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Citation: Rev. Sci. Instrum. **67**, 279 (1996); doi: 10.1063/1.1146548

View online: http://dx.doi.org/10.1063/1.1146548

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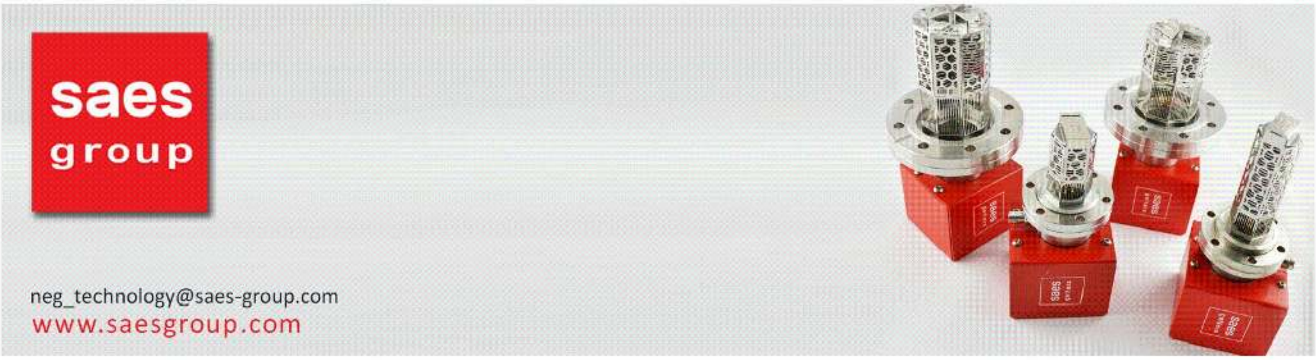

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Measurement of complex dielectric permittivity of partially inserted samples in a cavity perturbation technique

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(Received 3 February 1995; accepted for publication 26 September 1995)

Suitable correction factors are incorporated in the measurement of shift in resonance frequency and full width at half-maximum in the cavity perturbation technique when the sample is inserted partially into the cavity. The present approach is observed to be more accurate compared to the available theories and it does not have limitations concerning the shape of the sample. Several standard samples are taken for the present measurement and compared with the full insertion method. Some of the samples are partially inserted into the cavity step by step and the experimental results are compared with the present approach and that of Lehndroff. © *1996 American Institute of Physics.* [S0034-6748(96)00701-2]

I. INTRODUCTION

The measurement of complex dielectric permittivity at microwave frequencies is considered important since it gives the relaxation time, dipole moment in liquids, characterization of ferrites for device applications, microwave conductivity, momentum relaxation time, effective mass in semiconductors, etc. Depending on the sample conditions, the method of measurement varies: while Surber's plunger technique[1] is widely used for liquids, von Hippel's SWR technique[2] is used for solids.

The cavity perturbation technique was proposed by Montgomery in 1947[3] and further developments in both experimental and theoretical aspects have been made by several workers.[4–7] This technique is utilized for both liquids and solids.[8–10] Simple measurement procedure, high sensitivity, automated experiment facility, direct evaluation of complex dielectric permittivity, as well as magnetic permeability are some salient features of the technique. Though rigorous analysis of Maxwell's equations is available,[5] Murthy and Raman[11] proposed simpler approach by assuming the cavity to be a discrete resonant circuit and arrived at the theory for the evaluation of complex dielectric permittivity. The cavity perturbation technique is based on the change in the resonant frequency and quality factor of the cavity due to the insertion of a sample inside the cavity at the electric field maximum position or the magnetic field maximum position, depending on the nature of the parameter to be studied. All these theories deal with the case where the sample is fully inserted into the cavity, thus occupying the narrow dimension of the cavity, b (see Fig. 1). Therefore, the present measurement procedure which involves the filling of an entire cavity height b may give rise to a broad resonance curve for high loss samples. In order to avoid this problem, only a small amount of the sample is inserted into the cavity. In this case, the available theories for the calculation of complex dielectric permittivity have to be modified. This partial insertion case had been used by Champlin and Krongard[6] in the 1960s for semiconductor spheres. In 1992, Lehndroff[12] put forward a new theory for thin rodlike samples.

This paper presents a new methodology of calculating the complex dielectric permittivity of partially inserted samples having a rodlike structure as well as rectangular cross sections. A comparison between the experimental results obtained with Lehndroff's[12] approach and the present approach is also given.

II. THEORY

The evaluation of complex dielectric permittivity using the cavity perturbation technique involves the measurement of shift in the resonant frequency and decrease in the quality factor of the cavity due to the insertion of the sample at the electric field maximum position. For the full insertion case, the complex dielectric permittivity is given by

$$\epsilon' - 1 = \frac{V_c}{4V_s}\left[\frac{f_0^2}{f_1^2} - 1\right] \tag{1}$$

and

$$\epsilon'' = \frac{V_c}{4V_s}\frac{f_0^2}{f_1^2}\left[\frac{1}{Q_1} - \frac{1}{Q_0}\right], \tag{2}$$

where f_0 and f_1 are the resonant frequency of the empty and sample loaded cavity, respectively, Q_0 and Q_1 are the quality factors of the empty and sample loaded cavity, respectively, and V_c and V_s are the volume of the cavity and the sample, respectively.

Lehndroff[12] has given a solution for the evaluation of complex dielectric permittivity for a thin rodlike sample structure taking into consideration the induced electric field moment or the depolarization factor. The complex dielectric permittivity computed using Lehndroff's[12] method takes into consideration the variation of depolarization factor with the insertion length:

$$\epsilon' - 1 = \frac{1}{N}\left[\frac{\Delta F[(\eta/N) - \Delta F] - (1/Q_s^2)}{(1/Q_s^2) + [(\eta/N) - \Delta F]^2}\right], \tag{3}$$

$$\epsilon'' = \left(\frac{\eta}{N^2}\right)\left[\frac{1/Q_s}{(1/Q_s^2) + [(\eta/N) - \Delta F]^2}\right], \tag{4}$$

where $\eta(=2V_s/V_c)$ is the filling factor, $\Delta F[=(f_0 - f_1)/f_0]$ is the normalized shift in resonant frequency, and $1/Q_s$ is the

TABLE I. The complex dielectric permittivity measured in some solid samples. Here *a*, *t*, and *w* refer to the sample dimensions along broad, narrow, and length directions of the waveguide.

Sample	Dimensions $a\times t\times w$ (mm^3)	Partial ϵ'	Partial ϵ''	Full ϵ'	Full ϵ''
Teflon	3.6×4.4×1	2.2	⋯	2.1	⋯
Glass	4.1×2.6×1.5	5.5	0.1	5.2	0.1
Glass	5.5×2.5×1.5	5.5	0.1		
Porous silicon (2000 Ω cm)	3.4×3.4×0.5	7.3	0.1	7.7	0.1
Silicon (320 Ω cm)	3.8×3.1×0.5	9.4	0.6		
Silicon (80 Ω cm)	2.5×2.3×0.3	11.7	3.6		
Gallium arsenide (12 000 Ω cm)	2.9×2.2×0.5	13.1	⋯		

difference in the inverse of quality factors of the cavity with and without sample. N is the depolarization factor for a long thin rod of finite size as given by[12]

$$N=\frac{3}{2}\left(\frac{d}{t}\right)^2\left[\ln\left(4\frac{t}{d}\right)-\frac{7}{3}\right], \tag{5}$$

where t is the penetration length and d is the diameter of the rod.

The proposed new approach to calculate the complex dielectric permittivity applies correction factors to the measured value of the shift in resonant frequency and the change in quality factor. When $t=b$, the correction factors approach zero.

The variation in the shift in resonant frequency $\Delta f(=f_0-f_1)$ with the insertion length t is mainly due to the additional phase factor created by the sample length. Therefore, to evaluate the effective shift in resonant frequency, a correction factor is included:

$$\text{corr 1}(t)=\left(\frac{1}{4}\right)\left[\frac{b}{t}-1\right]\left[1-\frac{\sin(2\beta_a w)}{\sin(2\beta_s w)}\right], \tag{6}$$

where corr $1(t)$ is the correction factor, β_a and β_s are the phase factors of the air and sample in the waveguide, and w is the thickness of the sample. For full insertion $(t=b)$, the correction factor becomes zero. The factor (1/4) accounts for the effective interaction of electric field intensity on the sample, $(b/t-1)$ involves the normalized length of the sample, and the third factor takes into account the normalized phase factor due to the reduced length $(t<b)$. The correction factor is now incorporated into the evaluation of the effective shift in the resonant frequency as given below,

$$\Delta f_{\text{corr}}(t)=\Delta f_{\text{expt}}(t)\text{corr } 1(t). \tag{7}$$

For $t=b$, $\Delta f_{\text{corr}}(b)=\Delta f_{\text{expt}}$, which is the full insertion case. Now, the corrected value of the resonant frequency is given by

$$f_{\text{corr}}(t)=f_{\text{expt}}(t)-\Delta f_{\text{corr}}(t). \tag{8}$$

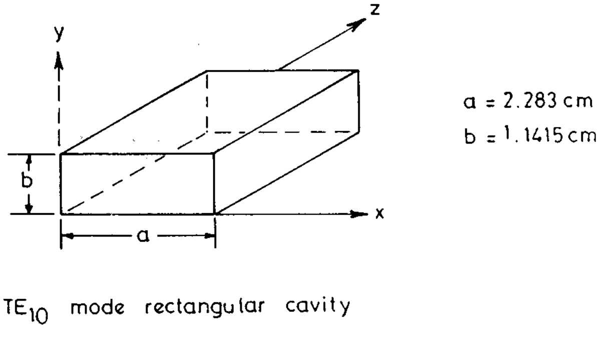

FIG. 1. The rectangular TE$_{10p}$ mode cavity where p is the number of half-wavelengths along the z direction (the TE$_{107}$ mode cavity is used for all the observations).

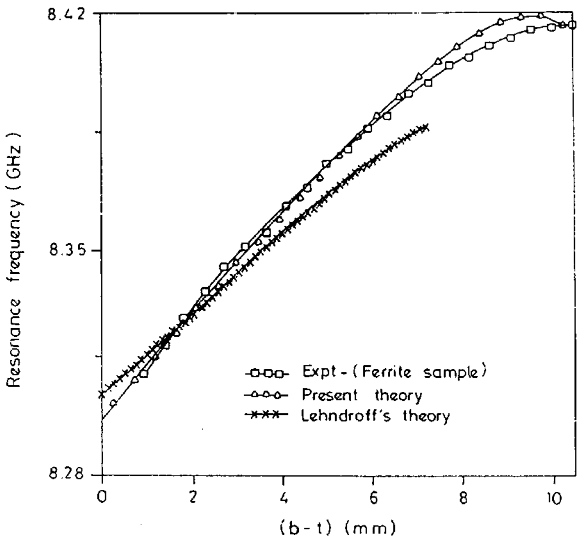

FIG. 2. The experimentally observed variation of resonance frequency with penetration depth of a ferrite sample having dielectric constant 30.5 and loss 7 measured using the full insertion method. The figure shows the theoretical variation by the present method and by Lehndroff's for comparison.

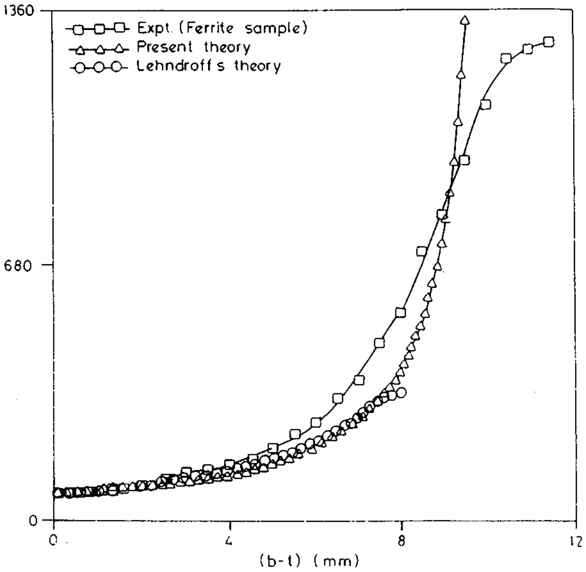

FIG. 3. The experimentally observed variation of quality factor with penetration depth of a ferrite sample having a dielectric constant of 30.5 and a loss of 7 measured using the full insertion method. The figure shows the theoretical variation by the present method and by Lehndroff's for comparison.

TABLE II. Calculated values of dielectric constant in Teflon (3 mm diam) for various insertion lengths.

Insertion length (cm)	Present approach ϵ'	Lehndroff ϵ'
Full	1.99	2.02
1.1165	1.98	2.01
1.0165	1.98	1.99
0.9165	1.97	1.97
0.8165	1.96	1.93
0.7165	1.94	1.87
0.6165	1.92	1.80

The evaluation of dielectric permittivity is given by

$$\epsilon' - 1 = \frac{V_c}{4[1-\text{corr }1(t)]V_s}\left[\left(\frac{f_0^2}{f_{\text{corr}}^2(t)}\right)-1\right]. \tag{9}$$

Here $4[1-\text{corr }1(t)]$ gives the corrected value for the value of 4 since it does not fully occupy the b dimension. For $t=b$, Eq. (9) reduces to Eq. (1).

Similarly, the correction value for full width at half-maximum in the observed resonance curve is

$$\text{corr }2(t) = \frac{1}{4}\left(\frac{b}{t}-1\right)[1-\exp(-2\alpha_s w)], \tag{10}$$

where α_s is the absorption coefficient of the sample in nepher/cm. Here the third factor accounts for the normalized value of the absorption amount due to the reduced length $(t<b)$. The effective value of the full width at half-maximum $[\delta f_{\text{corr}}(t)]$ is given by

$$\delta f_{\text{corr}}(t) = \delta f_{\text{expt}}\text{ corr }2(t), \tag{11}$$

where $\delta f_{\text{expt}}(t)$ is the measured value of full width at half-maximum from the resonance curve $[=f_{\text{expt}}(t)/Q_{\text{expt}}(t)]$.

The corrected quality factor is given as

$$Q_{\text{corr}}(t) = \frac{f_{\text{corr}}(t)}{\delta f_{\text{corr}}(t)+\delta f_{\text{expt}}(t)}. \tag{12}$$

The dielectric loss is given as

$$\epsilon'' = \frac{V_c}{4[1-\text{corr }1(t)]V_s}\left[\frac{f_0^2}{f_{\text{corr}}^2(t)}\right]\left[\frac{1}{Q_{\text{corr}}(t)}-\frac{1}{Q_0}\right]. \tag{13}$$

TABLE III. Calculated values of complex dielectric permittivity for Plexiglas (3.07 mm diam) for various insertion lengths.

Insertion length (cm)	Present approach		Lehndroff	
	ϵ'	ϵ''	ϵ'	ϵ''
Full	2.20	0.003	2.25	0.006
1.0745	2.16	0.003	2.19	0.007
0.9745	2.14	0.003	2.15	0.006
0.8745	2.12	0.003	2.10	0.007
0.7745	2.10	0.004	2.04	0.008
0.6745	2.06	0.004	1.94	0.007
0.5745	2.00	0.006	1.81	0.009

TABLE IV. Calculated values of complex dielectric constant for stycast (3 mm diam) for various insertion lengths.

Insertion length (cm)	Present approach		Lehndroff	
	ϵ'	ϵ''	ϵ'	ϵ''
Full	2.96	0.085	3.10	0.198
1.0935	2.99	0.086	3.12	0.198
0.9935	2.96	0.082	3.05	0.184
0.8935	2.90	0.074	2.94	0.159
0.7935	2.87	0.074	2.82	0.147
0.6935	2.81	0.068	2.63	0.118
0.5935	2.76	0.065	2.40	0.091

III. SAMPLES

The samples used in the present study are standard samples like Teflon™ and glass and semiconductor samples like porous silicon, two single crystal silicon having resistivities of 320 Ω cm (p type) and 80 Ω cm (n type), and gallium arsenide. These samples are rectangular in cross section. Apart from these, rodlike samples of Teflon, Plexiglas, stycast, rexolite, and some ferrite samples are also used.

IV. EXPERIMENT

A rectangular reflection type TE_{107} mode cavity oscillating at 9.8 GHz is connected to the one of the ports of the microwave vector network analyzer (HP 8720A). The cavity has a hole at the center corresponding to the maximum electric field position.

The cavity is connected to one of the ports of the microwave vector network analyzer. The analyzer is controlled by a microcomputer through an IEEE 488 interface bus.

The samples are inserted into the cavity and the resonant frequency $[f_{\text{expt}}(t)]$ and quality factor $[Q_{\text{expt}}(t)]$ are noted from the network analyzer. From these, $\Delta f_{\text{expt}}(t)$ and $\delta f_{\text{expt}}(t)$ are calculated. By initially assuming the values of ϵ', ϵ'', and f_0, β_a, β_s, and α_s are calculated using

$$f_0 = c\sqrt{\frac{\beta_a^2}{4\pi^2}+\frac{1}{\lambda_c^2}}, \tag{14}$$

$$\epsilon' = \lambda_0^2\left(\frac{1}{\lambda_c^2}+\frac{\beta_s^2}{4\pi^2}\right), \tag{15}$$

$$\epsilon'' = \frac{\lambda_0^2}{2\pi^2}\beta_s\alpha_s, \tag{16}$$

TABLE V. Calculated values of complex dielectric permittivity for rexolite (2.9 mm diam) for various insertion lengths.

Insertion length (cm)	Present approach		Lehndroff	
	ϵ'	ϵ''	ϵ'	ϵ''
Full	2.94	0.078	3.08	0.181
1.0865	2.87		2.99	0.168
0.9865	2.82		2.90	0.154
0.8865	2.77	0.067	2.80	0.144
0.7865	2.72		2.67	0.126
0.6865	2.68	0.062	2.51	0.111
0.5865	2.59		2.27	0.081

TABLE VI. Calculated values of complex dielectric permittivity for two ferrite samples (1 mm diam) for various insertion lengths.

Insertion length (cm)	Present		Lehndroff	
	ϵ'	ϵ''	ϵ'	ϵ''
	(a)			
Full	14.84	1.68	18.49	5.62
1.0215	15.59	1.73	19.44	6.26
0.9215	15.39	1.63	18.8	6.08
0.7215	14.25	1.34	15.78	4.89
0.5215	13.09	1.00	11.47	3.00
	(b)			
Full	17.25	2.87	21.75	10.55
1.0215	17.34	2.83	21.62	11.04
0.9215	17.54	2.72	21.55	11.22
0.7215	16.41	2.21	18.49	9.11
0.5215	18.49	2.07	17.95	8.83

where c is the velocity of light and λ_c is the cutoff wavelength.

The approximate values of β_s and α_s are put in Eqs. (6) and (10) to get corr $1(t)$ and corr $2(t)$. Applying these values in Eqs. (9) and (13), the first iterated value of ϵ' and ϵ'' are obtained. The first time iterated values are fed once again in Eqs. (15) and (16) and the first iterated values of β_s and α_s are obtained. This iteration process is repeated until the ϵ' and ϵ'' values are reproduced.

V. RESULTS AND DISCUSSION

The shift in the resonant frequency due to the introduction of the sample is on the order of 70–80 MHz (maximum), giving a fractional change ($\delta\omega/\omega$) of 0.008. Moreover, the maximum fractional change observed in the quality factor is 0.5–0.75. The sufficient criterion for perturbation is the shift in the resonant frequency upon introduction of sample and it should be small.[13] This condition is satisfied in all our measurements.

Table I presents the calculated values of ϵ' and ϵ'' using the full and partial insertion methods. For Teflon, glass, and porous silicon samples, the agreement between the two methods is quite good. In the case of semiconductor samples, the resistivity of 320 Ω cm corresponds to the ϵ'' of 0.6 and 80 Ω cm corresponds to the ϵ'' of 2.5. The literature value of dielectric permittivity ϵ' is 11.7.[14] The literature value of ϵ' for the gallium arsenide sample is 12.5.[14]

One of the ferrite samples is inserted into the cavity step by step at the interval of 0.5 mm at the electric field maximum of the cavity. The resonant frequency and the quality factor as a function of insertion length are observed and are plotted in Figs. 1 and 2, respectively. The dielectric permittivity of the sample is 30.5 and the loss is 7. The theoretical variations proposed by Lehndroff[12] and the present work for this sample are also plotted in Figs. 1 and 2.

Tables II–VI give the values of dielectric constant and loss calculated by the present approach and that of Lehndroff's[12] for Teflon, Plexiglas, stycast, ebonite, and two ferrite samples.

It may be observed from these experimental results that the error percentage for ϵ' measurement comes close to 10% until half of the insertion. The error percentage increases for lesser insertion and lossy samples. It may also be concluded from Fig. 1 that the present approach gives a better fit between the present theory and experiment.

In the case of dielectric loss measurements, both theoretical approaches give a high error percentage. Figure 2 and Tables II–VII show that the present approach is closer than that of Lehndroff's.[12]

It may be concluded that among the available measurement theories for the partial insertion method in the cavity perturbation technique, the present approach has a smaller percentage error. Therefore, even if the present approach gives a 10%–15% error, this gives a new opening to obtain a smaller percentage error with modified correction factors.

ACKNOWLEDGMENTS

One of the authors (V.S.) acknowledges the financial support provided by the CSIR, New Delhi in the form of a Senior Research Fellowship. The authors wish to thank T. K. Chongdar, NCML, Bombay for useful discussions.

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