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Flame blowout: Transition to an absorbing phase

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The turbulent flame inside a gas turbine engine is susceptible to local extinction leading to global extinguishment or blowout at fuel lean conditions. Flame blowout is traditionally viewed as a loss of static stability of the combustor. However, flames often exhibit rich dynamics as blowout is approached suggesting that a more comprehensive description of the dynamics of flame blowout, which could lead to reduced order models, is necessary. A turbulent flame can be considered as a collection of a large number of flamelets. The population dynamics of these flamelets could be used to model the overall flame behavior as a contact process. In this context, flame blowout can be viewed as the population of flamelets approaching zero, in other words, extinction of flamelets. In this paper, we employ a cellular automata based model to study the emergent dynamics of the population of such flamelets. We show that the model is able to qualitatively capture interesting dynamics that a turbulent flame inside a combustor exhibits close to flame blowout. Furthermore, we show that flame blowout is similar to a threshold-like transition to an absorbing phase. *Published by AIP Publishing.*
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A turbulent flame can be considered as a collection of mutually interacting flamelets. The interaction of these flamelets among themselves and with the walls of the combustor results in the turbulent combustor exhibiting various dynamics such as stable combustion, intermittency, thermoacoustic instability, flashback, and flame blowout. Such a system that consists of multiple parts interacting with each other and exhibiting an emergent collective behavior is called a complex system. In this paper, we argue that a turbulent combustor can be considered as a complex system and model the turbulent flame as a collection of flamelets using the framework of a finite state cellular automaton. The rules for the evolution of the cellular automaton are defined considering flame propagation as a contact process. We analyze the evolution of the cellular automaton to study the dynamics of population of flamelets and further compare it with the dynamics of the flame observed in experiments. In particular, we focus on the transition of the combustor from a state of stable combustion to flame blowout. In the context of population dynamics, flame blowout corresponds to a transition from an active regime of chemical reaction to an absorbing state, i.e., the extinct state of the population of flamelets which admits no further evolution in time.

I. INTRODUCTION

Over the past several decades, there have been various attempts to study lean blowout (LBO). Early studies by Longwell *et al.*[1] and Spalding[2] suggest that the Damköhler number (*Da*) which is the ratio of the residence time of the reactants to the reaction time scale is an important parameter that determines the lean blowout (LBO) margin. They conjectured that LBO occurs when *Da* of the reactive flow is less than a threshold value. Such a simple description could seldom characterize the blowout dynamics of a turbulent flame. For a turbulent flame, LBO could be described as a collective flamelet extinction event[3,4] and the ratio of the flow speed to the flame speed can be used to characterize blowout. When this ratio exceeds unity at critical locations inside the combustor, the flame blows out. Nair and Lieuwen[5] and Chaudhuri *et al.*[6,7] showed that the blowoff dynamics is far more complex and is characterized by multiple stages. Chaudhuri *et al.*[6,7] showed that the final stage of blowout is characterized by distinct extinction and reignition events causing the extinguishment of the flame. As such, these local extinction and reignition events are governed by various factors such as highly intermittent local strain rates arising from turbulence, coherent strain rates arising from vortex shedding, heat loss from the flames to the flame holder or to the flow, differential diffusion effects, the local boundary conditions, and the detailed chemistry of the reaction. The interaction effects of the above add further complexity to the problem.

This complex interaction between different processes causes a highly unsteady and aperiodic behavior of the flame prior to blowout. Muruganandam[8] and Nair[9] observed that both chemiluminescence and pressure fluctuations close to flame blowout exhibit "bursty" behavior. Gotoda *et al.*[10] further observed that these oscillations also exhibit multifractal characteristics. The flame dynamics close to blowout was investigated by De Zilwa *et al.*[11] using high-speed flame images and simultaneous pressure fluctuations. They observed that the flame exhibited highly unsteady behavior prior to lean blowout. Nair and Lieuwen[5] showed that the approach to blowout is through the increased appearance of flame holes in the reactive flow field in a random manner. Later, Chaudhauri *et al.*[6,7] and Tuttle *et al.*[12] investigated the interaction between flame-fronts and shear layer vortices by

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performing time-resolved chemiluminescence imaging along with simultaneous particle image velocimetry (PIV) and OH planar laser-induced fluorescence (PLIF). These studies suggested that close to blowout, it is the interaction between the shear layer vortices and flame fronts that causes the local stretch rates along the flame to exceed the extinction stretch rates leading to the local extinction of the flame and thus the production of a flame hole. The flame dynamics in specific combustor configurations and a more general review of the work on blowout of bluff body stabilized flames are given by Shanbhogue *et al.*[13]

There have been numerous efforts to capture the flame dynamics close to blowout using direct numerical simulations (DNS) and large eddy simulations (LES).[14,15] Similar to many experimental studies,[16,17] the primary goal for most of these studies was to obtain semi-empirical correlations that could identify the LBO margins for specific configurations of combustors. While they were largely successful in their pursuit of predicting stability margins, few models tried to capture the complex unsteady dynamics of the flame close to blowout. In this paper, we introduce a low-order phenomenological model that captures the flame dynamics of a turbulent combustor at the lean limit. The model considers the flame confined in the combustor as a collection of flamelets interacting with each other. This interaction between flamelets leads to the emergence of various dynamics at different conditions.

The rest of the paper is organized as follows: The model description is elaborated in Sec. II. The experimental setup and the measurement techniques employed in the present study is detailed in Sec. III. Section IV presents the results and discussion which is followed by concluding remarks in Sec. VI.

II. THE POPULATION DYNAMICS MODEL FOR FLAME BLOWOUT

Inside a combustor, the flame is the part of the reactive flow field where reactants react to form products, converting chemical energy to thermal energy in the process. When reactants burn (assuming the fuel is hydrocarbon/hydrogen), they produce hot radicals such as OH which participate in the key chain branching combustion reaction: $H + O_2 \longrightarrow OH + O$. The flame inside a combustor is sustained when the hot radicals formed at a particular instant interact with the unreacted air-fuel mixture that enters the combustor at the next instant, thus making sure that a sustained reaction is established inside the combustor. Thus, during stable combustion, there is always some amount of hot free radicals inside the combustor. The locations where these free radicals are present mark the spatial distribution of the flame inside the combustor. The concentration of the free radicals qualitatively indicates the corresponding rates of reaction. The experimental techniques (e.g., planar laser induced florescence—PLIF) that are used to capture the flame dynamics essentially map the concentration of CH or OH free radicals within the flow field.

Thus, the flame dynamics in a combustor is in essence equivalent to the dynamics of the concentration of the free radicals in the reactive flow field. In the model, for simplicity, we consider that the flame is comprised of multiple flamelets that contain free radicals. Here, the flamelet is a diffusive-reactive gas parcel where chemical energy is converted to thermal energy. The flame dynamics is the result of the local interaction between these multiple flamelets that compose the reaction field. In this narrative of flame dynamics, lean blowout is a scenario where the inability of the reactive flow to sustain the chain reaction results in the extinction of the population of flamelets in the flow field. Inspired by this connection between the flame dynamics and the population dynamics of the flamelets, we introduce a population dynamics model that captures the complex dynamics that the flame exhibits close to lean blowout.

Today, population dynamics models are widely used to study the dynamics of various natural phenomena such as spread of epidemics,[18] desertification of forests, extinction of species,[19] population dynamics of fisheries,[20] and societal and cultural collapse.[21] The effort to comprehend the dynamics of populations has been underway for more than two centuries beginning with the seminal work of Malthus[22] that suggested that population of a society undergoes an exponential growth in time eventually inducing a demand and supply mismatch of resources in the society. This model did not consider the effect of finiteness of resources and was soon replaced by the famous logistic model that considered population growth to be limited by the carrying capacity of the environment.[23] With further advent of mathematical biology, over the years, more sophisticated models started to emerge that would model the spatiotemporal aspects of population growth under various constraints imposed by its environment and the rules of interaction among the population.

While in each situation, the details of the model vary, the commonality is that, in every model, there is an interplay between the birth and death process that control the emergent global dynamics of the population. When we try to describe the population dynamics in a spatially extended system, the effect of spatial distribution of the population becomes a decisive factor in the birth and death process and hence in the population dynamics.[24] For example, the spread of an epidemic is at times controlled by minimizing the interaction between the infected population with the healthy population by spatially quarantining the infected population.[25]

In adapting a population dynamics model for describing the dynamics of a flame, we consider the process of ignition of the reactants as the birth process of a flamelet and the process of extinction of a flamelet as equivalent to the death of the flamelet. The ignition of an unburned gas parcel in a reactive flow is the result of its interaction with the neighboring flamelets. Hence, the spatial distribution of the flamelets determines the flame propagation within the reactive field which further controls the combustion dynamics. Considering these aspects, we define a cellular automaton (CA) for the population dynamics of flamelets that would mimic the flame dynamics inside a combustor. A cellular automaton is a discrete computational system that is useful in modeling a complex system. A generic cellular automaton consists of a regular array of cells (in most cases formed by a regular grid). At each time instant, each cell can assume any one of the finite number of states that are allowed. The states of the cells evolve parallelly at discrete time steps according to state update functions or dynamic transition rules. The update rules are a function of the state of the cell and its neighborhood. The characteristic feature of a cellular automata is their ability to display emergent dynamic behavior as a result of the local interaction between the states of the multiple cells that they are comprised of. Thus, they form an ideal basis for studying the behavior of a complex system. In this paper, we define a cellular automaton that mimics the flame dynamics inside a combustor. In defining the cellular automaton, we treat the combustor as a complex system in which the emergent dynamics of the flame is the result of local interaction between multiple flamelets, the unburned reactants, and the products of combustion.

Our primary objective is to model the flame dynamics close to flame blowout. First, a few simplifying assumptions are made. We consider the blowout characteristics of a bluff body stabilized premixed flame. A two-dimensional reaction field is considered for simplicity (Fig. 1). We assume that the premixed air-fuel mixture enters the combustor with a uniform velocity $2U$ across the inlet, the combustor has a width of W_c, and the bluff body in Fig. 1 has a width of $W_c/2$.

To construct the cellular automaton, first, we divide the reaction field inside the combustor into a uniform square grid (of size 400×400). The value assigned to each cell of the grid represents the state of the reactive flow in the corresponding spatial location. The states can be unburned (0), burning (1), or burned (2). The cell in a burning state represents a flamelet. In each step of the cellular automata, each cell interacts with their neighboring cells following a set of rules. This local interaction between the flowing fluid parcels which are unburned, burning, or burned leads to the propagation of the flame in the flow field. The specific characteristics of the interaction between the fluid parcels that we need to capture in this model are the local nature of the interaction and the effect of the underlying turbulent flow in the system. The rules of cellular automata that define flame propagation are given in Fig. 1. In each step of flame propagation, a burned site would remain burned (2 remains 2) and a burning site will become burned (1 becomes 2). An unburned site would burn with a probability P, if there is at least one site in its Moore neighborhood (i.e., the eight sites which surround it) which is burning. Here, P is the probability of flame propagation. When the probability of flame propagation P reduces, the average flame speed reduces. At lean equivalence ratios close to blowout, the flame speed reduces when the equivalence ratio of the combustible mixture reduces. Thus, reducing P is equivalent to reducing the equivalence ratio of the combustion and approaching lean blowout.

The previous steps simulate only the propagation of flame in the frame of reference moving along with the mean flow. The mean flow of the unreacted mixture is simulated by removing two columns of cells from the right side of the grid representing the reactive field and adding two columns of cells with unburned sites to the left of the grid in each step. This mimics the outflow of fluid parcels (burned, burning, or unburned) from the combustor and the influx of the unburned fluid parcels into the combustor. In order to initiate burning and to keep the flame anchored at the exit of the burner, burning fluid parcels are added at the top and bottom of the bluff body in every iteration of the cellular automata with a probability P (for reducing the complexity of the model, we take this probability as equal to the probability of flame propagation), provided there is at least N number of burning parcels in the reactive field. If the number of burning fluid parcels in the reactive field goes below N, no burning parcels are added on top and bottom of the bluff body and hence, the flame is no longer anchored at the burner. This corresponds to a situation when the flamelets fail to sustain the chain reaction,

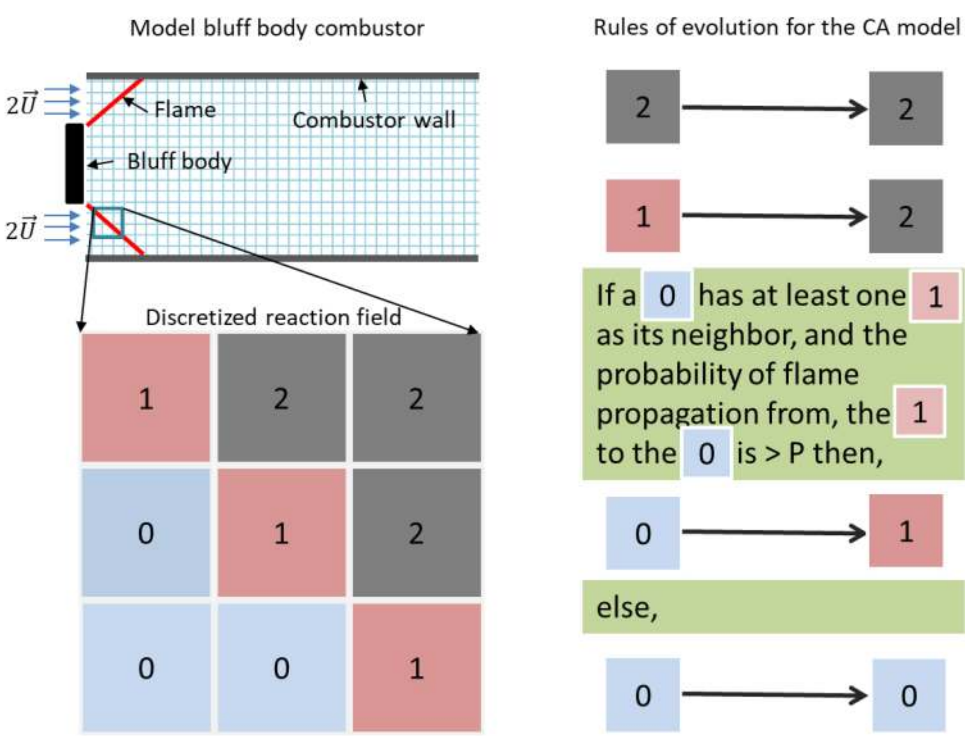

FIG. 1. The cellular automata model for a turbulent combustor with a bluff body stabilized flame. The reaction field is discretized into a uniform square grid. Fluid particle in each cell can be in one of the three states: unburned (0), burning (1), and burned (2). The states evolve in time according to the rules of evolution.

eventually leading to flame blowout. In this work, N is taken to be 3 which happens when there is at least one extra burning fluid parcel other than the two added at the inlet during every iteration of cellular automata. The value of N determines the value of P at which blowout happens. In experiments, this would correspond to the minimum reaction rate in the reaction field to ensure the chain reaction.

As explained in the previous paragraph, in order to simulate the flow from left to right, in every iteration of the cellular automata, the grid is moved by two columns (a distance of 2 units) toward the left. We consider the corresponding flow velocity to be $2U$. In every iteration, the most a flamelet can propagate is to its immediate Moore neighborhood. In the Moore neighborhood, the neighbors along horizontal and vertical directions are one unit distance away from the central site, and the neighbors along the diagonals are $\sqrt{2}$ unit distance away from the central site. This means, in every iteration, the flame travels $\sqrt{2}$ unit along the diagonal of the square grid and one unit along the horizontal and the vertical directions. As a result, the flame velocity is U along the x and y directions of the square grid of the cellular automata. Whereas, along the direction parallel to the diagonals of the grid, the flame propagates with a velocity of $\sqrt{2}U$. As a result, at equilibrium, when $P = 1$, the flame is stabilized at an angle of 45° to the mean flow (Fig. 3). The component of mean flow velocity along the diagonals of the square grid is balanced by the flame velocity in the same direction. As P reduces, the average flame velocity in all direction reduces, making the flame more and more parallel to the flow. Note that the mean flow has a significant effect on the dynamics of a flame close to blowout as it alters the manner in which the reacting gas parcels interact with their neighboring gas parcels. While this is an interesting problem, it is beyond the scope of this paper.

III. EXPERIMENTAL SETUP AND PROCEDURES

The experiments presented here were performed in a backward facing step, bluff body stabilized combustor. The experimental setup is the same as the one used in the study of Unni and Sujith.[26] The schematic of the experimental setup is shown in Fig. 2.

Air enters the test section through the settling chamber, which is provided to reduce the fluctuations in the inlet air. The fuel, liquefied petroleum gas (LPG, 40% propane and 60% butane), is introduced into the combustion chamber (90 mm × 90 mm × 1100 mm) via four radial holes (each of 1.7 mm diameter) using a central shaft. A circular disk (bluff body) of 40 mm diameter and 10 mm thickness mounted on the central shaft is used for flame stabilization. The partially premixed air-fuel mixture is spark ignited in the recirculation zone at the dump plane using a 11 kV ignition transformer. Mass flow controllers are used to measure and control the supply of fuel and air into the combustion chamber [Alicat Scientific, MCR Series, 100 SLPM model for fuel flow, 2000 SLPM for air flow; uncertainty is ±(0.8% of reading +0.2% of full scale)]. In the current study, the fuel flow rate ($\dot{m}_f$) is fixed and the air flow rate ($\dot{m}_a$) is gradually increased, decreasing the equivalence ratio [$Eq.Ratio = (\dot{m}_f/\dot{m}_a)_{actual}/(\dot{m}_f/\dot{m}_a)_{stochiometry}$]. The flow conditions are maintained such that the reactive flow is turbulent ($1.09 \times 10^5 < Re < 2.12 \times 10^5$). To facilitate optical diagnostics, a pair of quartz windows (400 mm × 90 mm × 10 mm) is provided in the side walls of the combustion chamber. The unsteady CH* chemiluminescence which is indicative of the unsteady reaction rate is measured using a photo-multiplier tube (PMT; Hamamatsu H10722-01) outfitted with a CH*-filter (λ = 432 nm and 10 nm $FWHM$). The CH* filter-PMT assembly has a rotationally symmetric viewing cone of 90° subtended angle. When this assembly is kept 50 cm away and along the central normal to the quartz window, PMT can capture the light coming through the entire quartz window. We used PMT in analog mode. The rise time of the PMT is 0.57 ns and the acquisition time is 10 μs.

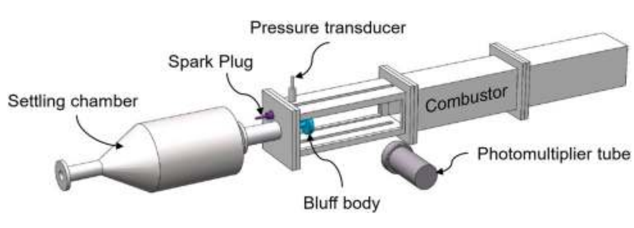

FIG. 2. The schematic of the backward facing step combustor used for the present study. A quartz window is used to facilitate measurement of unsteady reaction rate using a photo-multiplier. The design of the combustor was adapted from Komarek and Polifke.[27]

IV. THE FLAME DYNAMICS CLOSE TO LEAN BLOWOUT

The typical flame configurations observed at various values of P for the bluff body combustor is shown in Fig. 3. Reducing the value of P is equivalent to reducing the value of the equivalence ratio. Here, we observe that at lower values of P, flame holes are generated. Flame holes are the gaps in the flame front where the flamelets get extinguished. The

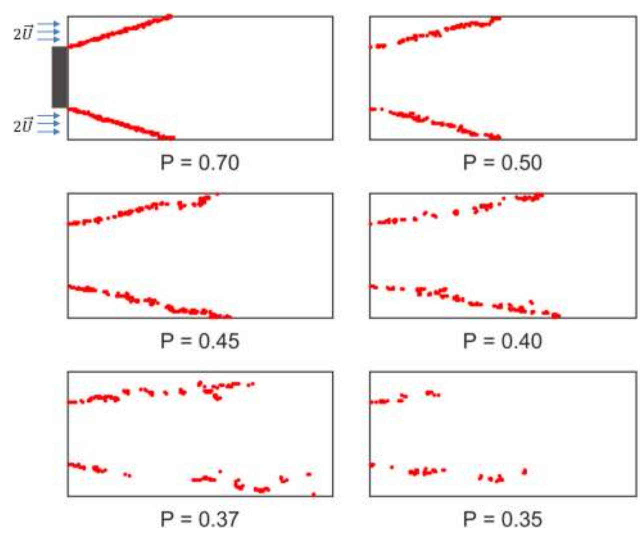

FIG. 3. Typical flame configuration obtained for different probability of flame propagation (P). As P reduces, flame-holes start to appear along the flame and the flame becomes more and more parallel to flow. Beyond a critical value of P, the flame extinguishes.

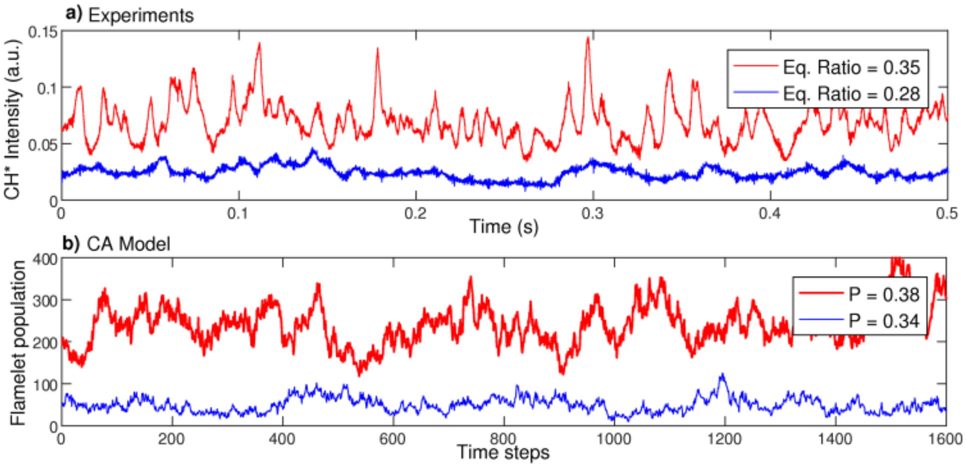

FIG. 4. (a) CH-chemiluminescence measured experimentally from the combustor at two different equation ratios, both away and near to flame blowout. (b) Flamelet population estimated from the model for two value of *P*s both away and near flame extinction.

processes of initiation and recovery of the flame holes create unsteadiness in the heat release rate. Nair and Lieuwen[5] observed these phenomena in experiments with bluff-body stabilized flames and called it the first stage of blowout. They reported that during this stage, even though the flame is unsteady due to the formation and recovery of flame holes, the overall position, and the qualitative behavior of the flame remains the same. Chaudhuri *et al.*[6] showed that during the first stage of blowout, the local flame extinction could be the result of the interaction of shear layer vortices with the flame front. However, during this stage, the recovery time for a flame hole is small enough that it ensures that the flame still remains in varicose mode (symmetrically distorted flame). A similar behavior is seen for the flame dynamics observed for P= 0.70, P = 0.50, and P = 0.45 in Fig. 3. Here, the flame is distorted due to the formation of flame holes. Nevertheless, the flame holes recover fast enough, ensuring that the flame dynamics remain largely symmetric.

At lower values of P, we see asymmetric behavior for the flame dynamics (cases of P = 0.40 and P = 0.37 in Fig. 3). This is because the reduced value of P results in larger flame holes that take longer time periods to recover. Energy released per unit time reduces with formation of flame holes. However, when the recovery of the flame hole happens, there is a burst of energy that is released. Since the formation of flame holes does not happen in a symmetric manner about the axis of the combustor, the bursts of energy also happen in an asymmetric manner. Similar asymmetrical behavior of flame is observed in the experiments.[9] However, in experiments, this asymmetry is enhanced by the formation of vortices along the shear layer at locations where the flame hole is formed due to the increased predominance of Kelvin-Helmholtz instability at these places.[6] This results in the dominance of absolute instability at the locations of flame holes, eventually leading to a sinus mode oscillation of the flame. This is also seen as the flapping of the flame.

Very close to blowout, at the final stage, the variation of the chemiluminescence fluctuation reduces. In Fig. 3, when $P = 0.35$, the flame is highly asymmetrical and flamelets are scattered across the flow field. The flame becomes columnar and parallel to the flow. This is indicative of the strain rates along the flame being close to extinction strain rates. In experiments, the formation of columnar flames before blowout was observed by Chaudhuri *et al.*[6] In the model, as the probability of the flame propagation reduces, the unburned parcel travels much more distance in the flow field before it starts to burn. This causes the average length of the flame to increase and the flame to be more and more parallel to the flow as we approach blowout.

The total CH* chemiluminescence intensity inside a bluff-body stabilized combustor was measured using a photomultiplier tube for different equivalence ratios. This measurement gives an indication of the unsteady reaction rate and hence the heat release rate inside the combustor.

We compare this measured chemiluminescence intensity fluctuations at different equivalence ratios to the fluctuation of the population of flamelets inside the reaction field in the cellular automata model. Population of flamelets gives an estimate of the reaction rate and hence the unsteady heat release rate inside the combustor.

We observe from Fig. 4 that there is a similarity between the time series of CH* chemiluminescence from the experiment and the flamelet population from the model. In both the cases, away from blowout, the variance in the time series is higher compared to that close to blowout (Fig. 5). In experiments, the increased variance of chemiluminescence fluctuations away from blowout is the result of vigorous flame flapping that is caused due to the growing tendency of the reactive flow to exhibit absolute instability.[6] We also observe (both in experiments and in the model) that close to blowout, the oscillations have increased low-frequency content which can be attributed to the longer recovery times for the flame holes as we approach blowout (Fig. 6). In experiments, the increase in low-frequency content for oscillations prior to blowout was reported previously by Nair and Lieuwen.[5] In the case of the model, away from blowout, the number of flamelets present in the reaction field is higher. When a flame hole is formed in this condition, the corresponding fluctuation in the number of flamelets is high compared to a condition very close to flame blowout. Close to flame blowout, the average number of flamelets present in the reaction zone is less and correspondingly the formation of a flame hole does not lead to a large fluctuation in the number of flamelets. Also, the increased lower frequency content in the fluctuations could be attributed to the increased length of the flame close to blowout.

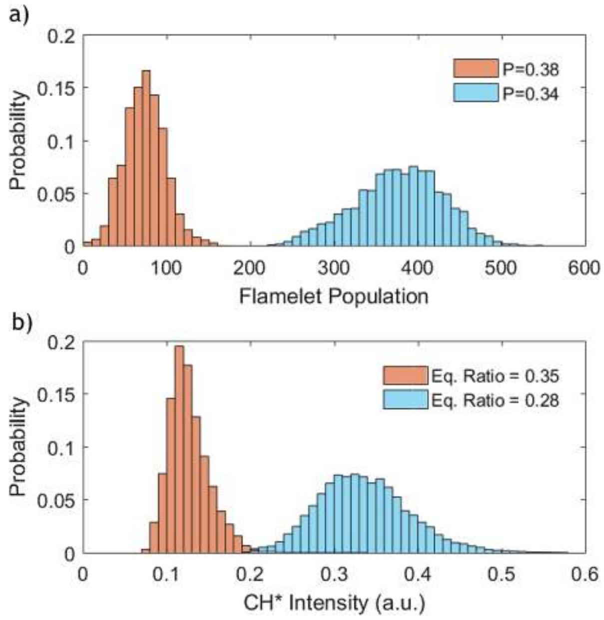

FIG. 5. Histograms corresponding to the oscillations in population of flamelets obtained from the model (a) and the heat release rate measured from experiments (b). As we approach leaner equivalence ratios, in both experiments and model, the variance of fluctuations decreases.

V. SIMILARITIES OF BLOWOUT WITH AN ABSORBING PHASE TRANSITION

Population dynamics models can either be mean field (MF) models such as a logistic model or models based on contact process.[28,29] In mean field models, the dispersion effect of the population does not affect the population growth rate. For example, in the logistic model, the growth rate depends on the carrying capacity of the environment and the total population at any moment, both of which are mean field measures. Whereas in a contact process, the population growth is dispersion limited and the spatial distribution of the existing population influences the growth of population. That is, birth and death process at any spatial location is influenced not only by the mean field measures but the local characteristics of the population distribution. Our model that describes the population of flamelets in a reactive flow field is a type of contact process. In the model, the birth of flamelet at any spatial location is influenced by the characteristics of the fluid parcel at that place and the states of the fluid parcels in its neighborhood with which the fluid parcel is in contact with (and hence a contact process).

In a contact process, the characteristics of extinction of a population depend upon the dispersion characteristics of the system. Close to population extinction, the distribution of population becomes highly fragmented. This fragmentation further reduces the stability of the population. In Fig. 3, we can observe that at low values of P, the flame becomes more and more fragmented. This dispersion limited nature of population dynamics warrants that for a contact process, the population extinction is a threshold like process and is analogous to a critical phase transition.[30] In Fig. 7, we see that as P reduces, the average density of population of flamelets reduces. We see that the reduction in population density is linear with P close to flame blowout. Furthermore, beyond a threshold value of P, the density of population goes to zero indicating an extinction of flamelets in the reactive field. Similar observations are made in our experiments where the average CH* chemiluminescence intensity from a bluff body stabilized combustor was measured using a photomultiplier tube. We observe that the average CH* intensity reduces as equivalence ratio is reduced and goes to zero beyond a threshold value of equivalence ratio. We further note that the variation in average CH* intensity with equivalence ratio or

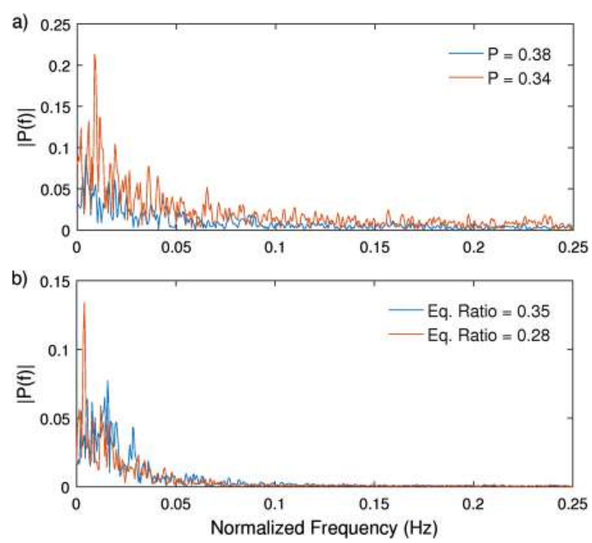

FIG. 6. The amplitude spectra of the population of flamelets obtained from model (a) and the heat release rate measured from experiments (b) indicate that as we approach flame blowout, the relative amplitude of low frequency oscillations increases. Here, the frequency is normalized with the data rate to enable comparison between the model and experiment. Further study is required to identify the optimal parameters for the cellular automata model to obtain a one to one correspondence between the results from the model and the experimental results.

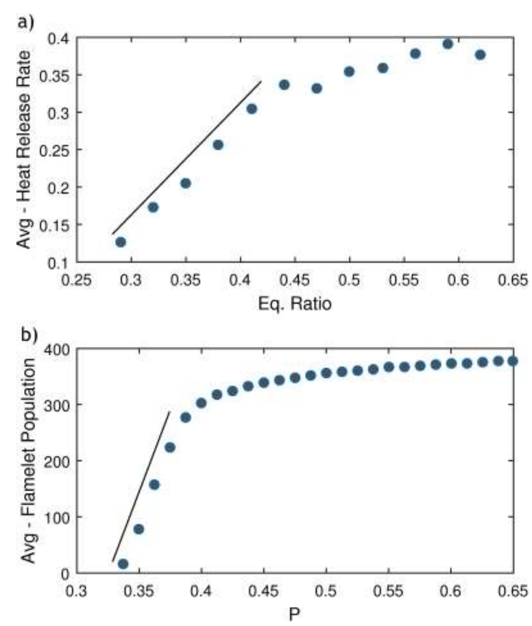

FIG. 7. The average heat release rate measured from experiments at various equivalence ratios (a). The variation of average population of flamelets as we approach flame blowout, estimated from the population dynamics model (b).

flamelet population with P is linear close to flame blowout. It is interesting to note that this system exhibits linear variation of population of flamelets with respect to the parameter P, since for most systems on the verge of extinction, the population density exhibits a power law behavior. This unique feature is possibly due to the fact that during flame propagation, even though the dynamics occur in a 2D field (in the case of the model), the flame is more or less confined along a line in the reaction field, unlike in the case of a regular contact process in a 2D grid, where the population becomes highly fragmented and spread out in the space as the system approaches population extinction.[30] Nevertheless, we observe that even in the present case, the transition to extinct phase happens beyond a threshold value of P, and hence the system seems to have the characteristics of a threshold like process analogous to a critical phase transition.

In the cellular automata model, the flame oscillation prior to blowout exhibit a meta-stable state characterized by intermittent formation of large flame holes, followed by rapid transition to a short-lived reactive state producing a burst of energy in the system. As a result, the system exhibits a switch between low reaction rate (or low flamelet population) and high reaction rate (or high flamelet population) [Fig. 4(b)]. This can also be observed in the behavior of CH* chemiluminescence intensity obtained from experiments just prior to flame blowout [Fig. 4(a)]. Dynamic characterization of such meta-stable states could provide viable precursors to flame blowout and could be taken up as a future study. As the equivalence ratio is varied further, the flame blows out. This flame blowout corresponds to the population extinction of flamelets in the CA model up on the reduction of P. In the context of population dynamics, the extinct phase is called an absorbing state. Once reached, an absorbing state cannot be escaped by a mere dynamic transition.[31] Transition to absorbing state is ubiquitous in problems of non-equilibrium phase transitions that fall under the universality class of directed percolation. As the flame blows out, the reaction field in the CA model transitions to a flamelet extinct phase. Upon reaching extinction, the system is frozen or trapped in the extinct phase. Thus, suggesting that reduction of probability of flame propagation P, beyond a threshold value causes the system to undergo a phase transition into an absorbing state. Description of flame blowout as a phase transition to an absorbing state seems more appropriate compared to considering flame blowout as a mere loss of static stability of the flame, since in the path to absorbing phase transition, the CA model exhibits various key dynamics of the flame close to a flame blowout as observed from the experiments.

VI. CONCLUDING REMARKS

Considering that a turbulent flame can be viewed as a collection of flamelets, we were able to recast the problem of flame stabilization in a turbulent combustor as a problem of population dynamics of flamelets. In this context, the birth process of a flamelet was considered to be equivalent to the process of ignition of the reactants and the process of extinction of flame equivalent to the death of a flamelet. Further, lean blowout corresponds to a scenario where the inability of the reactive flow to sustain the chain reaction results in the extinction of the population of flamelets in the flow field. A cellular automata model was constructed to capture the population dynamics of flamelets close to a flame blowout. We observed that the model exhibits various dynamic characteristics of the flame close to lean blowout such as the stretching of the flame, the increase in the number of flame holes in the flame, reduction of average intensity of the flame, etc. Furthermore, we observe that the cellular automata model that represents the dynamics of the turbulent combustor undergoes a phase transition to an absorbing state (the extinct phase) imitating flame blowout in the turbulent combustor. In the future, this study can also be extended to investigate the problem of flame ignition in combustors.

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