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New approach of measuring the Q factor of a microwave cavity using the cavity perturbation technique

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A new approach of measuring the quality factor of the reflection type microwave cavity resonator for calculating the complex dielectric permittivity of solid and liquid samples in the cavity perturbation technique is proposed. This approach, based on the measurement of reflected power from the cavity at resonance, effectively reduces the measurement procedure and increases the accuracy. As the quality factor can be measured very fast using this approach, this approach can be extended to monitor the conductivity decay of the semiconductor samples.

I. INTRODUCTION

Complex dielectric permittivity at microwave frequencies is measured using several methods based on reflection/transmission[1-3] and cavity resonance techniques.[4] In the cavity resonance technique two methods are available: one is the fixed cavity perturbation[5-7] and the other is the adjustable plunger cavity technique.[8] The former method is suitable for all types of samples. The latter method is suitable for liquid samples.

The cavity perturbation technique is sensitive and simple for the measurement of complex dielectric permittivity of solids and liquids. In this technique, the resonance curve is plotted and the resonance frequency (f) and the full width at half-maximum (FWHM; Δf) are measured for an empty as well as sample loaded cavity. The quality factor (Q) of the cavity, defined as $f/\Delta f$,[9] is also measured for an empty as well as a sample loaded cavity. The change in the resonance frequency and quality factor with and without the sample gives the complex dielectric permittivity. The calculation of the quality factor from the measurement of FWHM is a conventional process and takes a considerable amount of time. In order to avoid this lengthier process, this paper explains a new approach of measuring the quality factor of the cavity by observing the reflected power at resonance. This approach is simple and quick, and is suitable for studying the temperature variation of complex dielectric permittivity of solids, liquids, and liquid mixtures. Also, this approach helps us to monitor the minority carrier lifetime and conductivity of semiconductors. In the case of semiconductors, the microwave conductivity is calculated from the dielectric loss derived from the change in the quality factor. Photoconductivity or excess carrier lifetime in semiconductors can be observed by the change in quality factor while shining a light of energy greater than the band gap. As the conductivity of the sample increases due to the irradiation, the quality factor decreases. When the light is cut off, the conductivity decays exponentially and hence the quality factor increases. The measure of this decay directly gives the lifetime of the minority carrier. But it is not possible to monitor the change in the quality factor within the sampling time of 100 ns. The new approach makes this possible and also holds well for other solids and liquids.

II. THEORY

In general, the transfer function for a cavity resonator can be written as[10]

$$T(\omega) = 1/(\omega_0^2 - \omega^2 + j\omega\omega_0/Q), \qquad (1)$$

where ω_0 is the resonant radian frequency and Q is the quality factor. Writing in terms of frequency and magnitude of the transfer function, one gets

$$|T(f)| = (f_0^2/Q)[(f_0^2 - f^2)^2 + (ff_0/Q)^2]^{(-1/2)}, \qquad (2)$$

where $|T(f)|$ is a bell shaped curve whose form is highly Q dependent, and f_0 is the resonance frequency. Equation (2) is also meant for transmission type cavity. After modification in the case of reflection type cavity, including the practical consideration of the nonzero value of reflected power at resonance due to the coupling factor and walls of the cavity and also writing in terms of the power transfer function, the power transfer function for a reflection type cavity is written as

$$P(f) = 1 - (1-k)(f_0^2/Q)^2[(f_0^2 - f^2)^2 + (ff_0/Q)^2]^{(-1)}. \qquad (3)$$

Here k is the normalized reflected power at resonance. The area enclosing the power transfer function is always constant for a particular cavity-coupling hole combination in a fixed range of frequency. The limitation in the frequency range is due to the nonuniform power variation with frequency. The resonance frequency (f_0), quality factor (Q_0), and the normalized reflected power at resonance (k_0) are measured for an empty cavity or semiconductor loaded cavity at dark condition or cavity loaded with a standard sample. Q_0 is calculated using the conventional processes of measuring FWHM. This is taken as the reference. Using these data, the area under the resonance curve (A_0) is obtained by numerical integration of Eq. (3) for a fixed span of frequency:

$$A_0 = \sum P_0(f). \qquad (4)$$

Now the test sample is put inside the cavity or the conductivity of the semiconductor is changed by shining with light. The resonance frequency (f_s) and the normalized
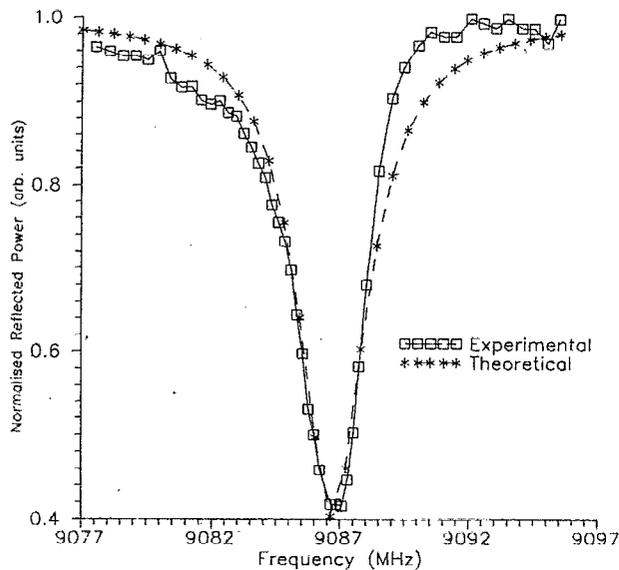
FIG. 1. Comparison between the resonance curve observed and the calculated curve based on Eq. (3).

reflected power at resonance (k_s) is measured. The area enclosing the new resonance curve (A_s) is equal to A_0:

$$A_s = \sum P_s(f) = A_0. \qquad (5)$$

Hence, the quality factor of the cavity loaded with a test sample (Q_s) is calculated by the substitution process. This approach is also extended for liquid mixtures and ferrites with suitable reference data. The quality factor (Q) referred to in this paper is the loaded quality factor.

III. EXPERIMENTAL PROCEDURE

All the experiments are performed using an X-band microwave bench. The microwave source is a X-13 Varian klystron. A rectangular reflection type TE_{105} mode cavity with a resonance frequency of 9.150 GHz is used. A suitable reference sample is chosen and loaded into the cavity at an electric field maximum. The resonance frequency, normalized reflected power at resonance, and the quality factor (by a conventional method of measuring FWHM) are noted. Using these reference parameters, the area enclosing the resonance curve (A_0) is calculated numerically for a frequency span of ± 20 MHz (in a step of 0.01 MHz) defined by Eqs. (3) and (4). The span may be increased if

TABLE I. Ferrite samples in the cavity with one of the ferrite as reference $(f_0 = 9.168\,10$ GHz: $k = 0.050;$ $Q_0 = 722.7)$.

f_s in GHz[a]	k_s[b]	Q_m[c]	Q_c[d]
9.167 97	0.052	720.7	720.3
9.168 26	0.054	717.6	717.6
9.168 19	0.054	717.6	717.7
9.168 16	0.055	715.9	715.7

[a] f_s: resonance frequency.
[b] k_s: the normalized reflected power at resonance.
[c] Q_m: Q measured using FWHM.
[d] Q_c: Q calculated based on Eq. (3).

TABLE II. (A) Samples are acetonitrile-benzene binary mixture in various proportions with sample holder as reference $(f_0 = 9.115\,40$ GHz; $k = 0.088;$ $Q_0 = 2532)$. (B) Samples are chlorobenzene-benzene binary mixture in various proportions with sample holder as reference $(f_0 = 9.115\,40$ GHz: $k = 0.1125;$ $Q_0 = 2532)$. (C) Samples are acetonitrile-carbon tetra chloride binary mixture of various proportions with sample holder as reference $(f_0 = 9.115\,40$ GHz: $k = 0.1125:$ $Q_0 = 2532)$. (D) Samples are dimethyl formamide-benzene binary mixture in various proportions with one of the mixture taken as reference $(f_0 = 9.122\,40$ GHz: $k = 0.213:$ $Q_0 = 2107)$.

	f_s in GHz[a]	k_s[b]	Q_m[c]	Q_c[d]
(A)	9.1018	0.338	1795	1806
	9.0983	0.463	1372	1407
	9.0947	0.556	1121	1107
	9.0910	0.619	926	903
	9.0886	0.65	806	799
(B)	9.1011	0.394	1569	1639
	9.0998	0.481	1398	1360
	9.0965	0.544	1058	1158
	9.0937	0.643	863	832
	9.0914	0.669	753	745
	9.1053	0.169	2365	2352
	9.1049	0.225	2168	2197
(C)	9.1026	0.306	1896	1917
	9.0947	0.537	1152	1178
	9.0937	0.631	874	871
(D)	9.1214	0.319	1734	1781
	9.1206	0.425	1380	1466
	9.1194	0.519	1131	1168
	9.1182	0.581	932	996
	9.1170	0.640	811	823

[a] f_s: resonance frequency.
[b] k_s: the normalized reflected power at resonance.
[c] Q_m: Q measured using FWHM.
[d] Q_c: Q calculated based on Eq. (3).

the FWHM exceeds 10 MHz. Now the test sample is loaded or the property of the sample (conductivity in the case of the semiconductor) is changed. This time, the normalized reflected power at resonance and resonance frequency only are measured. As the area remains constant, by using Eqs. (3)–(5), Q_s is calculated. In the case of semiconductor samples, the variation in the resonance frequency is not observed due to shining with light, hence measurement of reflected power at resonance itself is enough.

IV. ERROR ANALYSIS

The possible sources of error occur in the measurement of frequency and reflected power at resonance. The absolute error comes around ± 0.1 MHz in the frequency and ± 0.001 (in arbitrary units) in power measurement gives $\pm 3.5\% - 5\%$ error in calculating the quality factor using FWHM. But the area (A_0) calculated by the computation method is not much affected by the error in quality factor. The error in the area calculation is around $\pm 3\%$. Since this calculation of area is needed only once for a series of experiments, this does not affect the calculation of the quality factor of the sample loaded cavity. Considering all these factors, the net error in quality factor is around $\pm 3\%$ if there is a considerable shift (> 10 MHz) in the

TABLE III. Semiconductor samples, silicon (Si) and gallium arsenide (GaAs) under dark and illumination conditions. The reference is taken with sample under dark condition.

Sample	f_0[a]	k_0[b]	Q_0[c]	k_s[d]	Q_m[e]	Q_c[f]
Si	9.114 93	0.286	988	0.298	964	964
Si	9.118 18	0.419	1054	0.426	1036	1033
GaAs	9.072 05	0.209	947	0.213	935	940

[a] f_0: resonance frequency with sample (dark and illumination).
[b] k_0: normalized reflected power at resonance under dark condition.
[c] Q_0: Q factor measured using FWHM with sample under dark condition.
[d] k_s: normalized reflected power at resonance under illumination.
[e] Q_m: Q factor measured using FWHM with sample under illumination.
[f] Q_c: Q factor calculated based on Eq. (3) with sample under illumination.

resonance frequency with and without sample. This is due to the fact that the power output of the klystron is highly frequency dependent. Though the power measurements are normalized, more errors creep in the experimental procedure. In the case of semiconductors no variation in the resonance frequency is observed and hence the error in the quality factor considerably reduces to ±1%.

V. RESULTS AND DISCUSSION

Figure 1 shows the comparison between the experimentally observed cavity resonance curve and the curve defined by Eq. (3). The parameters, such as resonance frequency, normalized reflected power, and quality factor, used for obtaining the theoretical curve are taken from the experimental curve itself. It may be observed that the two curves are almost close.

Tables I–III present the results obtained on some solid, liquid, and semiconductor samples. Table I gives the result obtained for some ferrite samples. The reference sample is also a ferrite sample. The reference parameters used to calculate the area (A_0) is also presented along with the table caption. Tables II A–II D are for liquid mixtures. Here the reference is the sample holder. Table III is for some semiconductor samples with light shining on them. The reference is taken with the sample in the dark condi-

tion. In all these tables Q_m refers to the Q factor measured using FWHM, where Q_c refers to the Q factor calculated using f_s, k_s, and A_0. Equations (3)–(5) are used in the calculation of Q_c.

This new approach provides a simple and quick measurement procedure. In the case of measurement of minority carrier lifetime of semiconductor samples, this is the most feasible method to use for experiments.

Using this approach, the complex dielectric permittivity of liquids and solids can be measured accurately. Comparing this method with other reflected power techniques[1,2] shows that the cavity perturbation technique is more precise. Although there is a disadvantage in positioning the sample at the electric field maximum every time, the speed and accuracy involved provides overall advantage. Hence this approach can be utilized for all types of samples. Also, automation of this cavity perturbation technique is possible without using sophisticated instruments like a vector network analyzer.

ACKNOWLEDGMENTS

[1] W. H. Surber Jr., J. Appl. Phys. 19, 514 (1948).
[2] M. Jeyaraj, A. Kumarasamy, and J. Sobhanadri, J. Phys. E 12, 1179 (1979).
[3] T. W. Dakin and C. N. Works, J. Appl. Phys. 18, 789 (1947).
[4] H. K. Henisch and J. Zucker, Rev. Sci. Instrum. 409 (1956).
[5] K. S. Champlin and R. R. Krongard, IRE Trans. Microwave Theory Tech. MTT-9, 545 (1961).
[6] V. R. K. Murthy and R. Raman, Solid State Commun. 70, 847 (1989).
[7] V. Subramanian, B. S. Bellubbi, and J. Sobhanadri, Pramana J. Phys. (India) 41, 9 (1993).
[8] V. Subramanian, B. S. Bellubbi, and J. Sobhanadri, Rev. Sci. Instrum. 64, 231 (1993).
[9] S. F. Adam, *Microwave Theory and Applications* (Prentice–Hall, Englewood Cliffs, NJ, 1969), p. 271.
[10] D. Kajfez and P. Guillon, *Dielectric Resonators* (Artech House, Norwood, MA, 1986), p. 12.