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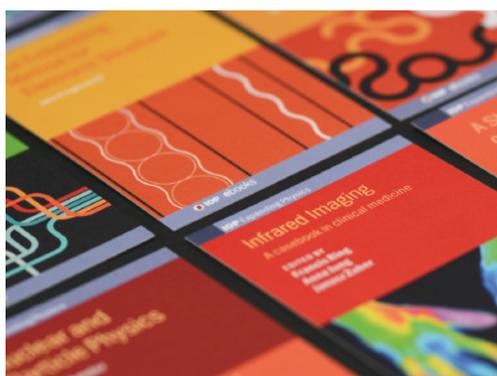
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Modelling transient behavior during stress relaxation

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Abstract. Stress relaxation test is widely used to estimate activation parameters during plastic deformation. The one-dimensional theoretical models used to predict the stress relaxation behavior uses the Orowan rate equation to derive the stress drop as a function of time. Classical models assume that the average mobile dislocation density and internal stress remain invariant during the test. Although subsequent works have extended the original models, the treatment is still incomplete. In the present work, two of the existing stress relaxation models, power law model and logarithmic model are analyzed using the experimental data obtained from SS 316. The models are modified based on the understanding of mechanics of deformation mechanism during stress relaxation. The modified power law model was found to be inconsistent mathematically whereas the modified logarithmic model was found to improve the accuracy of prediction.

1. Introduction

Stress relaxation test is one of the simplest transient tests. A typical stress relaxation test is performed by interrupting an uniaxial tensile test for a pre-defined time interval (in the order of 60s) at a fixed total strain. It has been shown that the stress relaxation phenomenon in materials has a major contribution in the formability improvement [1, 2]. Since then, efforts have been made to systematically quantify the contribution of stress relaxation in ductility improvement through uniaxial tensile tests[1]. It was shown that the improvement is pronounced at high pre-strain, strain rate and longer hold time. Similar improvements in other materials such as titanium [3] and stainless steel [4, 5] has been reported.

The stress relaxation test has been widely used in the past to determine the metallurgical parameters related to plastic deformation. The stress drops continuously with time and the stress- time relation has been used to estimate the activation volume[6, 7], long range internal stress[8] and other rate dependent constitutive relations [9, 10]. However, the structural changes and microscopic mechanisms that occur during stress relaxation have been largely ignored. Understanding these mechanisms [11] are essential to correlate the stress relaxation behaviour with ductility improvement. The phenomenological models of stress relaxation can give an insight on the effects of different physical mechanisms responsible for stress relaxation. The widely used models for stress relaxation are reviewed in the following section. The limitations of the models when attempting to account for the variation of internal stress and mobile dislocation density is highlighted and new modifications of the models are proposed.



2. Modelling stress relaxation

The flow stress of a rate dependent metallic material drops continuously during stress relaxation. Several models have been proposed in the past to predict the stress vs time during stress relaxation, many of which are summarized in the excellent review [12]. The flow stress (σ) in a dislocation controlled plastic deformation can be modelled as a sum of internal stress (σ_i) due to the long range obstacles and effective stress (σ^*) due to short range obstacles [13].

$$\sigma = \sigma_i + \sigma^* \quad (1)$$

Since the total strain remains constant ($\varepsilon_t = \varepsilon_e + \varepsilon_p = C$) during relaxation, $d\varepsilon_t = 0$.

$$\dot{\varepsilon}_p = -\dot{\varepsilon}_e \quad \text{or} \quad -\frac{\dot{\sigma}}{E} \quad (2)$$

where 'E' is the Young's Modulus. From Orowan's equation [14], the plastic strain rate is given as

$$\dot{\varepsilon}_p = \phi b \rho_m v \quad (3)$$

where ρ_m is the mobile dislocation density, b , Burgers vector, v , the average dislocation velocity and ϕ is a constant. Based on the empirical results, [15, 16], the average dislocation velocity can be modelled as

$$v = B(\sigma^*)^{m^*} \quad (4)$$

where m^* is the effective stress exponent obtained from data fitting. From equations 2, 3 and 4,

$$-\dot{\sigma} = E\phi b B \rho_m (\sigma^*)^{m^*} \quad (5)$$

Assuming constancy in ρ_m and σ_i , Gupta and Li [16] integrated eq.5 as

$$\sigma^* = K(t + a)^{-n} \quad (6)$$

where $K = E\phi b B \rho_m (m^* - 1)$, $n = \frac{1}{m^* - 1}$ and a is the integration constant.

An alternate method is considering a thermally activated dislocation movement, where the average dislocation velocity, v can be written in the Arrhenius rate form [12, 17] as

$$v = v_0 \cdot \exp \left[-\frac{\Delta G_0 - V^* \sigma^*}{kT} \right] \quad (7)$$

v_0 is a constant relating the average dislocation distance during each activation, ΔG_0 is the activation energy corresponding to zero stress, V^* , k and T are activation volume, Boltzmann constant and absolute temperature respectively.

Substituting eqns. 7 and 2 in the Orowan equation (eq. 3) and integrating will yield a logarithmic stress-time relation [18, 12, 17] during relaxation.

$$\sigma(t) = \sigma(0) - \alpha \ln(1 + \beta t) \quad (8)$$

where $\sigma(0)$ is the stress at the beginning of the relaxation when time, $t = 0$. α and β are constants [17]. The above equation (eq.8) is valid if the internal stress, σ_i and the mobile dislocation density, ρ_m does not change during relaxation.

3. New model for stress relaxation

The assumption of constant σ_i and ρ_m above is only a special case in the analysis of stress relaxation. The non-constancy of σ_i and ρ_m during stress relaxation were long known [19, 20, 12]. Their variation however was thought to be negligible [20]. Subsequent studies involving repeated relaxation tests [21, 22] and other microstructural studies [17, 23, 11] report that it is essential to consider the variation of mobile dislocation density when modelling stress relaxation.

An accurate description of stress relaxation should account for the variation of both the dislocation density and internal stress with time [17]. Limited attempts have been made in the past [20, 24, 21] to accommodate the effect of varying ρ_m and σ_i in the stress relaxation models. However, the assumptions in each of the modified models limits its applications for a wide range of relaxation behaviour. In the present work, the classical power law and logarithmic models are modified to include the transient effect of ρ_m and σ_i .

Xiao and Bai [24] proposed hyperbolic function for the reduction in mobile dislocation density with time. However its less flexible. The variation of ρ_m is expected to share a similar mathematical form of v [21, 7],

$$\frac{\rho_{m(t)}}{\rho_{m0}} = (v/v_0)^p \tag{9}$$

where $\rho_{m(t)}$ and ρ_{m0} represent the time dependent and initial mobile dislocation densities, and p is the exponent.

Substituting eq.9 in eq.4,

$$\rho_{m(t)} = \rho_{m0} B' (\sigma^*)^{pm*} \tag{10}$$

The reduction of mobile dislocation density during relaxation can be either due to mutual annihilation or by conversion to sessile dislocations. The fraction of mobile dislocations that becomes immobile contributes to an increase in internal stress (σ_i). The plastic strain continuously increases during stress relaxation (eq.2) and the strain hardening due to plastic strain further increases the internal stress. The time dependent recovery [11] decreases the internal stress with time. The net change in internal stress during relaxation can be assumed to be small and linearly related [12] with the strain change.

$$\Delta\sigma_{i(t)} = \theta_r \Delta\varepsilon(t) = \frac{\theta_r \Delta\sigma(t)}{E} \tag{11}$$

Combining eq.1 and eq.11,

$$\frac{d\sigma^*}{dt} = \frac{d\sigma}{dt} \left(1 - \frac{\theta_r}{E} \right) \tag{12}$$

Substituting eqns.12,10 in eq.6 and integrating yields,

$$\sigma = \sigma_{i(o)} + K'(t + a)^{-n*} \tag{13}$$

$\sigma_{i(o)}$ is the internal stress at the beginning of the relaxation. Eq.13 is similar to eq.6, except that the exponent, $-n^* = \frac{1}{m^*(1+p) - 1}$ accommodates the change in mobile dislocation density and K' the variation of internal stress with time.

Similar modifications can be proposed for the logarithmic stress-time relation (eq.8). Mohebbi et al. [17] proposed a generalized modification of logarithmic model as follows

$$\sigma_0 - \sigma = \alpha \ln \left\{ 1 + \beta \int \frac{\rho_{m(t)}}{\rho_{m0}} dt \right\} \tag{14a}$$

$$\dot{\sigma}^* = -\alpha\beta \exp \left(\frac{\sigma^*(t) - \sigma^*(0)}{\alpha} \right) - \dot{\sigma}_{i(t)} \tag{14b}$$

Eqns.14a and 14b independently and respectively accommodated the variation of ρ_m and σ_i during stress relaxation. In the present work, the variation of internal stress with time is modelled as indicated by eq.12. The constant, β in eq.8 is directly proportional to the mobile dislocation density. In a recent study [17], three different trends of $\frac{\rho_m(t)}{\rho_{m0}}$ vs time in logarithmic scale is discussed, viz, concave up, concave down and linear. An exponential function of the form $e^{\lambda t}$ $\{\lambda < 0\}$ can be used to model such variation of mobile dislocation density with time. Therefore, the modified model is given as,

$$\sigma(t) = \sigma(0) - \bar{\alpha} \ln(1 + \bar{\beta}(t)t) \quad (15)$$

where $\bar{\alpha}$ and $\bar{\beta}$ are constants. $\bar{\alpha}$ is scaled using eq.12 to model the internal stress. The time dependent evolution of mobile dislocation density is modelled using the constant, $\bar{\beta}$ given by $\bar{\beta} = \bar{\beta}_0.e^{\lambda t}$.

4. Results and Discussion

The modified power law model and logarithmic model proposed in the present work is evaluated using the stress relaxation test results of SS316 and AA 8011 alloy published in [4] and [25] respectively. A representative fit of the data using modified power law and modified logarithmic law are shown in Fig.1 and Figure 2 respectively.

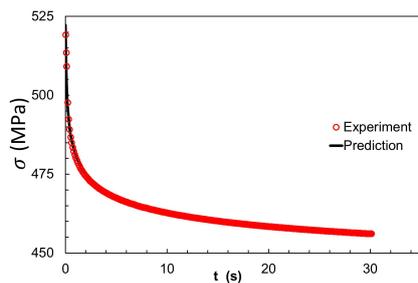


Figure 1: Stress relaxation model using eq.13 at $\varepsilon = 3\%$, $\dot{\varepsilon} = 1 \times 10^{-2} s^{-1}$, $t = 30s$, SS 316

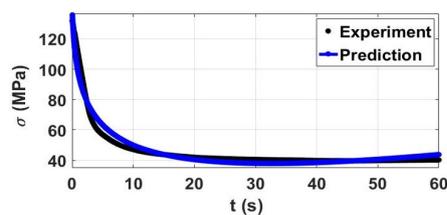


Figure 2: σ vs t predicted using eq.15 compares well with the experimental data at $\varepsilon = 8.5\%$, $\dot{\varepsilon} = 5 \times 10^{-3} s^{-1}$, $t = 60s$, AA 8011 alloy

Both the models fit the experimental data very well. The parameters obtained for different conditions are tabulated in Table.1. The exponent $-n^*$ in eq.13 can be estimated from the log-log plot of stress rate vs time [16, 17], as shown in Fig.3. While estimating the parameters, it is noticed (Table.1) that the magnitude of the slope (Fig.3), $|S| < 1$ at larger strain of 25% in SS 316 and at all the conditions in AA 8011. This will yield a positive value for $-n^*$, which cannot explain the decrease of stress with time using eq.13. Thus eq.13 is feasible only when $|S| > 1$.

The logarithmic model on the other hand is found to be applicable for a wide range of experimental conditions. The modified logarithmic law proposed allows additional flexibility so that the restriction of $|S| = 1$ in the original logarithmic model can be overcome.

Table 1: Modified Stress relaxation models, $\dot{\epsilon} = 1 \times 10^{-2} s^{-1}$ and $t=30$ s for SS 316 and $\dot{\epsilon} = 5e^{-3} s^{-1}$ and $t=60$ s for AA 8011 alloy

$\epsilon(\%)$	Slope [$\ln(\dot{\sigma})$ vs $\ln(t)$]	Modified Power law	Modified logarithmic law
SS316			
15	-1.287	$641.2 + 43.42(t + 0.14)^{-0.287}$	$718.8 - 9.8 \ln[1 + 37.4te^{-0.028t}]$
25	-0.808	Not feasible	$852.3 - 12.2 \ln[1 + 62.1te^{-0.06t}]$
AA 8011			
8.5	-0.87	Not feasible	$1355 - 24.76 \ln[1 + 4.05te^{-0.03t}]$

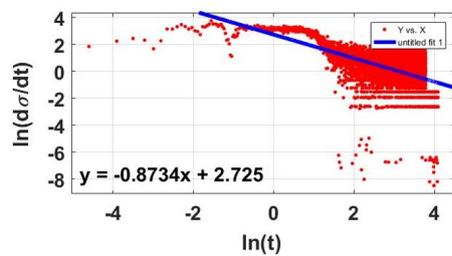


Figure 3: log-log plot of $\dot{\sigma}$ vs time, AA 8011 alloy

5. Conclusion

In the present work, two new models to predict the stress relaxation behaviour have been proposed. The new models are modification of the widely used power law and logarithmic models. Both the models overcome the limitations of the existing models in accounting the variation of mobile dislocation density and internal stress during relaxation. The modified power law model, however yielded physically infeasible results under certain conditions of high plastic strain and therefore is not recommended. The modified logarithmic model is found to be suitable for a wide range of test parameters.

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