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Multi-objective optimum design of an aero engine rotor system using hybrid genetic algorithm

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Abstract. A working aero engine rotor system is subjected to multi-objective optimisation using Genetic Algorithm based optimisation. A Hybrid Genetic Algorithm (HGA) is introduced to reduce the weight and unbalance response of the rotor system with constrain on critical speed. The existing aero engine gear box casing vibration is found to be within the critical speed constraint and additional constraints are imposed to move the critical away from this zone. Bearing–pedestal model and Rayleigh damping model are used for accurate results. The optimisation resulted in Pareto optimal solutions and best solution selected using utopia point concept. The outcome of the paper is a comparative study which highlights the advantages of HGA over Controlled Elitist Genetic Algorithm (CEGA) and Goal Programming (GP).

1. Introduction

Aero engines are the backbone of an aircraft. Rotor dynamics analysis of the aero engine rotor system will ensure that unwanted vibrations are not hampering the satisfactory operation of the engine. Optimisation is introduced to rotor dynamics problems to improve the design of rotor systems. When more than one objective is to be addressed at a time, multi-objective optimisation is the preferred method. Initial studies on optimisation of the rotor system considered a single objective. Weight was optimized by programming the optimisation problem by converting it into a non-linear mathematical problem [1]. Further studies on weight optimisation introduced additional constraints and design variables [2]. Another weight optimisation problem discussed in detail the gyroscopic effects on optimisation [3]. All these studies employed weight as single objective and constraints are imposed for other potential design variables.

Simultaneously optimizing different design variables has its own advantage and researchers explored the same over time. The weighted function of vibration due to unbalance and stability limit is studied and support structure is modified to achieve positive results [4]. Pareto front is generated for the same problem to arrive at a set of alternate solutions for the designer [5]. Weight is combined with the force transmitted to bearing locations and multi-objective optimisation is carried out [6]. Immune Genetic Algorithm was applied to the same problem and resulted in better solutions [7]. This study explored the potential of the immune system and applied the same to the genetic algorithm. The initial convergence of the genetic algorithms is prevented and the search is extended to different optimal solutions. The emphasis of the algorithm was on the selection of the population. HGA used for the present problem concentrate on reducing the convergence time with improvements over the solutions obtained [8]. The novel elements of the present study are discussed here. An algorithm better equipped to reach optimum solutions in an efficient manner is introduced to multi-objective optimisation of an

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aero engine rotor system. The effectiveness of the algorithm is proved by comparing it with existing proven algorithms.

2. Optimisation Methods

Optimisation is the minimisation or maximisation of different functions by altering variables influencing the functions. Multi-objective optimisation results in Pareto optimal solutions which represent a trade-off between the objectives considered and are superior to all other possible solutions. Point close to the Utopia point, which represents minimum value of the objectives considering one at a time, is the optimum solution [9].

2.1. Controlled Elitist Genetic Algorithm(CEGA)

The theory about Genetic Algorithms is found in abundance in the literature and [10] is one of the best references among them. Non-dominated Genetic Algorithm (NSGA) is an algorithm to generate Pareto front for multi-objective optimisation and it works on the concept of non-domination of solutions [11]. The drawbacks such as difficulties associated with computation of solution, absence of elitism and the requirement of choosing an optimal parameter value for sharing parameter, led to the development of NSGA-II [12]. By incorporating control over the elite members, NSGA-II is modified and Controlled Elitist Genetic Algorithm is developed [8].

2.2. Goal Programming (GP)

The goal programming allows the user to set goals or targets for the objective functions and they are achieved satisfactorily [13]. Hence, this method is simpler compared to other traditional optimization algorithms, and results are attained quicker. Once the goals are set, the objective functions are transformed into constraints. The objective function is also restructured accordingly. Its restructuring methodology categorises the goal programming into scaled and unscaled versions. The present work uses the scaled version of the goal programming technique [14].

2.3. Hybrid Genetic Algorithm(HGA)

CEGA takes numerous function assessments to attain convergence although it reaches the area next to an optimal Pareto front comparatively fast. Hence CEGA is used for a diminutive number of generations to reach close to an optimum front and this solution is the initial point for goal programming solver that is quicker and proficient for a local search. The solutions obtained from HGA are combined with the existing population and a new Pareto front is obtained. Goals required for Goal Programming are provided by CEGA as the extreme points from the Pareto front established in previous run [8].

3. Rotor dynamics Model

The Rayleigh circular beam element is used for modeling the distributed shaft. Rigid disks representing the mass and inertia properties of compressor and turbines are attached to respective locations on shaft. Procedure for modeling is detailed in [15]. The single spool rotor system is made of stepped shaft carrying three disks and supported on two bearings. A segment of the turbines are selected as rotor and steel alloy hollow cylinder is used for connecting the two turbines. The schematic of the rotor system representing with the disks, bearings and their properties are shown in Figure-1. Initial configuration data and the material characteristics of the rotor are shown in Table-1.

Flexible pedestal of mass M_1 and stiffness of K_1 is connected to bearing of stiffness K_2.Stiffness of the bearing is in series with stiffness of pedestal. This forms four degrees of freedom system. This system is converted to two degrees of freedom system with stiffness K_{eq} as shown below:

$$K_{eq} = \begin{bmatrix} \dfrac{K_2\left(K_1 - \omega^2 M_1\right)}{K_2 + K_1 - \omega^2 M_1} & 0 \\ 0 & \dfrac{K_2\left(K_1 - \omega^2 M_1\right)}{K_2 + K_1 - \omega^2 M_1} \end{bmatrix} \tag{1}$$

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ω is the angular velocity of the rotor system.

The proportional damping model known as Rayleigh damping [16] shown below is used.

$$[C] = \alpha[M] + \beta[K_{shaft} + K_{eq}] \tag{2}$$

[C] is damping matrix, α and β are Rayleigh damping coefficients,*[K_{shaft}]* is the stiffness due to shaft.

Table 1. Initial design for distributed rotor

Element	inside diameter (m)	outside diameter (m)	length (m)	Material	Density (kg/m^3)	Elastic modulus (N/m^2)
1	0	0.020	0.02	Steel alloy	7760	$2.1\text{x}10^{11}$
2	0	0.020	0.025	Steel alloy	7760	$2.1\text{x}10^{11}$
3	0	0.014	0.028	Steel alloy	7760	$2.1\text{x}10^{11}$
4	0	0.020	0.022	Steel alloy	7760	$2.1\text{x}10^{11}$
5	0	0.015	0.0055	Steel alloy	7760	$2.1\text{x}10^{11}$
6	0	0.015	0.0055	Steel alloy	7760	$2.1\text{x}10^{11}$
7	0	0.020	0.07	Steel alloy	7760	$2.1\text{x}10^{11}$
8	0	0.015	0.0055	Steel alloy	7760	$2.1\text{x}10^{11}$
9	0	0.015	0.0055	Steel alloy	7760	$2.1\text{x}10^{11}$
10	0	0.020	0.01	Steel alloy	7760	$2.1\text{x}10^{11}$
11	0	0.040	0.015	Nimonic alloy	8180	$2.05\text{x}10^{11}$
12	0	0.040	0.01	Nimonic alloy	8180	$2.05\text{x}10^{11}$
13	0.020	0.050	0.02	Steel alloy	7860	$2.0875\text{x}10^{11}$
14	0	0.040	0.01	Nimonic alloy	8180	$2.05\text{x}10^{11}$
15	0	0.040	0.01	Nimonic alloy	8180	$2.05\text{x}10^{11}$

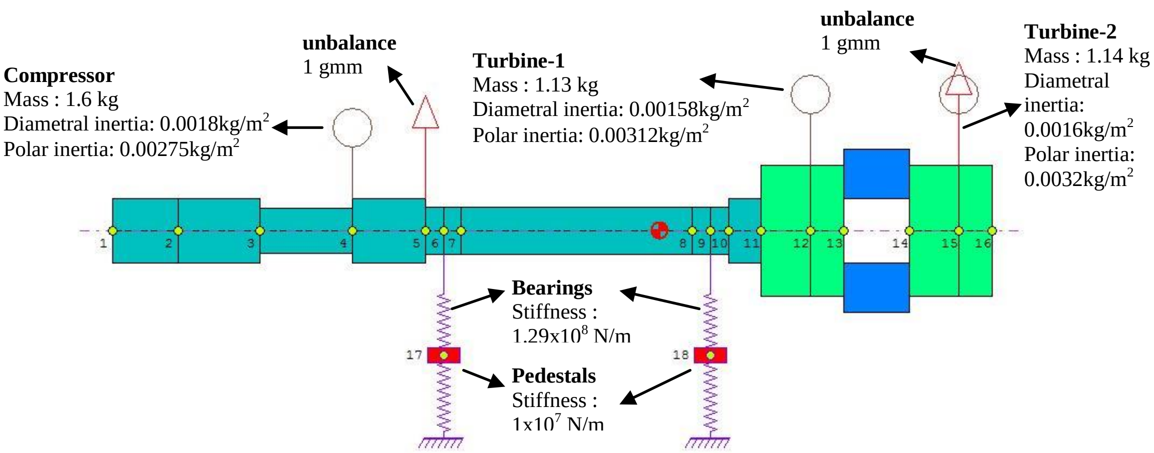

Figure 1. Rotor system schematic

Formulation of the multi-objective optimisation problem

3.1. Step-1: Selection of Pedestal stiffness of both bearing locations and diameters of the shaft as design variables

Design variables are selected based on numerical experiment. Diameters of the shaft at the bearing and disk locations are not considered as design variables to simplify the design modifications.

3.2. Step2: Selection of the second critical speed as constraint

$$\Omega_{low} \le \omega_2^c \le \Omega_{high} \tag{3}$$

ω_2^c is the second critical speed and Ω_{low} =1800 rad/s and Ω_{high} =3000 rad/s are the limiting frequencies due to self sustaining speed and idle speed. In this particular case the existing engine gear box casing vibration is found to be in at 420 Hz and it is decided to shift the critical away from this zone by keeping Ω_{high} at 2518 rad/s.

The equation of the rotor system in standard matrix form considering equations (1) and (2):

$$[M]\{\ddot{p}\}+(\alpha[M]+\beta[K]+\omega[G])\{\dot{p}\}+\lfloor K_{shaft}+K_{eq}\rfloor\{p\}=\{F\} \tag{4}$$

[M} is the mass matrix; *{p}* and *{F}* are the response and force vectors respectively.

Converting equation (4) into state space form,

$$\begin{bmatrix} (\alpha[M]+\beta\lfloor K_{shaft}+K_{eq}\rfloor+\omega[G]+\omega[M]) & [M] \\ [M] & 0 \end{bmatrix}\frac{d}{dt}\begin{Bmatrix} p \\ \dot{p} \end{Bmatrix}+\begin{bmatrix} \lfloor K_{shaft}+K_{eq}\rfloor & 0 \\ 0 & -[M] \end{bmatrix}\begin{Bmatrix} p \\ \dot{p} \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{5}$$

Solving equation (5) damped natural frequencies are obtained.

3.3. Step3:Selection of maximum amplitude of vibration due to unbalance and weight of shaft as the objective functions

Consider a solution of the form given below

$$\{p\}=\{p_s\}\cos\omega t+\{p_c\}\sin\omega t \tag{6}$$

Substituting in equation (6) in equation (4) yields

$$\begin{Bmatrix} p_c \\ p_s \end{Bmatrix}=\begin{bmatrix} ([K_{shaft}+K_{eq}]-\omega^2[M]) & \omega^2[G]-\omega[\alpha[M]+\beta[K_{shaft}+K_{eq}]] \\ -\omega^2[G]+\omega[\alpha[M]+\beta[K_{shaft}+K_{eq}]] & ([K_{shaft}+K_{eq}]-\omega^2[M]) \end{bmatrix}\begin{Bmatrix} F_C \\ F_s \end{Bmatrix} \tag{7}$$

Using the above solution, vibration amplitude due to unbalance is written in the form below,

$$p=\sqrt{(p_c^2+p_s^2)} \tag{8}$$

Weight of the rotor system is calculated using equation (9)

$$W(x)=\sum_i^n \frac{\rho_i * \pi * (do_i^{\,2}-di_i^2) * h_i}{4} \tag{9}$$

ρ is the density of shaft , *di* is the inner diameter, *do* the outer diameter and *h* the length of each element. *n* is the number of elements.

3.4. Step4: Deciding the bounding limits of design variables

$$\begin{aligned} &0.01 \le do_i \le 0.03 (i=1,2,4,7,10,13) \\ &0.01 \le di_i \le 0.03 (i=13) \\ &1\text{x}10^6 \le K_{eq} \le 1\text{x}10^9 \end{aligned} \tag{10}$$

4. Result and Discussions

The Pareto fronts generated by CEGA alone and by means of HGA are shown in Figure-2 and they are compared against each other using the spread and the average distance of the solutions. The average distance of the solutions on the Pareto front are enhanced by the use of hybrid function. The spread determines change in two fronts and that is superior for Pareto front generated by HGA. This indicates that the front has changed considerably from that obtained by CEGA with no hybrid function. It is clear that by using the HGA an optimal Pareto front is obtained but the diversity of the solution is reduced. This is pointed out by a higher value of the average distance measure and the spread of the front.

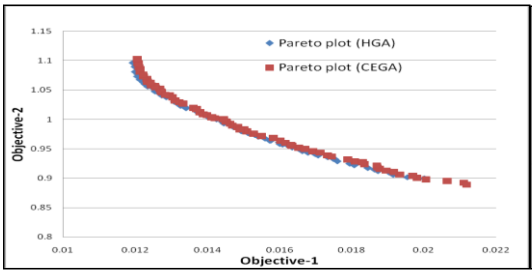

Figure 2. Pareto front for CEGA and HGA

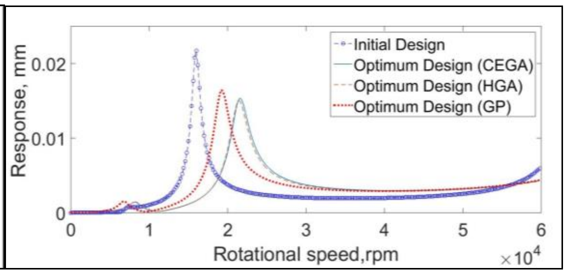

Figure 3. Unbalance responses of designs

The post optimal analysis is carried out to choose an optimal solution for implementing the results of optimisation. Utopia point method is used for the selection of a final solution. Amplitude of vibration for of initial design and optimum designs corresponding to HGA, CEGA and GP are shown in Figure-3. Table-2 and 3 represent the initial design and optimum designs corresponding to HGA, CEGA and GP objective function and constrain values.

Table 2. Different designs- Diameter of shaft element and bearing stiffness

Design variable	Element	Initial design	HGA	CEGA	GP
Outside diameter, m	1	0.020	0.0293	0.0280	0.0287
	2	0.020	0.0239	0.0252	0.0231
	4	0.020	0.0187	0.0179	0.0186
	7	0.020	0.0115	0.0115	0.0103
	10	0.020	0.0173	0.0169	0.0143
	13	0.050	0.0401	0.0401	0.0400
Inside diameter, m	13	0.020	0.0300	0.0299	0.0300
Pedestal Stiffness, N/m	Station 6	$1x10^7$	$4.73x10^8$	$4.73x10^8$	$4.0x10^8$
	Station 9	$1x10^7$	$3.37x10^8$	$2.57x10^8$	$2.5x10^8$

Table 3. Different designs- critical speed, shaft weight and amplitude of vibration

Parameter	Initial design	HGA	CEGA	GP
Critical speed, rpm	16500	19200	21600	21400
Amplitude of vibration, mm	$2.17x10^{-2}$	$1.50x10^{-2}$	$1.53x10^{-2}$	$1.64x10^{-2}$
Weight, kgf	1.120	0.978	0.975	0.950

Comparing with initial design,

- HGA optimum design: Amplitude reduced by 30.8% and the weight decreased by 12.6%.
- CEGA optimum design: Amplitude reduced by 29.4% and the weight decreased by 12.9%.
- GP optimum design: Amplitude reduced by 24.4% and the weight decreased by 15.6%.

HGA solution is found to be superior to CEGA as it showed better reduction in amplitude of vibration and comparable improvement in weight. Though GP optimum solution showed better improvement in weight, the corresponding reduction is amplitude was less. A better way to compare the results of three optimization methodologies is to identify the improvements of one objective for constant value of other objective. Table-4 shows such a comparison. For the same weight HGA has shown better reduction in amplitude of vibration compared to CEGA and GP.

Table 4. Different designs- shaft weight and amplitude of vibration

Parameter	HGA	CEGA	GP
Weight, kgf	0.95	0.95	0.95
Amplitude of vibration, mm	$1.62x10^{-2}$	$1.63x10^{-2}$	$1.64x10^{-2}$

5. Conclusions

Multi-objective optimisation of a rotor system for aero engine application is carried out through in-house MATLAB coding with additional constrain imposed by gear box casing vibration. The Pareto Front generated by HGA is found to be superior in terms of average distance measure and spread of solutions compared to CEGA. Optimum solutions are found out from HGA and CEGA Pareto Fronts separately using Utopia point method. HGA optimum solution resulted in better optimisation compared to initial design with amplitude of vibration reduced by 30.8% and the weight decreased by 12.6%. CEGA optimum design resulted in the amplitude of vibration reduction of 29.4% and the weight reduction of 12.9%. GP optimum design resulted in the amplitude of vibration reduction of 24.4% and the weight reduction of 15.6%. The optimal solution using HGA showed better improvement in amplitude of vibration compared to CEGA and GP optimisation optimum solutions. Weight reduction was more in GP solution but with less optimal reduction in amplitude of vibration. HGA and CEGA optimal solutions had similar weight reduction. Comparing the reduction in amplitude for same weight, it is confirmed that HGA solution is superior to CEGA and GP solutions.

6. Acknowledgments

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