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Modified signal-averaging technique to reduce measurement time

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A simple method of reducing the total measurement time in signal averaging is described. The method uses averaging over autocorrelation estimates rather than over the signal directly. The method may be useful in certain cases where the repetition rate is small.

Signal averaging has been widely used to recover signals entirely buried in noise. This technique can be used whenever the signal is repetitive (not necessarily periodic) and a reference trigger source is available. The noisy waveform is sampled at predetermined instants of time after each trigger pulse and these sample points obtained from successive occurrences of the signal are averaged. The noise, being uncorrelated to the trigger, averages out in an incoherent fashion while the signal builds up coherently.[1]

It is well known that the SNIR (signal-to-noise improvement ratio) of such an averaging procedure is $\sqrt{n}$, where n is the number of signals averaged. This is true in the case of white noise, though there are deviations from this behavior in the case of nonwhite noise.[2] In any case, this SNIR is obtained at the expense of time efficiency. Even when multipoint averaging techniques are used, the total measurement time T_{tot} is

$$T_{\text{tot}} = nT, \tag{1}$$

where T is the mean time between triggers.

On the other hand, correlation techniques have also been used to obtain signal-to-noise enhancement. The autocorrelation of a signal usually contains most of the information required to reconstruct the original signal. Of course, a certain amount of information is lost, but in many cases this does not matter. In the case of a sinusoidal signal, the amplitude and frequency can be inferred from the autocorrelation function though the phase information is lost.[1,3]

In case of the autocorrelation of a noisy signal, for large delays the noise contribution dies down so that this provides an efficient method of improving the signal-to-noise ratio. However, if the autocorrelation estimate is obtained from a finite number of samples of the noisy waveform, the SNIR obtained may not be sufficient.

Our aim here is to point out that signal averaging can be effectively used to further improve the signal-to-noise ratio of the autocorrelation estimate. $2N$ samples of the noisy signal obtained after each trigger pulse could be used to find an autocorrelation estimate. The signal averaging principle could then be applied to these autocorrelation estimates. The variance of the averaged autocorrelation estimate would be $1/M$ times the variance of a single estimate when the averaging is done over M triggers. This method is in a sense analogous to Bartlett's method of averaging periodograms.[4]

The technique can be implemented quite easily on signal averaging computers and is in fact an extension of the boxcar averager principle. With the availability of microprocessors, the modification of the digital boxcar averager circuits would be fairly simple, and in fact signal analysis systems with all the required capabilities are available (e.g., Hewlett-Packard model 3721A).

The noisy signal $V(t)$ may be written as

$$V(t) = S(t) + n(t), \tag{2}$$

where $S(t)$ is the "clean" signal, while $n(t)$ is the noise which will be assumed to be white, Gaussian [the Gaussian assumption can be released, making Eq. (5) approximate[4]], ergodic, stationary, and having a zero mean. The autocorrelation estimate $\hat{R}(m)$ is

$$\hat{R}(m) = \frac{1}{N} \sum_{k=0}^{N-1} V(k)V(k+m), \tag{3}$$

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whose expectation value is

$$E[\hat{R}(m)] = \frac{1}{N}\sum_{k=0}^{N-1} S(k)S(k+m) + R_{nn}(m), \tag{4}$$

where $R_{nn}(m)$ is the noise autocorrelation function.

The variance of the autocorrelation estimate [Eq. (3)] is

$$\mathrm{Var}[\hat{R}(m)] = \frac{\sigma^2}{N^2}\Big(N\sigma^2 + \sum_{k=0}^{N-1} S^2(k) + \sum_{k=0}^{N-1} S^2(k+m) + 2\sum_{k=0}^{L} S(k)S(k+2m)\Big), \tag{5}$$

where

$$L = \begin{cases} N-1 \text{ for } m \leq N/2 \\ 2(N-m)-1 \text{ for } m > N/2 \end{cases} \quad \text{and} \quad \sigma^2 = \mathrm{Var}[n(t)].$$

For sufficiently large N and m, we can simplify Eq. (5) in certain cases to get

$$\mathrm{Var}[\hat{R}(m)] \approx \frac{\sigma^2}{N}[\sigma^2 + 2S^2] \tag{6}$$

Therefore, the SNIR from a single trigger is

$$(\mathrm{SNIR})_s = (S/\sigma)(E[\hat{R}(m)]/\sqrt{\mathrm{Var}[\hat{R}(m)]})^{-1}. \tag{7}$$

Thus for a given value of SNIR, the averaging has to be done over fewer samples, resulting in a reduction of the total measurement time by a factor of $(\mathrm{SNIR})_s$. The total measurement time becomes significant when the dead time between samples is large, and under such circumstances the correlation estimate $\hat{R}(m)$ may be computed by simple but slow hardware. The signal parameters can be inferred from the autocorrelation estimate after ignoring small values of m.

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Signal to noise enhancement of lock-in amplifiers

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A lock-in amplifier can be considered to be a special case of the more general box-car averager. Expressions for the output noise voltage of lock-in amplifiers are deduced from the corresponding results derived earlier for box-car averagers. Deviations from the simple $(4\,RC)^{1/2}$ law are shown to exist in the case of nonwhite noise.

Lock-in amplifiers are generally used to recover sinusoidal signals buried in noise. The instrument can also be used when the signal is periodic with an arbitrary but known waveform. A box-car averager, on the other hand, is employed to extract repetitive (though not necessarily periodic) signals of unknown waveshapes. It has been pointed out by Blume[1] that a lock-in amplifier can be viewed as a special case of the box-car averager when the signal is periodic with a period T which equals exactly twice the gate-width ϵ.

For white noise, Ernst[2] has shown that in the case of *linear* signal averaging, the signal to noise improvement ratio (SNIR) is proportional to $\sqrt{n}$, where n is the number of samples used. However, box-car averagers and lock-in amplifiers use an exponential averaging process. Recently, Neelakantan and Dattagupta[3] have calculated the SNIR of box-car averagers. (This paper is hereafter referred to as I). The analysis indicates that the analog of the $\sqrt{n}$ law, i.e., SNIR $= (2\,RC/\epsilon)^{1/2}$, is valid only for white noise, and important deviations from this law exist for nonwhite noise sources. The purpose of this brief note is to point out that for the lock-in amplifiers (with a first order filter), the result[4] for the output noise quoted as being proportional to $(4\,RC)^{1/2}$, is valid again for white noise only. Additional expressions for the SNIR of lock-in amplifiers in the case of other noise sources of practical interest are also presented.

The analysis is identical to that of the box-car averager except that the gate-width ϵ is to be set equal to $T/2$, where T is the signal time-period. Using Eq. (20) of I, we have

$$(\mathrm{SNIR})_{\text{white noise}} = (4\,RC/T)^{1/2} \tag{2}$$

the familiar result for lock-in amplifiers.[4]

In real life one often encounters band-limited noise such as the one with an exponential correlation function with a time constant α^{-1}, say [cf., Eq. (21) of I]. Using Eq. (22) of I, the output noise voltage in this case can be written