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Large Elasto-Plastic Deflection of Thin Beams with Roller Support Contact

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Abstract

Here, an explicit numerical approach is presented to solve the problem of a three-point bending of a thin elasto-plastic beam undergoing large deflection supported on cylindrical rollers with radius comparable to the deflection. There are three sources of non-linearity in this problem: roller to beam contact, plastic deformations and large deflections. An incremental form of moment-curvature based constitutive law is derived from a uniaxial linearly hardening stress-strain material model. The governing boundary value problem in terms of its slope is transformed into an initial value problem, with the domain up-to the point of contact. The non-linear initial value problem is linearized about the current step and solution for the subsequent step is obtained by employing the classical Runge-Kutta fourth-order method. This solution procedure is repeated for a range of lengths and end angles of beam. Subsequently, a feasible data set is created from the solution space which satisfied the contact configuration condition. It is found that the increase in the radius of roller introduces a stiffening effect in the force response of the structure. A springback analysis is also performed for the beam data from a feasible set which satisfies plastic deformation condition. It is found that springback decreases with the increase in the radius of roller supports.

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Keywords: Large deflection; Bilinear hardening; Elasto-plastic deformation; Non-linear differential equation; Three-point bending

1. Introduction

Bending experiments are generally easier to conduct than uniaxial tests. Among the bending tests, the three-point bending test is one of the most widely conducted experiments for determining mechanical properties of materials. If a relatively thin specimen is bent beyond its elastic limit, then large deflection is expected. The study of elastic large deflection of slender beams also known as 'elastica' is an extensively studied area, see [1]. The solution of elastic problems may be analytically obtained in the form of elliptic or Jacobian integral forms, see [2] and the literature therein. [3] solved the elastica problem under follower load, completely numerically in an explicit manner employing a variable transformation rule which transformed the boundary value problem into an initial value problem. On the

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other hand, the elasto-plastic large deflection of beams have relatively less literature and are solved semi-analytically or numerically, see [4],[5],[6] etc.

In a three-point bend test, where support rollers are of dimension comparable to the displacement that the beam undergoes, is a problem which involves three kinds of non-linearity: material non-linearity due to plastic deformation, geometric non-linearity due to slip of beam over the roller supports and large deflection (where slope of beam cannot be neglected with respect to 1). Though a FEM based approach of modeling contact, plasticity and geometric non-linearity is accurate to simulate the response, it is computationally expensive, see [7]. A mechanics of materials based non-FEM method which is computationally economic possessing fair predictive capability was introduced in [8]. This approach was employed to solve the elasto-plastic air-bending of a slender beam, i.e without the contact non-linearity. Employing a modified version of the above mentioned approach, the elasto-plastic vee-bending of a slender beam was solved in [9]. On the other hand, the elastic bending of a slender beam on finite dimensional roller supports, was solved efficiently in [10].

What remains in hand is that of solving the large deflection three-point bending of an elasto-plastically deforming beam supported on rollers of dimension comparable to the deflection. The present problem is also solved by employing the approach presented in [9] which is simple, economic and with fair predictive capability. In this approach the non-linear governing differential equation of an end loaded cantilever is solved by linearizing about the current pseudo time step and subsequently solving the linearized initial value problem by Runge-Kutta fourth-order classical solver. This process is repeated for a range of beam lengths and end angles. Those data which satisfied the kinematic condition of contact configuration are picked to form the feasible data set. In the present work, the dependence of the deformed shape of the beam upon removal of load, on various bending parameters is also studied from its springback response.

2. Formulation

In Fig. 1 Left a long slender beam obeying Euler-Bernoulli hypothesis is supported on two friction-less cylindrical roller supports of radius R each. A central load denoted by F acts vertically downward in a quasi-static manner. Owing to symmetry of support and loading, it is easy to observe that an equivalent problem involving a cantilever loaded by a cylindrical roller punch of radius R as shown in Fig . 1 Right exists. The locus of the center of the roller punch in this equivalent problem, remains at a constant horizontal distance L from the fixed end of the cantilever.

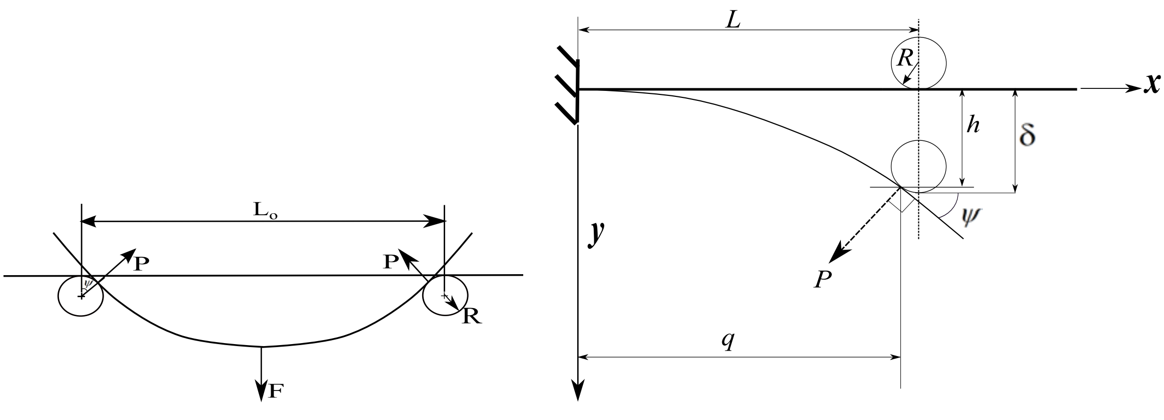

Fig. 1. Left: A long thin beam supported on finite dimensional cylindrical roller supports. Right:A long cantilever under friction-less roller punch induced large deflection

The kinematic and kinetic relationships connecting the original problem to the equivalent cantilever problem are:

$$2L = L_o \tag{1}$$

And

$$2P\cos\psi = F \tag{2}$$

The roller punch is assumed to be rigid and friction-less. When the roller has descended by δ, the beam also deflects and the position of the beam-roller contact point in the deformed configuration is defined by its coordinates: $x, y = q, h$. the kinematic condition of contact configuration between beam and roller punch are mathematically represented by:

$$\delta = h + R(1 - cos\psi) \tag{3}$$

And

$$L = q + Rsin\psi \tag{4}$$

Considering a coordinate system measured along the deformed beam, starting from the fixed end up to the point of contact, we can obtain the governing differential equation in terms of the slope angle of the deformed beam. Assuming that the new coordinate is defined by s, the angle the tangent at any s makes with the horizontal x axis to be $\phi(s)$ and the point of contact to be at $s = l$ (not L), the governing equation is given by: (as in [9])

$$D\frac{\partial^2\phi}{\partial s^2} + Pcos(\psi - \phi) = 0, \quad 0 \leq s \leq l, \quad \psi = \phi|_{s=l} \tag{5}$$

The boundary conditions are given by: $\phi|_{s=0} = 0$, $\frac{\partial\phi}{\partial s}|_{s=l} = 0$, in which D is the tangent modulus of rigidity or in other words the slope of moment-curvature response of the beam. Here elastic-perfectly plastic material model is chosen for analysis. The uniaxial bi-linear stress-strain law is converted to an incremental moment-curvature based isotropic hardening constitutive law as in [9]. The law is pictorially represented in Fig. 2, where M_{y0} and κ_{y0} are the initial yield moment and curvature respectively. In this figure, the quantity $'m'$ designates the ratio of tangent modulus to Young's modulus of the material. Clearly, $0 \leq m \leq 1$ with lower and upper bounds of m implying elasto-perfectly plastic and completely elastic material models respectively.

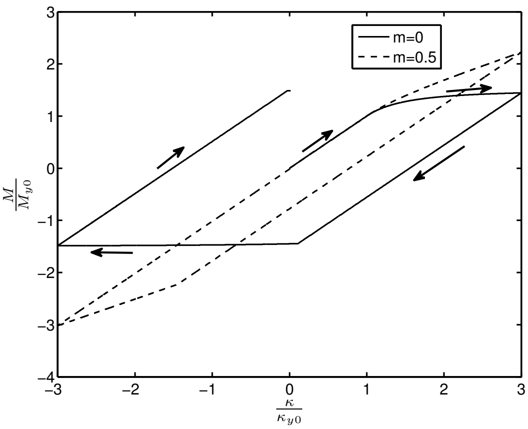

Fig. 2. Constitutive law for rectangular beam cross section

3. Numerical Methodology

The governing Eq. 5 which is a boundary value problem is normalized and transformed into an initial value problem (IVP) and is given by:

By a transformation of variable as given in [3], the boundary value problem defined by Eq. 5, is transformed into an initial value problem which reads:

$$D\frac{\partial^2\alpha}{\partial s^2} - Pcos\alpha = 0 \tag{6}$$

With initial conditions: $\alpha|_{s=l} = 0$, $\frac{\partial\alpha}{\partial s}|_{s=l} = 0$, in which the variable transformation rule is given by, $\alpha = \psi - \phi$

By linearizing Eq. 6[1] at the current step an incremental form of IVP is obtained. This incremental IVP is solved for increment of α by assuming the coefficients to be at their respective values of the previous step, which are known. Classical Runge-Kutta fourth-order method is used to solve the IVP for any given beam length l and end angle ψ. This process of solving the incremental IVP is repeated for a range of l and $\psi \leq \frac{\pi}{2}$, and only those solution data which satisfied the kinematic condition as given in Eqs. 3 and 4 are considered to form the feasible solution set.

In the three-point plastic bending process, the roller punch first monotonically reaches a maximum displacement of δ_f and then it retracts back to unload the beam completely. Since permanent shape change takes place in this loading followed by unloading process, a measure to quantify the shape change and its dependence on the various process parameters are of interest. Springback is a way of quantifying the shape change in beams, defined as:

$$\eta = \frac{\psi_l - \psi_{ul}}{\psi_l} \tag{7}$$

where ψ_l denotes the maximum loaded end angle when the roller punch is displaced by δ_f and ψ_{ul} denotes the unloaded end angle of the beam.

In the process of normalization of pertinent variables, an interesting non-dimensional parameter denoted by ζ is obtained. It has both geometric and material properties and is given by: $\zeta = \kappa_{y0}L$. The force and springback response on various bending process parameters viz. $'\zeta'$, radius $'R'$ of roller punch and non-dimensional tangent modulus $'m'$ are studied in the next section.

4. Results and Discussion

Employing the present approach, the elastic results are validated with [10] by setting $m = 1$ in the formulation. Assuming a very small radius of roller and considering elastic -perfectly plastic material model (i.e. $\frac{R}{L} = 0.003$, $m = 0$), the force response predicted by present method is validated with the air- bending response of [8].

In Fig. 3 the force-displacement response of the structure under various conditions is presented. It can be seen that the initial part of each of the response curves are linear, which conforms with the small deflection linear elastic beam theory. As expected, the elastic ($m = 1$) response is stiffer than its elaso-plastic ($m < 1$) counterpart with other process parameters remaining same. Additionally, instability or negative stiffness is seen to develop in the elasto-plastic beam for a much smaller displacement than its elastic counter part. Increasing the roller radius is observed to increase the stiffness of the structure. It may be noted here that the response curve corresponding to the higher radius ($m = 0, \frac{R}{L} = 0.3, \zeta = 0.5$) is seen to intersect lower radius response curves ($m = 1, \frac{R}{L} = 0.03, \zeta = 0.5$) and ($m = 0.5, \frac{R}{L} = 0.03, \zeta = 0.5$). This implies, a given solution of force-displacement data point may be obtained from various combinations of process parameters. Also it may be seen that lower the magnitude of the parameter ζ, lower is the stiffness.

In Fig. 4 the springback-displacement response of the structure under various conditions is presented. In each of the response curves it is observed that with increase in displacement the springback decreases. At a given displacement, springback is observed to be lower for larger radius. Similarly for a given displacement, higher ζ accounts for more springback. Since the springback response is meaningful only when plastic deformation takes place (for elastic case $\zeta = 1$), it can be seen from the figure that higher ζ ensures the start of plastic deformation at higher displacement. Similar to that observed in force-displacement plot, in the springback response also it can be seen that a given springback-displacement data point may be obtained from different combinations of process parameters. This phenomenon is observed for the case with same radius ($\frac{R}{L} = 0.03$) but $\zeta = 1, 0.5$ and $m = 0, 0.5$, which produces a

[1] detailed in [9]

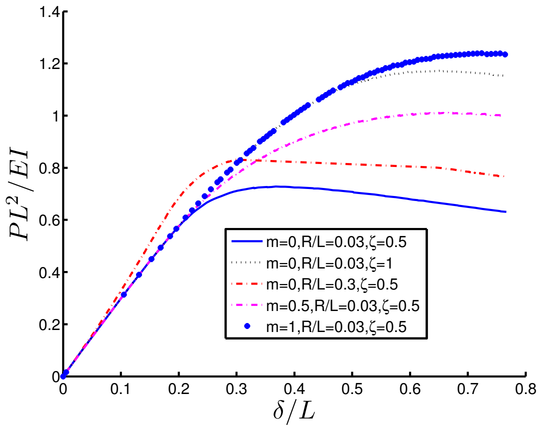

Fig. 3. Force-displacement response for various process parameters

springback of around 0.8. This observation may encourage a designer to chose material and dimension of beam and fixture in such a way to achieve a given springback.

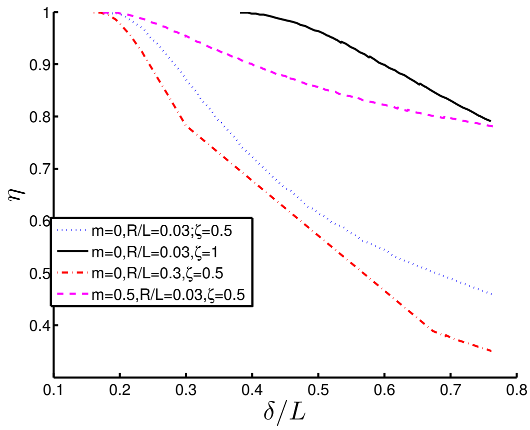

Fig. 4. Spingback response for various process parameters

5. Concluding Remarks

The approach presented in [9] is employed here to solve the problem of large deflection of an elasto-plastic beam undergoing three-point bending on finite dimensional roller supports. A full fledged contact modeling with material and geometric non-linearity modeling would have been much more expensive than the method used here.
The force response showed stiffening effect with increase in radius of roller. The springback response showed that increasing the radius reduces the springback. It is also observed that in both force and srpingback responses, the same solution from various combination of process parameters is possible to be obtained.

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