Mixed convection flow over a vertical power-law stretching sheet

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Abstract

Purpose – The purpose of this paper is to consider steady two-dimensional mixed convection flow along a vertical semi-infinite power-law stretching sheet. The velocity and temperature of the sheet are assumed to vary in a power-law form.

Design/methodology/approach – The problem is formulated in terms of non-similar equations. These equations are solved numerically by an efficient implicit, iterative, finite-difference method in combination with a quasi-linearization technique.

Findings – It was found that the skin-friction coefficient increased with the ratio of free-stream velocity to the composite reference velocity and the buoyancy parameter while it decreased with exponent parameter. The heat transfer rate increased with the Prandtl number, buoyancy parameter and the exponent parameter.

Practical implications – A very useful source of information for researchers on the subject of convective flow over stretching sheets.

Originality/value – This paper illustrates mixed convective flow over a power-law stretched surface with variable wall temperature.

Keywords Convection, Flow, Friction, Heat transfer

Paper type Research paper

Nomenclature

C_{fx}	local skin-friction coefficient	Pr	Prandtl number
C_p	specific heat at constant pressure	Re_x	local Reynolds number
f	dimensionless stream function	T	temperature
g	acceleration due to gravity	$U(x)$	composite reference velocity
Gr_x	local Grashof number	$U_w(x)$	moving plate velocity
Nu_x	local Nusselt number	$U_\infty(x)$	free stream velocity
m	exponent of velocity and temperature	u	velocity component in the x direction

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v velocity component in the y direction
x, y Cartesian coordinates

Greek symbols

α thermal diffusivity
β coefficient of thermal expansion
ξ, η transformed variables
λ buoyancy parameter
μ dynamic viscosity
ν kinematic viscosity
ρ density
ψ stream function

Subscripts

w, ∞ conditions at the wall and infinity, respectively
ξ, η denote the partial derivatives w.r.t. these variables, respectively

1. Introduction

The problem of flow and heat transfer over a continuous moving vertical surface with velocity U_W parallel the free-stream velocity U_∞, finds numerous and wide ranging applications in many engineering and manufacturing processes. Examples of practical applications are the cooling of an infinite metallic plate in a cooling bath, the boundary layers along material handling conveyers and along a liquid form in condensation processes, glass blowing, continuous casting spinning of fibres, etc. The study of problems involving viscous fluid flow over a stretching sheet is an important type of flow occurring in many engineering manufacturing processes. Such processes are wire and fibre coating, food stuff processing, heat treated materials travelling between a feed roll and a wind-up roll or materials manufactured by extrusion, glass fibre and paper production, cooling of metallic sheets or electronic chips, crystal growing, drawing of plastic sheets, etc. In these processes, the quality of the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching. In view of such applications Crane (1970) initiated the analytical study of boundary layer flow of a Newtonian fluid over a linearly stretching surface. Gupta and Gupta (1977) have analysed heat and mass transfer from an isothermal stretching sheet with suction or blowing effects. Chen and Char (1988) extended the works of Gupta and Gupta (1977) to that of a non-isothermal stretching sheet. Grubka and Bobba (1985b) have studied the heat transfer by considering the power-law variation of surface temperature. Afzal (1993) has been studied the heat transfer effects from a stretching surface. Ali and Al-Yousef (1998) have investigated the problem of laminar mixed convection adjacent to a uniformly moving vertical plate with suction or injection. In this investigation, similarity solutions were reported for the boundary layer flow subject to power-law velocity and temperature boundary conditions. Tsou *et al.* (1967) showed experimentally that the flow and heat transfer problem from a continuously moving surface is a physically realizable one and studied its basic characteristics. Later, various aspects of the problem have been treated by many authors. Soundalgekar and Murty (1980) have studied the effects of power-law surface temperature variation on the heat transfer from a continuous moving surface with constant surface velocity. The effects of variable surface temperature and linear surface stretching were examined by Grubka and Bobba (1985). Similarity solutions were reported by Ali (1994) for the case of power-law surface velocity and three different thermal boundary conditions. Ali (1995) has extended his work for the stretching surface subject to suction or injection. Moutsoglou and Chen (1980) have considered buoyancy effects on flow and heat transfer from an inclined continuous sheet with either uniform wall temperature or uniform surface heat flux. An analysis of mixed convection heat transfer from a vertical continuously stretching sheet has been presented by Chen (1998). In this presentation, similarity solutions were obtained for an isothermal sheet moving with surface velocity proportional to $x^{1/2}$ and a linearly stretching sheet subject to a linear wall temperature distribution. A detailed numerical study of the problem of mixed convection flow adjacent to an inclined continuously stretching sheet has been reported by Chen (2000a).

Abdelhafez (1985) and Chappidi and Gunnerson (1989) have studied laminar boundary layer for two cases where $U_w > U_\infty$ and $U_w < U_\infty$ are treated separately and formulated two sets of boundary value problems. Afzal *et al.* (1993) formulated a single set of governing equations by employing composite velocity as $U = U_w + U_\infty$ irrespective of whether $U_w > U_\infty$ or $U_w < U_\infty$. Lin and Haung (1994) were analysed for horizontal isothermal plate moving in parallel or reversibly to a free stream where similarity and non-similarity equations are used to obtain the flow and thermal fields. Considering the reference velocity $U(x) = U_w(x)$, the flow on a continuously stretching sheet has been studied by Afzal and Varshney (1980) as mentioned in Aziz and Na (1986). Afzal (2003) has obtained the similarity solutions of laminar boundary layer driven by the stretching surface and pressure gradient, each proportional to the same power-law of the down-stream coordinate based on composite reference velocity. Heat transfer characteristics of a non-isothermal surface moving parallel to a free stream are studied by Chen (2000b). In this study, heat transfer results are predicted two surface heating conditions, power-law variation in wall temperature and uniform surface heat flux. The detailed analyses laminar fluid flow problems due to the combined motions of a bounding surface and free stream in the same direction have been discussed by Abraham and Sparrow (2005) and Sparrow and Abraham (2005). They have used the relative velocity model where one of the participating media is in motion. Ishak *et al.* (2007) examined the boundary layer flow over a continuously moving thin needle in a parallel stream. The effects of transpiration on the flow and heat transfer over a moving permeable surface in a parallel stream are analysed by Ishak *et al.* (2009a, b). The development of the boundary layer on a fixed or moving surface parallel to a uniform free stream in presence of surface heat flux has been investigated by Ishak *et al.* (2009).

The present numerical study investigates the steady mixed convection flow along a semi-infinite vertical power-law stretching sheet. The stretching sheet is considered to move with a power-law velocity parallel to the power-law free-stream velocity and it is assumed to subject to a power-law wall temperature. The coupled non-linear partial differential equations governing the flow have been solved numerically using an implicit finite difference method in combination of quasi-linearization technique (Inouye, 1974; Roy and Saikrishnan, 2003). Results are compared with some results reported by Tsou *et al.* (1967), Soundalgekar and Murty (1980), Ali (1995), Moutsoglou and Chen (1980) and Chen (1998) and are found to be in excellent agreement.

2. Mathematical formulation

Consider a steady two-dimensional incompressible viscous mixed convection boundary layer flow along a semi-infinite vertical power-law stretching sheet moving with velocity $U_W(x) = U_{0w}x^m$ in the x-direction. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to it. Figure 1 shows the coordinate system and physical model for the flow configuration. The free-stream

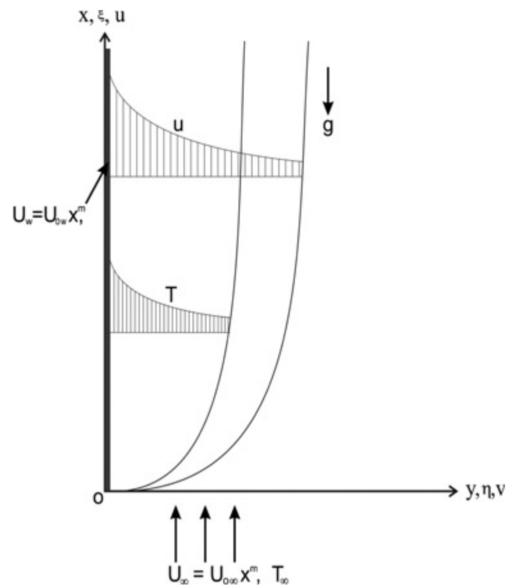

Figure 1.
Physical model and coordinate system

velocity $U_\infty(x) = U_{0\infty}x^m$ and plate velocity $U_W(x) = U_{0w}x^m$ are varying in the same direction. The stretching sheet is assumed to be subject to power-law wall temperature $T_W(x) = T_\infty + Ax^{2m-1}$. The buoyancy force arises due to the temperature difference in the fluid. All thermophysical properties of the fluid in the flow model are assumed constant except the density variations causing a body force in the momentum equation. The Boussinesq approximation is invoked for the fluid properties to relate density changes, and to couple in this way the temperature field to the flow field (Schlichting, 2000). Under these assumptions, the equations of conservation of mass, momentum and energy governing the mixed convection boundary layer flow over a moving vertical plate are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \pm g\beta(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

The physical boundary conditions for the problem are given by:

$$\begin{aligned} &y = 0 : u = U_W(x),\ \mathrm{v} = 0,\ T = T_W(x), \\ &y \to \infty : u \to U_\infty(x),\ T \to T_\infty, \end{aligned} \tag{4}$$

where the stretching sheet velocity $U_W(x)$, the free-stream velocity $U_\infty(x)$ and the wall temperature $T_W(x)$ are given by:

$$U_W(x) = U_{0w}x^m, \quad U_\infty(x) = U_{0\infty}x^m, \quad T_W(x) = T_\infty + Ax^{2m-1}.$$

Applying the following transformations

$$\begin{aligned}
\xi &= \left(\frac{U(x)x}{\nu}\right)^{1/2}, \quad \eta = \left(\frac{U(x)}{\nu x}\right)^{1/2} y, \quad \psi(x,y) = (\nu U(x)x)^{1/2} f(\xi,\eta), \\
u &= \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x}, \quad T - T_\infty = (T_W(x) - T_\infty)\Theta(\xi,\eta), \\
U(x) &= U_W(x) + U_\infty(x), \quad u = U_0 x^m f_\eta, \quad \Pr = \frac{\nu}{\alpha}, \ \lambda = \frac{Gr_x}{\mathrm{Re}_x^2}, \qquad (5)\\
v &= -(\nu U_0)^{1/2} x^{(m-1)/2}\left\{\left(\frac{m+1}{2}\right)[f(\xi,\eta) + \xi f_\xi] + \left(\frac{m-1}{2}\right)\eta f_\eta\right\}, \\
\mathrm{Re}_x &= \frac{U_0 x^{m+1}}{\nu}, Gr_x = \frac{g\beta(T_W(x) - T_\infty)x^3}{\nu^2},
\end{aligned}$$

to Equations (1)-(3), we find that Equation (1) is identically satisfied, and Equations (2) and (3) reduce to:

$$F_{\eta\eta} + \left(\frac{m+1}{2}\right) f F_\eta - mF^2 \pm \lambda\Theta = \left(\frac{m+1}{2}\right)\xi(FF_\xi - f_\xi F_\eta), \qquad (6)$$

$$\Theta_{\eta\eta} + \Pr\left(\frac{m+1}{2}\right) f\Theta_\eta - \Pr(2m-1)F\Theta = \Pr\left(\frac{m+1}{2}\right)\xi(F\Theta_\xi - f_\xi\Theta_\eta), \qquad (7)$$

where $f = \int_0^\eta F d\eta + f_w; f_w = 0$.

In Equation (6), λ that represents the buoyancy force effect on the flow field has $\pm$ signs; the plus sign indicates the buoyancy-upward (or buoyancy-assisted) flow, while the negative sign stands for buoyancy-downward (or buoyancy-opposed) flow.

The non-dimensional boundary conditions become:

$$\begin{aligned}
F &= 1 - \varepsilon, \quad \Theta = 1 \quad \text{at } \eta = 0 \\
F &\to \varepsilon, \quad \Theta \to 0, \quad \text{as } \eta \to \infty,
\end{aligned} \qquad (8)$$

where $\varepsilon = (U_\infty(x)/(U_{0\infty}(x) + U_{0w}(x)))$ corresponds to the ratio of free-stream velocity to the composite reference velocity.

The local skin-friction coefficient C_{fx} is defined as:

$$\begin{aligned}
C_{fx} &= \mu\frac{\partial u/\partial y}{\frac{1}{2}\rho U^2} = 2\mathrm{Re}_x^{-\frac{1}{2}} F_\eta(\xi, 0) \\
&\text{i.e. } C_{fx}\mathrm{Re}_x^{\frac{1}{2}} = 2F_\eta(\xi, 0).
\end{aligned} \qquad (9)$$

The local heat transfer rate in terms of Nusselt number can be expressed as:

$$Nu_x = -x \frac{\partial T/\partial y}{(T_w(x) - T_\infty)} = -\mathrm{Re}_x^{\frac{1}{2}} \Theta_\eta(\xi, 0)$$
$$\text{i.e. } Nu_x \, \mathrm{Re}_x^{-\frac{1}{2}} = -\Theta_\eta(\xi, 0). \tag{10}$$

3. Method of solution

The non-linear coupled partial differential Equations (6) and (7) subject to the boundary conditions (8) have been solved numerically using an implicit finite-difference scheme in combination with the quasi-linearization technique (Inouye and Tate, 1974). An iterative sequence of linear equations carefully constructed to approximate the non-linear Equations (6) and (7) for achieving quadratic convergence and monotonicity. Using the quasi-linearization technique, the non-linear coupled partial differential Equations (6) and (7) with boundary conditions (8) are replaced by the following sequence of linear partial differential equations:

$$F_{\eta\eta}^{i+1} + A_1^i F_\eta^{i+1} + A_2^i F^{i+1} + A_3^i F_\xi^{i+1} + A_4^i \Theta^{i+1} = A_5^i, \tag{11}$$
$$\Theta_{\eta\eta}^{i+1} + B_1^i \Theta_\eta^{i+1} + B_2^i \Theta^{i+1} + B_3^i \Theta_\xi^{i+1} + B_4^i F^{i+1} = B_5^i. \tag{12}$$

The coefficient functions with iterative index i are known and the functions with iterative index $(i + 1)$ are to be determined. The corresponding boundary conditions are given by:

$$F^{i+1} = 1 - \varepsilon, \quad \Theta^{i+1} = 1 \quad \text{at } \eta = 0,$$
$$F^{i+1} = \varepsilon, \quad \Theta^{i+1} = 0, \quad \text{at } \eta = \eta_\infty. \tag{13}$$

The coefficients in Equations (11) and (12) are given by:

$$A_1^i = \left(\frac{m+1}{2}\right)(f + \xi f_\xi), \quad A_2^i = -\left(\frac{m+1}{2}\right)\xi F_\xi - 2mF,$$
$$A_3^i = -\left(\frac{m+1}{2}\right)\xi F, \quad A_4^i = \lambda, \quad A_5^i = -\left(\frac{m+1}{2}\right)\xi F F_\xi - mF^2,$$
$$B_1^i = \mathrm{Pr}\left(\frac{m+1}{2}\right)(f + \xi f_\xi), \quad B_2^i = -\mathrm{Pr}(2m-1)F, \tag{14}$$
$$B_3^i = -\mathrm{Pr}\left(\frac{m+1}{2}\right)\xi F, \quad B_4^i = -\mathrm{Pr}(2m-1)\Theta - \mathrm{Pr}\left(\frac{m+1}{2}\right)\xi\Theta_\xi,$$
$$B_5^i = -\mathrm{Pr}(2m-1)F\Theta - \mathrm{Pr}\left(\frac{m+1}{2}\right)\xi\Theta_\xi F.$$

Since the method is presented for ordinary differential equations by Inouye and Tate (1974) and also presented for partial differential equations in a recent study by Roy and Saikrishnan (2003), its detailed description is not provided for the sake of brevity. At each iteration step, the sequence of linear partial differential Equations (11) and (12) were expressed in difference form using central difference scheme in the η-direction and backward difference scheme in ξ-direction. Thus in each step, the resulting equations were then reduced to a system of linear algebraic equations with a block tri-diagonal matrix, which is solved by Varga's algorithm (2000). To ensure the convergence of the numerical solution to the exact solution, step sizes $\Delta\eta$ and $\Delta\xi$ are optimized and taken as 0.01 and 0.005, respectively. The results presented here are independent of the step sizes at least up to the fourth decimal place. A convergence criterion based on the relative difference between the current and previous iteration values is employed. When the difference reaches 10^{-4}, the solution is assumed to have converged and the iteration process is terminated. Accuracy of the presented approach is verified by direct comparison with the results previously reported by Tsou *et al.* (1967), Soundalgekar and Murty (1980), Ali (1995), Moutsoglou and Chen (1980) and Chen (1998). The results of this comparison are presented in Table I and are found to be in excellent agreement.

4. Result and discussion

Computations have been carried out for various values of Pr$(0.7 \leq \Pr \leq 7.0)$, $\lambda(-1.0 \leq \lambda \leq 5.0)$, $\varepsilon(0.1 \leq \varepsilon \leq 0.9)$ and $m(0 \leq m \leq 1)$. The edge of the boundary layer (η_∞) has been taken between 5.0 and 8.0 depending on the values of the parameters.

The effects of buoyancy parameter (λ) and Prandtl number (Pr) on the velocity and temperature profiles ($F(\xi, \eta)$, $\Theta(\xi, \eta)$) are presented in Figures 2-5. The velocity profiles $F(\xi, \eta)$ are displayed in Figures 2 and 3 for different values of exponent parameter $m = 0$ and $m = 1.0$. When $m = 0$, results correspond to uniform motion while $m = 1.0$, corresponds to linear stretching surface. In buoyancy aiding flow ($\lambda > 0$), the buoyancy force shows the significant overshoot in the velocity profiles near the surface for lower Prandtl number fluid ($Pr = 0.7$, air), whereas for higher Prandtl number fluid ($Pr = 7.0$, water) the velocity overshoot is not present. The magnitude of the overshoot increases with the buoyancy parameter $\lambda(\lambda > 0)$ while decreases as the Prandtl number (Pr) increases. The physical reason is that the buoyancy force (λ) affects more in smaller Prandtl number fluid (air, $Pr = 0.7$) due to the less viscosity of the fluid. Hence, the velocity increases within the moving boundary layer as the assisting buoyancy force acts like a favourable pressure gradient. Thus, the velocity overshoot occurs and for higher Prandtl number fluids the overshoot is not present because higher Prandtl number (Pr) (water, $Pr = 7.0$) means more viscous fluid which makes it less sensitive to the buoyancy force. Comparative studies in Figures 2 and 3 indicate that the magnitude of the velocity overshoot decreases remarkably when $m = 1.0$ i.e. linear stretching surface within the boundary layer. It is interesting to note from the Figures 2 and 3 that for opposing buoyancy flow, i.e. for negative value of buoyancy parameter ($\lambda < 0$), the buoyancy opposing force reduces the magnitude of the velocity

Pr	0.7	1.0	2.0	7.0	10	100
Tsou *et al.* (1967)	0.3492	0.4438	–	–	1.6804	5.545
Soundalgekar and Murty (1980)	0.3508	–	0.6831	–	1.6808	–
Ali (1995)	0.3476	0.4416	–	–	1.6713	–
Moutsoglou and Chen (1980)	0.34924	–	–	1.38703	–	–
Chen (1998)	0.34925	0.44375	0.68324	1.38619	1.68008	5.54450
Present work	0.35004	0.44401	0.68314	1.38625	1.68011	5.54610

Table I. Comparison of $-\Theta_\eta(0)$ for $\lambda = 0$, $\xi = 0$, $\varepsilon = 0$, $m = 0$ and $n = 0$ and selected values of Pr to previously published work

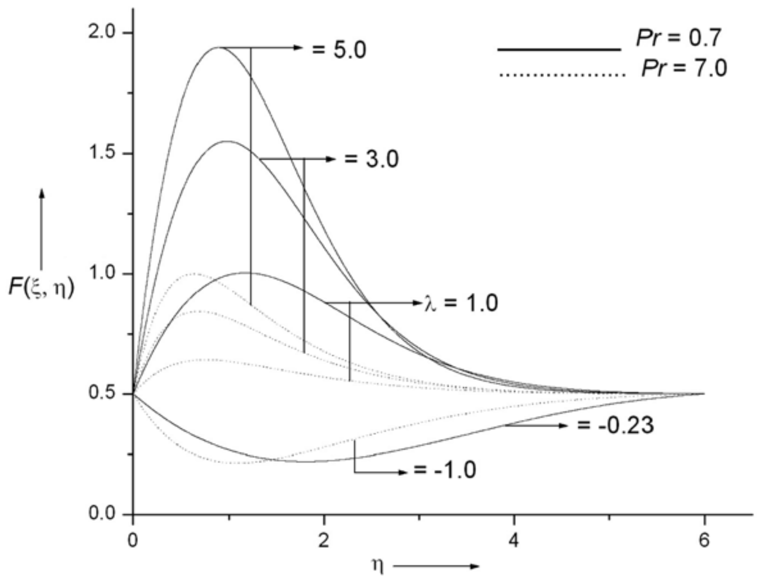

Figure 2.
Effects of λ and Pr on velocity profile for $\varepsilon = 0.5$, $\xi = 0.5$ and $m = 0.0$

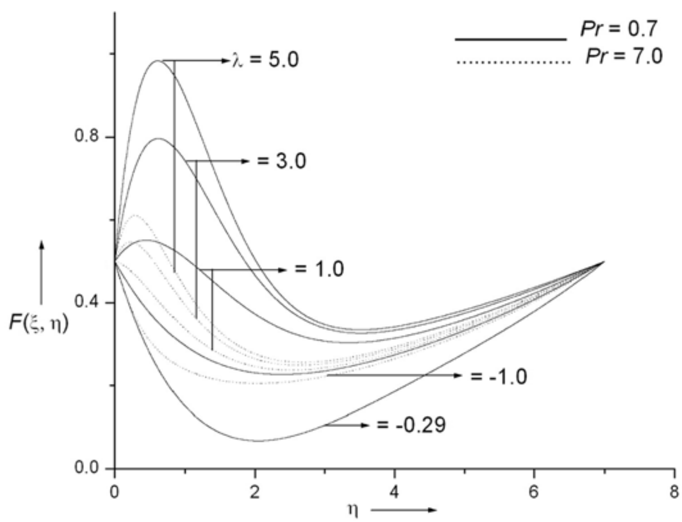

Figure 3.
Effects of λ and Pr on velocity profile for $\varepsilon = 0.5$, $\xi = 0.5$ and $m = 1.0$

significantly within the boundary layer for low Prandtl number fluid ($Pr = 0.7$, air) as well as for high Prandtl number fluid ($Pr = 7.0$, water).

The effect of the buoyancy parameter (λ) on temperature profile $\Theta(\xi, \eta)$ is comparatively less as shown in the Figures 4 and 5. Further, it is observed from the Figures 4 and 5 that an increase in the (higher) Prandtl number (Pr) (water, $Pr = 7.0$) clearly induces a strong reduction in the temperature of the fluid and thus results into the thinner thermal boundary layer. Prandtl number Pr is inversely proportional to thermal conductivity and lower Prandtl number (Pr) fluids will possess higher thermal conductivities and therefore diffuses heat energy more than momentum. Comparative studies on Figures 4 and 5 indicate that the magnitude of the temperature decreases

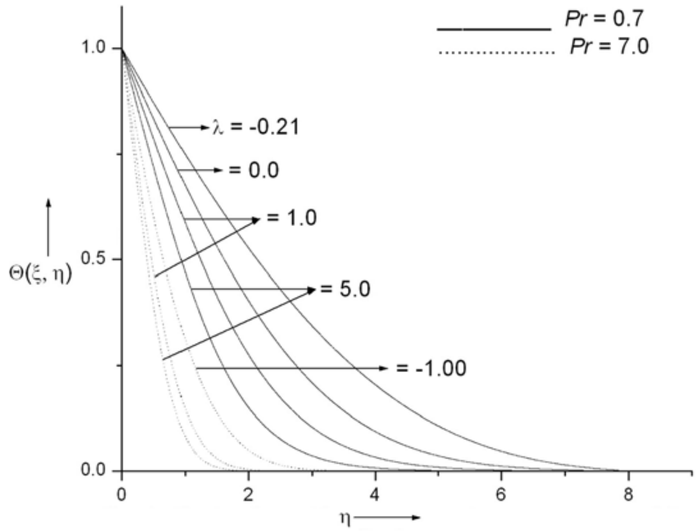

Figure 4.
Effects of λ and Pr on temperature profile for $\varepsilon = 0.5$, $\xi = 0.5$ and $m = 0.0$

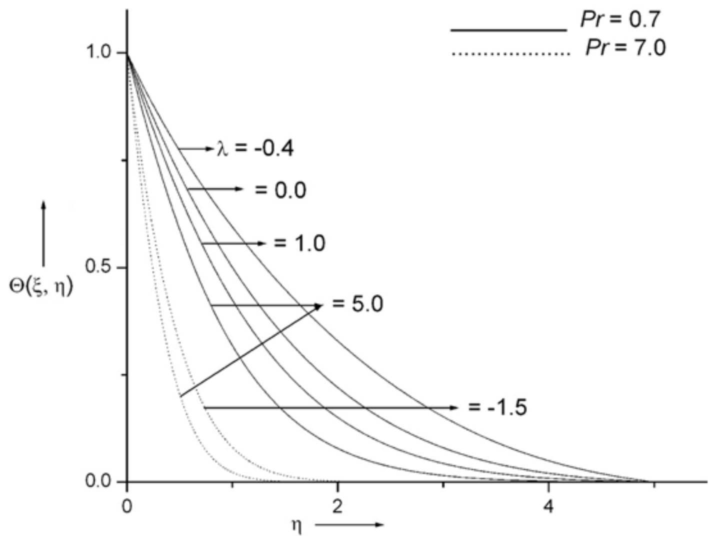

Figure 5.
Effects of λ and Pr on temperature profile for $\varepsilon = 0.5$, $\xi = 0.5$ and $m = 1.0$

significantly when $m = 1.0$ i.e. linear stretching surface within the thermal boundary layer.

Figure 6 depicts the effects of ε (the ratio of free-stream velocity to the composite reference velocity) and m (exponent parameter) on the velocity profile $F(\xi,\eta)$ for $\lambda = 1.0$, $\xi = 0.5$ and $Pr = 0.7$. The velocity is strongly depending on ε because it occurs in the boundary condition for $F(\xi,\eta)$. It has been observed that the magnitude of the velocity within the boundary layer increases with the increase of ε while decreases as m increases from $m = 0.0$ to $m = 1.0$. The physical reason is that the assisting buoyancy force due to thermal gradients acts like a favourable pressure gradient which accelerates the fluid for uniform motion when $m = 0.0$ causing the velocity

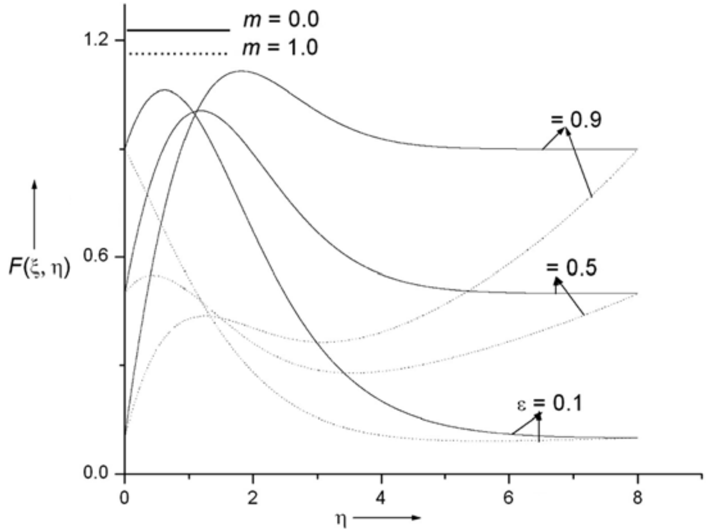

Figure 6.
Effects of ε and m on velocity profile for $Pr = 0.7$, $\xi = 0.5$ and $\lambda = 1.0$

overshoot near the surface within the moving boundary layer. The velocity overshoot reduces significantly when $m = 1.0$ (linear stretching surface) as ε increases.

The effects of ε (the ratio of free-stream velocity to the composite reference velocity) and velocity exponent parameter m on the skin-friction coefficient ($C_{fx}\mathrm{Re}_x^{1/2}$) when $\lambda = 2.0$ and $Pr = 7.0$, are shown in Figure 7. Results indicate that the skin-friction coefficient ($C_{fx}\mathrm{Re}_x^{1/2}$) increases with ε but decreases when velocity exponent parameter m increases from $m = 0.0$ to $m = 1.0$ for a fixed value of ε. This is due to the fact that the increase of ε enhances the fluid acceleration and hence skin-friction coefficient increases. In particular for $m = 0.0$, skin-friction coefficient ($C_{fx}\mathrm{Re}_x^{1/2}$) increases approximately 53 per cent as ε increases from 0.50 to 0.90 and when $m = 1.0$, skin-friction coefficient ($C_{fx}\mathrm{Re}_x^{1/2}$) increases approximately about 238 per cent as ε increases from 0.50 to 0.90. It is clearly

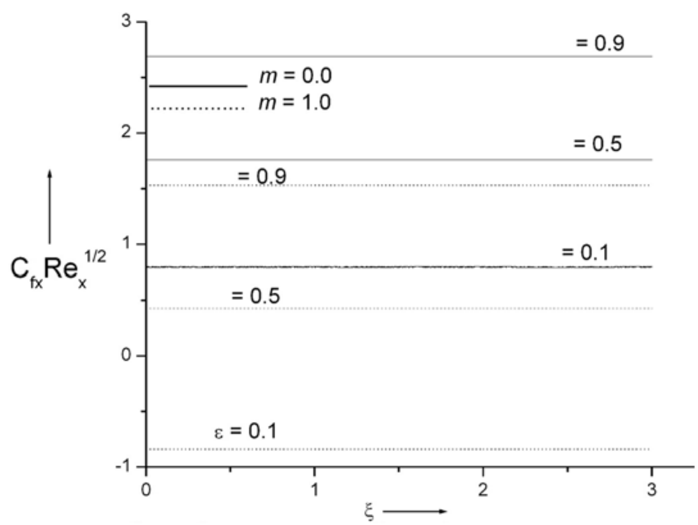

Figure 7.
Effects of ε and m on skin friction coefficient for $\lambda = 2.0$ and $Pr = 7.0$

evident from the fact that the effect of power-law stretching surface is more prominent on the skin-friction coefficient$(C_{fx}\mathrm{Re}_x^{1/2})$ as compared to uniform motion ($m = 0.0$).

Figure 8 displays the effects of buoyancy parameter (λ) and exponent parameter m on the skin-friction coefficient$(C_{fx}\mathrm{Re}_x^{1/2})$ with the stream wise distance ξ for, $\varepsilon = 0.4$ and $Pr = 7.0$. The skin-friction coefficient $(C_{fx}\mathrm{Re}_x^{1/2})$ increases with buoyancy parameter (λ) while it decreases with the exponent parameter m. The physical reason is that the power-law stretching reduces the gradient of the velocity on the surface $F_\eta(\xi,0)$. The gradient of the velocity on the surface $F_\eta(\xi, 0) < 0$ implies that the fluid is being dragged by the plate and $F_\eta(\xi, 0) > 0$ implies that the plate is being dragged by the fluid. Due to this negative values of skin-friction coefficient $(C_{fx}\mathrm{Re}_x^{1/2})$ have been occurred at $\lambda = 1.0$ when $Pr = 7.0$, $\varepsilon = 0.4$ and $m = 1.0$. As λ increases from $\lambda = 1.0$ to $\lambda = 3.0$, the positive skin-friction coefficient $(C_{fx}\mathrm{Re}_x^{1/2})$ is obtained. In particular for $Pr = 7.0$ and $\varepsilon = 0.4$, the skin-friction coefficient $(C_{fx}\mathrm{Re}_x^{1/2})$ increases about 60 and 200 per cent for m = 0.0 and m = 1.0, respectively, when buoyancy parameter (λ) increases from $\lambda = 3.0$ to $\lambda = 5.0$. This signifies the importance of the power-law stretching surface on the velocity field.

The effects of Prandtl number Pr, buoyancy parameter (λ) and exponent parameter m on the heat transfer rate $(Nu_x\mathrm{Re}_x^{-1/2})$ are presented with the stream wise distance ξ for $\varepsilon = 0.4$ in Figure 9. The heat transfer rate $(Nu_x\mathrm{Re}_x^{-1/2})$ increases with buoyancy parameter (λ), Prandtl number Pr and the exponent parameter m. In particular, for $\varepsilon = 0.4$ and $\lambda = 3.0$, the heat transfer rate $(Nu_x\mathrm{Re}_x^{-1/2})$ increases about 160 per cent as Prandtl number Pr increases from 0.7 to 7.0 when $m = 0.0$. Further, it is observed that for $Pr = 7.0$ as λ increases from $\lambda = 1.0$ to $\lambda = 5.0$, the heat transfer rate $(Nu_x\mathrm{Re}_x^{-1/2})$ increases approximately by 12 and 10 per cent, respectively, when $m = 0.0$ and $m = 1.0$. Furthermore, the heat transfer rate $(Nu_x\mathrm{Re}_x^{-1/2})$ increases approximately about 105 per cent as exponent parameter m increases from $m = 0.0$ to $m = 1.0$ for $\lambda = 1.0$ and $Pr = 7.0$.

5. Conclusions

A detailed numerical investigation was carried out for the mixed convection flow over a vertical power-law stretching sheet. The governing boundary layer equations were

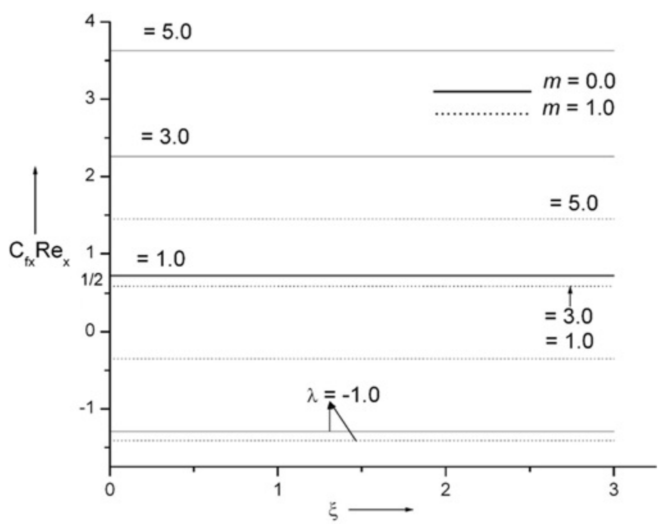

Figure 8.
Effects of λ and m on skin friction coefficient for $\varepsilon = 0.4$ and $Pr = 7.0$

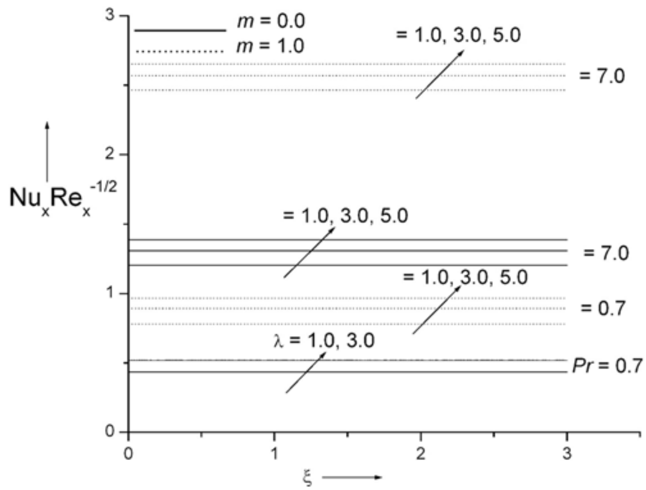

Figure 9.
Effects of λ, Pr and m on heat transfer rate for $\varepsilon = 0.4$

transformed into a set of coupled non-linear partial differential equations subject to relevant boundary conditions. The final set of coupled non-linear partial differential equations was solved using an implicit finite-difference scheme in combination with the quasi-linearization technique. Conclusions of the investigation are as follows:

- The buoyancy force caused overshoot in the velocity profile for lower Prandtl number fluid (air, $Pr = 0.7$) but for higher Prandtl number fluid (water, $Pr = 7.0$) the velocity overshoot was not present.
- The effect of the ratio of free-stream velocity to the composite reference velocity (ε) was significant on the velocity profile.
- The exponent parameter m resulted in thinner momentum and thermal boundary layers and reduced the surface temperature.
- The skin-friction coefficient increased with the ratio of free-stream velocity to the composite reference velocity ε and the buoyancy parameter λ while it decreased with exponent parameter m.
- The heat transfer rate increased with the Prandtl number Pr, buoyancy parameter λ and the exponent parameter m.

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