8) we find, after a bit simplification:

$$2(p^0)^3 \chi x^4 \omega + (1 - (p^0)^2\omega^2)(X_1 - 2p^0 \chi x^2 \omega) + (p^0)^2 x\, \omega Y_1 = 0 \quad (4.10)$$

$$2x^2 X_1^2 - 12p^0 X_1 \chi x^4 \omega + 12(p^0)^2 \chi^2 x^6 \omega^2 + x^2 Y_1^2 - (p^0)^2 \chi^2 x^8 + 2p^0 \chi Y_1 x^5 + 2\chi^2 x^6 + 2(p^0)^2 \chi^2 x^6 \omega^2 + \left(2Y_1/p^0\right)\chi x^3 (1 - (p^0)^2\omega^2) - \left(\chi/p^0\right)^2 x^4 (1 - (p^0)^2 \omega^2)^2 = 0 \quad (4.11)$$

We reproduce the expressions for X_1 and Y_1 after substituting the ansatz (4.9) in (4.5):

[1] Note that it is not possible to redefine the charges p^a to get rid of the $I^a{}_b$ dependence because of the shift involved in defining ω^a.

$$X_1 = q_0 - (2D/(p^0)^2) + (3\chi/p^0)\,\omega + 3\chi\, p^0 \omega x^2 - p^0 D\omega^3$$
$$Y_1 = -\,(3\chi/p^0)\, x\big(1-(p^0\omega)^2\big) - p^0\chi x^3 \,.$$

Here for easy reading of the equations we have defined $\chi = D_a I^a{}_b p^b$. This gives a considerable simplification as we need to solve them only for the variables x and ω in terms of the quantities p^0, q_0, D and χ. These equations can further be simplified by noting that they contain only even powers of x. Setting $x^2 = y$ and eliminating y and ω respectively we find the following factorized form:

$$f_1(\omega) f_3(\omega) F_3(\omega) = 0\,, \tag{4.12}$$

and

$$g_1(y) g_3(y) G_3(y) = 0\,. \tag{4.13}$$

Here $f_k(\omega), g_k(y)$ are polynomials of degree k with respect to their arguments. Their explicit expressions are given by

$$f_1(\omega) = 2\chi p^0 \omega + (p^0)^2 q_0 - 2D\,,$$
$$g_1(y) = 4\chi^2 (p^0)^2 y - \hat{s}\,,$$

and

$$f_3(\omega) = (2\chi^2 + \hat{s})(p^0)^3\omega^3 - 3\chi (p^0)^2\big(2D - (p^0)^2 q_0\big)\omega^2 + 6\chi^2 p^0 \omega - \chi\big(2D - (p^0)^2 q_0\big)$$
$$g_3(y) = \chi^2 (p^0)^6\big(2\chi^2 + \hat{s}\big)^2 y^3 - 9\chi^4 (p^0)^4 \hat{s} y^2 + 6\chi^2 (p^0)^2 (\hat{s})^2 y - (\hat{s})^3$$

with $\hat{s} = \big(2D - (p^0)^2 q_0\big)^2 - 4\chi^2$. Solving the linear equations $f_1(\omega) = 0 = g_1(y)$ gives rise to the supersymmetric attractors described in [10]. We will now focus on the cubic polynomials. The discriminants of $f_3(\omega)$ and $g_3(y)$ are given by $-27\chi^2(\hat{s})^3(p^0)^6$ and $-27\chi^4(p^0)^{12}(\hat{s})^7(2\chi^2+\hat{s})^2\big(2D-(p^0)^2 q_0\big)^2$ respectively. Both become negative for $\hat{s} > 0$ and hence $f_3(\omega) = 0 = g_3(y)$ admit unique real valued solutions for ω and y. It is straightforward to verify that the resulting ω, y indeed provides a non-susy solution for the equations of motion (4.10), (4.11). Further, it can be verified that the cubic polynomials $F_3(\omega)$ and $G_3(y)$ do not provide any solution for the equations of motion.

To express the non-supersymmetric solution orderly in a closed form we will make the following rescaling of the variables:

$$\omega \to \tilde{\omega}/p^0,\; y \to \tilde{y}/(p^0)^2,\; q_0 \to (\tilde{q}\chi + 2D)/(p^0)^2\,. \tag{4.14}$$

The equations $f_3(\omega) = 0 = g_3(y)$ now take the simple form

$$(\tilde{q}^2 - 2)\tilde{\omega}^3 + 3\tilde{q}\tilde{\omega}^2 + 6\tilde{\omega} + \tilde{q} = 0$$
$$(\tilde{q}^2 - 2)^2\tilde{y}^3 - 9(\tilde{q}^2 - 4)\tilde{y}^2 + 6(\tilde{q}^2 - 4)^2\tilde{y} - (\tilde{q}^2 - 4)^3 = 0$$

and the corresponding attractor solution is given by

$$\tilde{\omega} = \frac{f_-(\tilde{q}) - f_+(\tilde{q}) - 2^{1/3}\tilde{q}}{2^{1/3}(\tilde{q}^2 - 2)}\,, \tag{4.15}$$

$$\tilde{y} = \frac{g_+(\tilde{q}) - g_-(\tilde{q}) + 2^{1/3}3(\tilde{q}^2 - 4)}{2^{1/3}(\tilde{q}^2 - 2)^2}\,, \tag{4.16}$$

with

$$f_\pm(\tilde{q}) = \big((\tilde{q}^2 - 2)(\tilde{q}^2 - 4)^{3/2} \pm \tilde{q}(\tilde{q}^2 - 4)^2\big)^{1/3}\,,$$
$$g_\pm(\tilde{q}) = (\tilde{q}^2 - 4)\left(\tilde{q}(\tilde{q}^2 - 2)^3\sqrt{\tilde{q}^2 - 4} \pm (\tilde{q}^8 - 8\tilde{q}^6 + 6\tilde{q}^4 + 40\tilde{q}^2 - 2)\right)^{1/3}\,.$$

The non-supersymmetric attractors can now be constructed from the above using (4.9) and the rescaling (4.14).

5. Conclusion

In this paper we have studied non-supersymmetric attractors in four dimensional $N = 2$ supergravity coupled to n vector multiplets with the purely cubic pre-potential. We have expressed the most general form of the equations of motion in terms of a set of convenient variables involving the moduli fields (ω^a and x_2^a). We have used a generalized ansatz involving a constrained involutory matrix to solve the equations of motion. This gives rise to new branches of non-supersymmetric attractors for every consistent choice of involutions.

It was possible to obtain an exact analytic expression for the solution because of the factorization in (4.12) and (4.13). It would be interesting to see if it is possible to obtain an analogous expression without assuming any ansatz for the moduli. This will help in classifying all non-supersymmetric attractors in these type of supergravity theories. It would also be interesting to consider the flow equations and obtain generalized attractor equations to solve them. A first step towards this would be to construct a fake superpotential for these non-supersymmetric attractors. Incorporating stringy corrections to the pre-potential too gives rise to rich structures. An issue of greater import is to look into the microscopic description of these new branches of attractors in both supersymmetric as well as non-supersymmetric cases. Localization proves to be a powerful technique to obtain the exact partition function which captures sub-leading corrections to the entropy. It is worth exploring whether it can be used to understand the origin of these new branches in $N = 2$ theories.

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