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Fluid flow with variable properties in a two-dimensional channel

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Asymmetric velocity distribution for a fluid with a highly temperature sensitive viscosity coefficient in a two-dimensional channel is obtained as a function of the temperature difference between channel plates. Numerical results for three liquids have been obtained.

Two-dimensional channel flows have been examined by many authors in investigating many fluid-mechanical problems. One such problem is the stability of flow in a two-dimensional channel. If the flow velocity profile is symmetric with respect to the channel axis, and for constant fluid properties flow, the critical Reynolds number based on maximum fluid velocity and channel semiwidth has been found to be[1,2] around 5300. For asymmetric velocity profiles, the critical Reynolds number is increased considerably. For one particular asymmetric velocity profile, the critical Reynolds number is reported to be[3] as high as 12 000, and thus the high stabilizing effect of an asymmetric velocity profile has been established. Fu and Joseph[4] studied the linear instability of asymmetric flows, in which the stabilizing effect of asymmetry and the dominating destabilizing effects of the inflection of the basic velocity profiles were obtained. Chen[5] treated the asymmetric hydrodynamically developing channel flows. The results show greater stability for more skewed inlet velocity profiles. However, how the asymmetric velocity profile is to be obtained was never discussed in Refs. 3–5.

One way of obtaining an asymmetric velocity profile in a two-dimensional channel is by having a fluid with a highly temperature sensitive viscosity coefficient and an asymmetric temperature profile by having two different temperatures at two walls. While the boundary conditions at two walls are thus fixed, an actual temperature profile is dependent on the ratio of heat transfer by convection to the heat transfer by conduction. It is evident that the analysis is simplified for fully developed temperature profiles. For large Reynolds numbers, which are necessary to investigate flow instabilities, fully developed temperature profiles are obtained for small Prandtl numbers, or for large viscous dissipation. In the present case, the analysis is strictly applicable for small Prandtl numbers and negligible viscous dissipation, although the results are qualitatively applicable for all two-dimensional, laminar channel flows. In investigating the effect of variable fluid properties on the stability of a two-dimensional channel flow, mean viscosity and density profiles need be considered, although their fluctuations are neglected to avoid too bulky expressions. This is strictly true for the low Mach number case. By considering the mean viscosity and density variations, but not their fluctuations, it is possible to obtain a linearized flow stability equation, which for the constant properties case reduces to the well-known Orr–Sommerfeld equation. The present note represents only one part of the investigation, in which the effect of variable fluid properties on the mean velocity profile has been investigated. The results are useful for experimental verification of a theoretical investigation of effects of variable fluid properties on critical Reynolds number, which will be the subject of a future report.

The method of attack is based on relating the inverse of the dimensionless viscosity coefficient to a third order polynomial of temperature. The coefficients of the polynomial are obtained by the least square method with maximum error of 3% in the case of glycerine and 6% in the case of lubricating oil (SAE 50). The effect of variable fluid properies on mean velocity profile is determined with the help of closed form solutions of the Navier–Stokes equation with a linear temperature distribution. Numerical results are obtained for aniline, glycerine, and lubricating oil, and the effect is found to be largest in the case of glycerine for the same temperature difference between the plates. The main advantage of the present method is that a closed form solution of an asymmetric velocity profile as a function of temperature difference is obtained, which can easily be used in further calculations.

The flow of a viscous incompressible fluid confined between two parallel horizontal plates, which are a distance b apart has been studied with local velocity u and pressure p. Let x and y denote the coordinates along and normal to the flow direction, respectively, the latter being measured from the lower plate. The upper plate is kept at a temperature T_2 and the lower plate at T_R, the reference temperature. The nondimensional form of the Navier–Stokes equation for this system is represented by

$$\mathrm{Re}(dp^*/dx^*) = d[u^*(du^*/dy^*)], \qquad (1)$$

where

$$u^* = u/U; \quad y^* = y/b; \quad p^* = p/\rho U^2,$$
$$x^* = x/b; \quad \mu^* = \mu/\mu_R, \quad \mathrm{Re} = \rho U b/\mu_R,$$

U being the maximum fluid velocity occurring at y_m

TABLE I. Values of the coefficients α–δ for three liquids.

Liquids	α	$\beta \times 10^2$	$\gamma \times 10^4$	$\delta \times 10^5$
Aniline	1.06	1.9218	5.0750	0.00
Lubricating oil SAE 50	1	7.5370	21.8479	1.5390
Glycerine	1	10.0200	15.4560	7.4579

and μ_R being the dynamic viscosity coefficient of fluid at the reference temperature. In Eq. (1), the inverse of μ^* is given by a third-order polynomial as

$$(\mu^*)^{-1} = \alpha + \beta\theta + \gamma\theta^2 + \delta\theta^3. \qquad (2)$$

For the case in which the reference plate temperature T_R is taken to be 300°K, the values of the coefficients $\alpha - \delta$ have been determined for three liquids and have been given in Table I. Equation (1) is now integrated subject to no-slip conditions at the wall, as well as the conditions that at $y^* = y_m^*$, $u^* = 1$. Assuming a linear temperature distribution, $\theta = T - T_R = \theta_2 y^*$, $\theta^* = \theta/\theta_2 = y^*$, θ_2 being equal to $T_2 - T_R$, the resulting equations are

$$y_m^* = [(\alpha/2) + (\beta\theta_2/3) + (\gamma\theta_2^2/4) + (\delta\theta_2^3/5)]/[\alpha + (\beta\theta_2/2) + (\gamma\theta_2^2/3) + (\delta\theta_2^3/4)] \qquad (3)$$

$$u^* = [\alpha y^* y_m^* + (\beta\theta_2 y_m^* - \alpha)(y^{*2}/2) + (\gamma\theta_2^2 y_m^* - \beta\theta_2)(y^{*3}/3) + (\delta\theta_2^3 y_m^* - \gamma\theta_2^2)(y^{*4}/4) - \delta\theta_2^3 y^{*5}/5]/ y_m^{*2}[\alpha/2 + (\beta\theta_2 y_m^*/6) + (\gamma\theta_2^2 y_m^{*2}/12) + (\delta\theta_2^3 y_m^{*3}/20)]. \qquad (4)$$

The velocity distribution across the channel width can now be written in the form

$$u^* = A_0 y^* - A_1 y^{*2} - A_2 y^{*3} - A_3 y^{*4} - A_4 y^{*5}. \qquad (5)$$

Values of the coefficients, as well as y_m^* for liquid aniline, lubricating oil, and glycerine for various temperature differences have been calculated, but for lubricating oil only these values are given in Table II.

Velocity distribution for three liquids with strong temperature-dependent viscosity coefficients have

TABLE II. Values for the coefficients for lubricating oil.

Temperature difference	A_0	A_1	A_2	A_3	A_4	y_m
10	3.1212	1.6279	1.1817	0.2943	0.0172	0.5565
20	2.4569	0.1954	0.3416	0.8189	0.1008	0.6000
30	1.9530	−0.6645	1.0464	1.3145	0.2565	0.6326
40	1.5682	−0.1710	0.5703	1.6989	0.4700	0.6572
50	1.2722	−1.4563	0.0472	1.9574	0.7239	0.6761

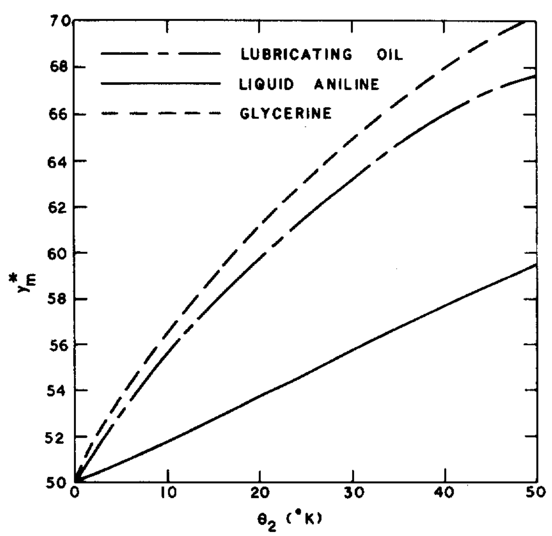

FIG. 1. Position of maximum velocity vs temperature difference.

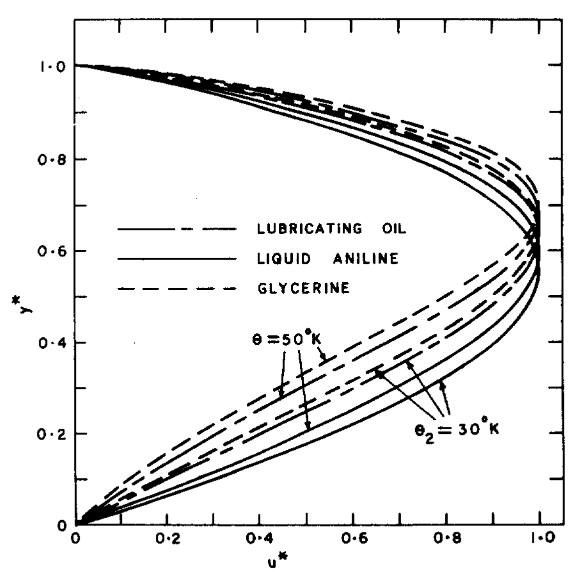

FIG. 2. Velocity distribution.

been calculated under the restriction that the temperature distribution is linear. Figure 1 shows clearly how the maximum velocity shifts to the side of the hotter channel wall. The temperature difference between the two plates need not be too large, and a temperature difference of only 50°C is enough to show a shift in the position of maximum velocity by as high as 20% of the channel width. Figure 2 shows these clearly for three liquids. The error in evaluating the coefficients in Eq. (5) clearly depends on the accuracy with which the viscosity coefficient is reproduced by Eq. (2). This is conveniently done by the method of least squares, and the viscosity coefficients have a maximum error of 6%. From the above analysis it is concluded that with relatively small temperature differences between two plates, it is possible to develop a highly asymmetric velocity profile, which is quite stable.

This note presents part of the results of research by the first author (MKB) carried out at the Department of Aeronautical Engineering, Indian Institute of Technology, Madras, India.

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