Technical Note

Lower bound of the fracture toughness relation in biaxial loading

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Abstract: Many studies have been carried out to relate K_I and K_{II} at fracture in biaxial loading. In the present investigation, the Mohr type of stress intensity factor (SIF) circles in uniaxial and pure shear type of loading are presented and discussed for the 'no fracture' relation between K_I and K_{II} for inclined cracks in biaxial loading. This 'no fracture' relation forms the lower bound for the fracture toughness relation between K_I and K_{II}. The experimental data points are seen to lie outside this lower bound for different materials.

Keywords: biaxial fracture, SIF semicircle

1 INTRODUCTION

Fracture of bodies containing inclined cracks and subjected to the biaxial type of loading has been investigated by several researchers [**1–6**], based on the stress distribution and strain energy in the core region ahead of the crack. The size and shape of the core region and consequently the strain energy absorbed vary depending on the assumptions made. In his review paper Theocaris [**7**] discussed the shortcomings of these theories based on different criteria. The role of frictional stresses in mode II loading and the effect of T-stress (normal stress acting parallel to the crack plane) are not clear in the fracture process and hence it is difficult to correlate the fracture toughness relation between K_I and K_{II} and the experimental data.

In the present investigation the relationship between K_I and K_{II} acting on an inclined crack in biaxial loading is discussed based on the stress intensity factor (SIF) circles in uniaxial and pure shear type of loading. The analysis gives a 'no fracture' condition which forms the lower bound for a theoretical approach based on any criterion.

2 STRESS INTENSITY FACTOR SEMICIRCLES

Theocaris and Michopoulos [**8**] have shown that the stress intensity factors associated with σ_x, σ_y and τ_{xy} acting on a body containing a through crack can be represented by a Mohr type of circle. Adopting this procedure [**9**], a body containing a through crack and loaded uniaxially in the principal direction 1 is considered. The loading condition and the SIF circle are shown in Fig. 1. The stress intensity factors K_{I1} and K_{I2} correspond to the principal stresses σ_1 and σ_2 acting in the principal directions. When the biaxiality ratio λ ($= K_{I2}/K_{I1}$) is equal to zero and the crack in the body is parallel to the principal direction 2, the maximum K_{I1} for fracture is K_{Ic}, as indicated by point A on the X axis. Thus the critical crack angle α_c for the loading condition $\lambda = 0$, $K_I = K_{Ic}$ is equal to zero.

Any other point (say point C) on the SIF circle will give the normal SIF, K_I, and the SIF due to shear, K_{II}, for an inclined crack. The crack angle α with respect to direction 2 is defined by the line OC. The lower portion of the SIF circle will give the other normal notional SIF, K_I'. This K_I' acts parallel to the crack plane and hence will not influence the brittle fracture of the body. Therefore the lower portion of the SIF circle is shown in dotted lines here and only the upper semicircle is considered for discussion. With the present loading condition $K_{I1} = K_{Ic}$ and with the material fracture toughness ratio m ($= K_{IIc}/K_{Ic}$) greater than 0.5, the inclined crack will not cause fracture. K_{I1} has to be increased further beyond K_{Ic} to cause fracture for the inclined crack condition, and the SIF semicircle for inclined cracks causing fracture will be different. The point C, or any other point on this semicircle ($\lambda = 0$, $K_I = K_{Ic}$) that gives K_I and K_{II} on the inclined crack, does not give the fracture condition. Since the points (K_I,K_{II}) on the semicircle represent a 'no fracture' condition on the K_I versus K_{II} plane, they will

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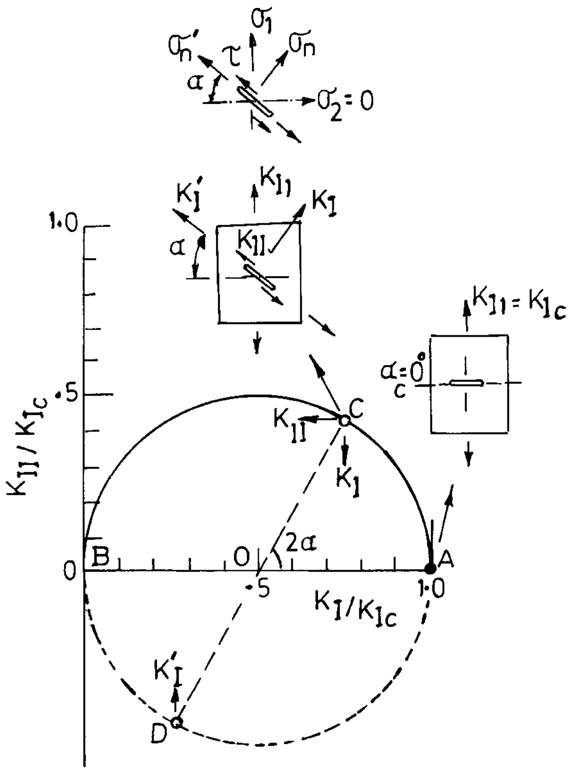

Fig. 1 SIF semicircle for uniaxial loading with $\lambda = 0$

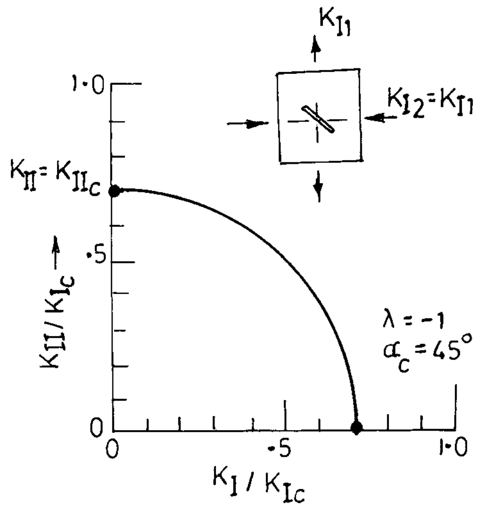

Fig. 2 SIF semicircle for shear loading with $\lambda = -1$

represent the same condition even when the loading conditions (i.e. λ values) are changed.

Similarly, in biaxial loading with $K_{I1} = -K_{I2}$, the biaxiality ratio is $\lambda = -1$. When the crack angle $\alpha = 45°$ to both directions 1 and 2, fracture occurs at $K_{II} = K_{IIc}$ $(= K_{I1} = -K_{I2})$. The SIF semicircle corresponding to this loading condition is shown in Fig. 2. Thus, for $\lambda = -1$ and $K_{II} = K_{IIc}$, the critical crack angle α_c is equal to 45° to the principal direction. Any other point on the SIF semicircle can give K_I and K_{II} for the crack of other inclinations, but it does not give the fracture condition in the present loading $K_{II} = K_{I1} = -K_{I2} = K_{IIc}$.

A point (K_I, K_{II}) on the K_I–K_{II} plane giving a 'no fracture' (or fracture) condition will represent the same condition irrespective of the loading system (λ value) which produces these components K_I and K_{II} (assuming the effects of frictional stress and T-stress to be negligible). Thus the points on the above two semicircles ($\lambda = 0$, $K_I = K_{Ic}$ and $\lambda = -1$, $K_{II} = K_{IIc}$), represent K_I and K_{II} values for the 'no fracture' condition and will continue to be so even under different loading conditions.

3 SIF SEMICIRCLE AND THE FRACTURE TOUGHNESS LINE

Now the two SIF semicircles and the fracture toughness relation line between K_I and K_{II} can be combined together to find out whether the theoretical relation that was developed based on some criterion could be correct or whether the assumptions made require reconsideration.

Figure 3a shows a hypothetical fracture toughness relation line (FTL) of a material with $0.5 < m < 1$, along with the SIF semicircle ($\lambda = 0$, $K_I = K_{Ic}$ and $\alpha_c =$ zero). The 'fracture' and 'no fracture' zones separated by the FTL are also shown. In the present example, the SIF semicircle intersects the FTL at C. The crack plane corresponding to point C makes an angle of α with direction 2. The points on the semicircle to the right of point C are in the 'fracture' zone and those on the left-hand side are in the 'no fracture' zone. This means that points on the right-hand side of C on the semicircle ($\lambda = 0$, $K_{I1} = K_{Ic}$), which indicate the 'no fracture' condition, are made to fall in the 'fracture' zone due to the hypothetical FTL cutting the semicircle. As this is not possible, the other alternative is that the FTL should not intersect the SIF semicircle defined by $\lambda = 0$, $K_I = K_{Ic}$ and $\alpha_c = 0$, but it should just touch the SIF semicircle at $K_I = K_{Ic}$ and both will have a common tangent at the point $K_I = K_{Ic}$ on the X axis.

Figure 3b shows the FTL and the SIF semicircle ($\lambda = -1$, $K_{II} = K_{IIc}$, $\alpha_c = 45°$). If the FTL intersects the SIF semicircle (say at point C), it means that points left of point C on the semicircle ($\lambda = -1$, $K_{II} = K_{IIc}$) are brought into the 'fracture' zone by the hypothetical FTL, though actually they represent a 'no fracture' relation between K_I and K_{II}. Hence the FTL should not intersect the SIF semicircle ($\lambda = -1$, $K_{II} = K_{IIc}$, $\alpha_c = 45°$). Both of them should just touch each other at $K_{II} = K_{IIc}$ and have a common tangent at the point $K_{II} = K_{IIc}$ on the Y axis.

Thus the two SIF semicircles give the possible lower limit for the fracture toughness relation in biaxial loading in the range of uniaxial tensile loading ($\lambda = 0$) to pure shear loading ($\lambda = -1$).

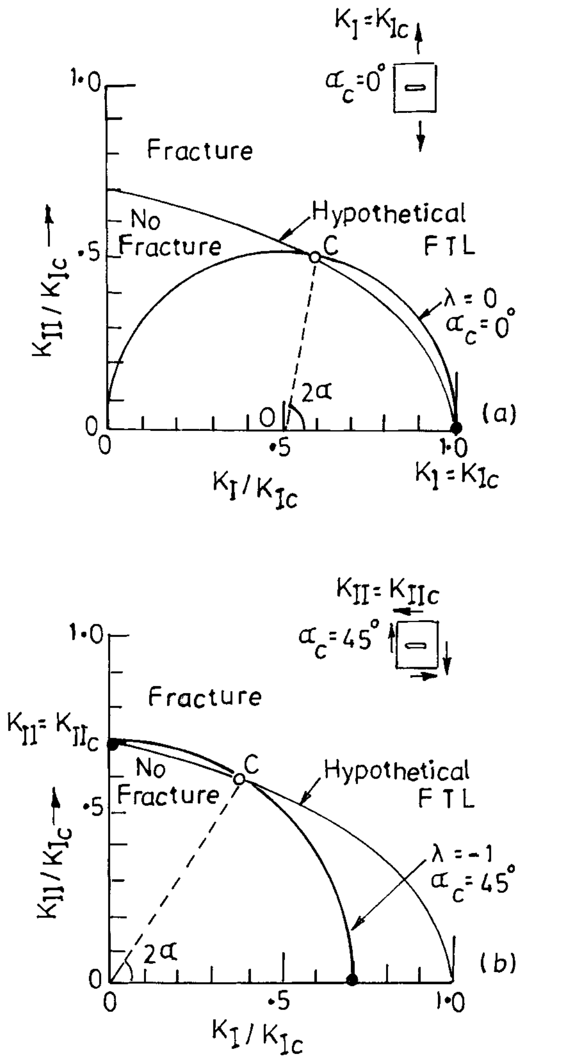

Fig. 3 Validation of the fracture toughness relation in biaxial loading

4 LOWER BOUND OF THE K_I–K_{II} RELATION

Many investigations have been made to correlate K_I and K_{II} in biaxial loading. In this section the lower bound of such correlation along with some experimental data are discussed based on the SIF semicircles corresponding to uniaxial and pure shear types of loading conditions.

Figure 4 shows the data points, with the base values taken from reference [**2**], for three materials indicated. The SIF semicircles corresponding to $\lambda = -1$ and $\lambda = 0$ are also shown in the figure, with the fracture toughness ratios $m = 0.8$ and $m = 1.0$. Since the SIF semicircles show a 'no fracture' relation between K_I and K_{II}, the experimental points must lie above these lines. The data points do indicate that they generally obey this lower bound condition for the value of m lying between 0.5 and 1.0. The lower bound SIF semicircles imply that any theoretical correlation developed to relate K_I and K_{II} should not intersect these semicircles.

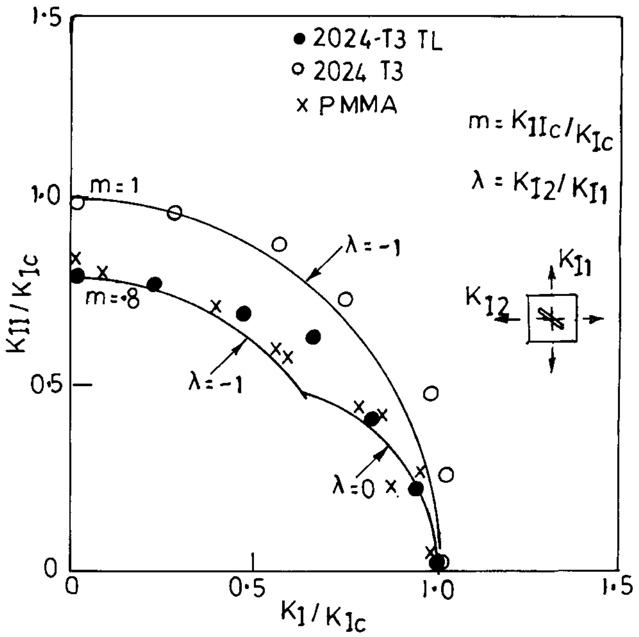

Fig. 4 Lower bound and data correlation for $m = 0$ and $m = 1$

Figure 5 shows the data points for material 7073-T6 Al alloy tested in different conditions [**3**]. The fracture toughness ratio m for this material is greater than unity. All the data points are lying outside the lower bound SIF semicircle corresponding to $\lambda = -1$. However, since the value of K_{IIc} is higher than that of K_{Ic}, below a certain angle α the normal SIF on the inclined crack K_I will be equal to or more than K_{Ic}. Thus, for example, in material M1, K_I can cause fracture in the presence of slanting

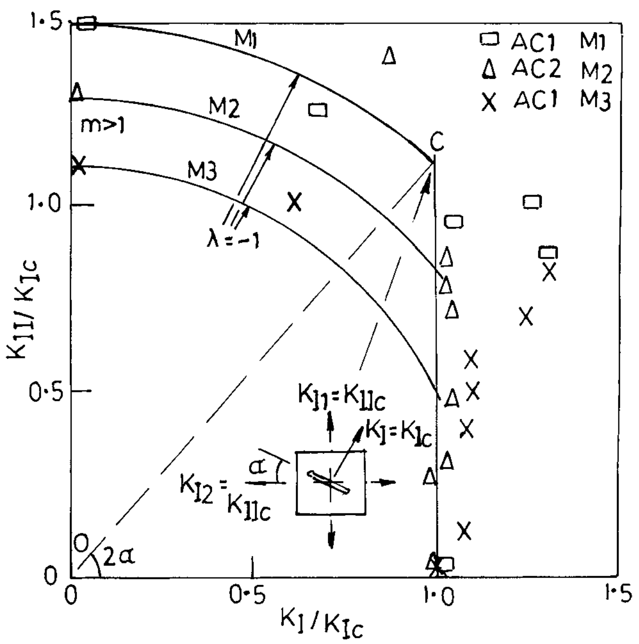

Fig. 5 Lower bound and data correlation for $m > 1$

cracks with angles less than α defined by line CO, as indicated in the figure. Thus, in such cases with $m > 1$, the lower bound line for crack angles less than $\propto$ will be a vertical line passing through $K_I = K_{IC}$, as shown in the figure.

5 CONCLUSIONS

The Mohr type of SIF semicircles for uniaxial and pure shear types of loading are developed which give the 'no fracture' relation between K_I and K_{II} for biaxial loading. These circles are clubbed with the K_I–K_{II} fracture toughness line to show that the theoretical relation K_I versus K_{II} and the experimental points must lie above these semicircles, which indicate the lower bound. The lower bound lines are discussed for different fracture toughness ratios m (K_{IIC}/K_{IC}).

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