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FINITE ELEMENT ANALYSIS OF TWO-DIMENSIONAL TURBULENT ASYMMETRIC NEAR AND FAR NON-PERIODIC AND PERIODIC WAKES BY κ-ϵ MODEL OF TURBULENCE

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ABSTRACT

Two dimensional time averaged, steady incompressible, adiabatic turbulent asymmetric near and far non-periodic and periodic wake flow problems are solved by Galerkin Finite Element Method. A primitive-variables formulation is adopted using Reynolds-averaged momentum equations, with standard k-ε turbulence model. Finite element equations are solved by Newton-Raphson technique with relaxation, using frontal solver. Periodic boundary condition is specified on the periodic lines of the cascade, and asymptotic boundary condition is specified at the exit. These boundary conditions are applied without much difficulty which are not so straight forward in finite volume (FV) method. The results show good agreement with FV prediction and experimental data.

NOTATION

A	:	Flow region.
C_1, C_2, C_μ	:	Turbulent model Constants.
ch	:	Blade chord.
E	:	ε matrix.
GU	:	Function used in FE equations.
GV	:	Function used in FE equations.
GK	:	Function used in FE equations.
GE	:	Function used in FE equations.
GP	:	Function used in FE equations.
k	:	Turbulent kinetic energy.
k	:	k matrix.
m	:	$n / 2$
M	:	Bilinear shape function.
M	:	M matrix (1 x m).
$\mathbf{M}^T$	:	Transpose of **M**.
n	:	Number of nodes in each element.
ne	:	Number of elements.
n_x	:	Direction cosine of outward normal on the boundary.
n_y	:	Direction sine of outward normal on the boundary.
N	:	Quadratic shape function.
N	:	N matrix (1 x n).
$\mathbf{N}^T$	:	Transpose of **N**.
p	:	Pressure.
p	:	p matrix (m x 1).
Re	:	Reynolds number.
Tu	:	Turbulent intensity at inlet.
u	:	Reynolds-averaged velocity in x-direction.
u_i	:	Fluctuating velocity tensor.
u	:	u matrix (n x 1).
U_i	:	Reynolds-averaged velocity tensor.
v	:	Reynolds-averaged velocity in y-direction.
v	:	v matrix (n x 1).
x	:	Cartesian co-ordinate.
x_i	:	Cartesian tensor.
y	:	Cartesian co-ordinate.
δ	:	Boundary layer thickness at the trailing edge.
δ_{ij}	:	$= 1$ for $i = j$, $= 0$ for $i \neq j$.
ε	:	Turbulent energy dissipation rate.
ϕ	:	u, v, k and ε.
ψ	:	x and y.
Γ	:	Flow boundary.
κ	:	Constant.
μ_t	:	Turbulent viscosity.
σ_k, σ_ε	:	Constants.

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Subscripts and superscripts

e : Element.
i : Varies from 1 to n. / For tensor 1, 2.
j : Varies from 1 to n. / For tensor 1, 2.
k : Varies from 1 to m. / Turbulent kinetic energy.
l : Varies from 1 to m. / For tensor 1, 2.
ε : Turbulent energy dissipation rate.

INTRODUCTION

The study of the characteristics of the wakes of a cascade of turbomachinery blades has a wide range of significant, scientific and engineering applications. It has direct application to turbomachinery in the aerodynamic design of turbomachinery components for better efficiency. With the knowledge of the mean and turbulence properties of the cascade wake, lift, drag and noise generated due to the incidence of wake and turbulence on a rotor can be controlled. The turbulent flow in the near wake of a streamlined body is quite complex even in the absence of any flow separation. This is due to the abrupt change in boundary condition at the trailing edge as well as a sudden transition from wall turbulence to free turbulence in the near wake. The flow in the near wake of a streamlined two-dimensional body, provides a simple yet critical test of the generality of turbulence models and calculation procedure. Almost all the turbulence models are developed from the extensive data base in the boundary layer and fully developed free shear flows, such as wakes and jets. The near wakes being the region of adjustment between two extreme states, there offers an independent test of the generality of turbulence models.

Here wake behind a flat plate of different roughness on either side and wake behind a cascade of blades of double circular arc (i.e. both the surfaces of the blades having different radii of curvature) are considered for investigation. The mean velocity profiles of the wake behind a flat plate of different surface roughness on either side and the cascade wakes are two-dimensional and asymmetric in general. The asymmetric nature in the cascade wake is due to the loading on the blade and different thickness of boundary layers on the pressure and suction surfaces of the blades. The asymmetric nature of the wake disappears after about 1.5 chord downstream from the trailing of the blade. For most engineering applications, the Reynolds number, based on the chord length of the airfoil, is large enough for the boundary layer near the trailing edge to be considered fully turbulent. Therefore, the subsequent wake is also turbulent. In flat plate wake, free stream velocity affects the nature of the flow. If there is an incidence, pressure gradient will exist in the free stream in the main flow direction and would affect the wake pattern. Here, zero pressure gradient is considered for the present investigation. The cascade wake has a periodic distribution of flow quantities with a period equal to the spacing of the blades. Here, the neighbouring blades do exert influence on the wake characteristics of any blade. The static pressure is considered to have an adverse gradient in the streamwise direction because the edge velocity in the cascade wake decreases downstream. Since the cascade wake is periodic along the lines which are parallel to the line passing through trailing edges of the cascade (the trailing edge line), we call it as periodic wake. The flat plate wake is called as non-periodic wake.

The earliest detailed two-dimensional near wake study was made by Chevray and Kovasznay (1969), who made measurements of mean-velocity and turbulence profiles in the symmetric wake of a thin flat plate. Their data have been used by a numerous workers to test the performance of various turbulence models. Several subsequent experimental investigations have been carried out e.g. Pot (1979), Andreopoulos and Bradshaw (1980), Ramaprian et al. (1981), Nakayama (1985), Hah and Lakshminarayana(1982). Attempts have also been made to predict these flows, e.g. Patel and Scheuerer (1982), Hah and Lakshminarayana (1982), Chang et al. (1986), Tulapurkara et al. (1991).

The availability of cascade near wake data is limited in the literature. Raj and Lakshminarayana (1973), Hobbs et al. (1980), more recently Zierke and Deutsch (1989) have made measurements in cascade wakes. A very few researchers have done numerical work on cascade wakes. Hah and Lakshminarayana (1982) predicted the cascade near wake numerically. They used different turbulence models and solved by finite difference method. But their wake exit boundary condition is not appropriate since the experimental values supplied by them do not allow the wake to decay according to trailing edge condition and the turbulence model used. Afterwards Hah (1984), in one of his papers, considered asymptotic boundary condition at the wake exit. Several researchers have calculated the flow through turbomachinery cascades taking the far wake flow region into consideration. Among those are Hah (1984), Hwang et al. (1988), Kirtley and Lakshminarayana (1988), Agouzoul and Camarero (1988), Gorski (1988), Kunz and Lakshminarayana (1991), Hobson and Lakshminarayana (1991), Suryavamshi and Lakshminarayana (1992). But unfortunately according to the authors' knowledge only Kirtley and Lakshminarayana (1988), and Suryavamshi and Lakshminarayana (1992) have shown wake pattern or the wake centerline decay of mean velocity.

THE PHYSICAL NATURE OF ASYMMETRIC WAKE

The characteristics of an asymmetric wake are best analysed by dividing the wake flow into two regions, namely (a) near wake, and (b) far wake.

(a) Near wake

In the first part of this region, the viscous sublayer on the blades or the flat plate is not completely mixed with the surrounding inertial sublayer. The molecular viscosity has a substantial effect on the flow distribution in the wake centre region. This region is confined to the trailing edge of the solid body and the velocity defect is large.

In the latter part of this region, the physical characteristics of the blade and aerodynamic loading on the blade have substantial effects on the characteristics of the wake. For turbulent wake, the effect of molecular viscosity is negligible. After the first part of the near wake, the viscous sublayer is mostly mixed up with the neighbouring inertial sublayer. The wake defect is of the same order as the mean velocity in this region.

(b) Far wake

In this region the wake structure is almost symmetric and the physical characteristics and aerodynamic loading have almost negligible effects on the development of the wake. The velocity defect is small and "history effects" (such as blade shape or loading) have vanished.

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GOVERNING EQUATIONS: TRANSFORMATION TO FINITE ELEMENT FORM

Some previous calculations by Ramaprian et al. (1981) demonstrated that one equation turbulence model, based on the turbulent kinetic energy equation with a prescribed length scale distribution, failed to predict the essential features of near wake and grossly underestimated the far wake. It was also shown that two-equation k-ε (turbulent kinetic energy and its dissipation rate) turbulence model predicts better.

A numerical analysis of the two dimensional time averaged, steady incompressible, adiabatic turbulent flow by Galerkin Finite Element Method for solving asymmetric near and far non-periodic and periodic wake flow adopting a primitive-variables formulation is presented here, using Reynolds-averaged momentum equations, with standard k-ε turbulence model. The k-ε model, proposed by Launder and Spalding (1972), is the most widely applied two-equation eddy-viscosity model and is employed in the present study. Eight noded isoparametric quadrilateral elements are used. Biquadratic interpolation for velocity, turbulent kinetic energy and the rate of turbulent energy dissipation are used whereas bilinear interpolation is used for pressure. Finite element equations are solved by Newton-Raphson technique with relaxation, as the governing equations are non-linear in nature. The sets of linear equations are solved by frontal solver.

The two dimensional time-averaged Navier-Stokes equations and the equation of continuity for steady, incompressible, adiabatic turbulent flows in cartesian tensor notation can be given in a non-dimensional form as follows :

$$U_j \frac{\partial U_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\frac{1}{Re} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \right] - \frac{\partial \left(\overline{u_i u_j}\right)}{\partial x_j} \qquad \ldots(1)$$

$$\frac{\partial U_i}{\partial x_i} = 0 \qquad \ldots(2)$$

Here U_i and p are the local Reynolds-averaged values of the velocity components in x_i direction and pressure respectively. u_i is the fluctuating value of velocity in x_i direction. Re denotes the Reynolds number. The term $\left(\overline{u_i u_j}\right)$ is called Reynolds stress.

For cascade the reference velocity is absolute velocity at the cascade inlet and reference length is the blade chord. For flat plate freestream velocity at the wake inlet and length of the plate is taken as reference velocity and reference length respectively.

The main assumption in the turbulent viscosity model of turbulence is to replace the Reynolds stress with the following expression :

$$-\overline{u_i u_j} = \frac{\mu_t}{Re} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij}\, k \qquad \ldots(3)$$

In the above equation μ_t denotes the eddy or turbulent viscosity, non-dimensionalised by the dynamic viscosity, which can be determined using k-ε turbulence model. The formulation for the eddy viscosity μ_t, used in the standard k-ε model, is obtained, based on Jones-Launder length scale model and Prandtl-Kolmogorov formula, as

$$\mu_t = C_\mu Re\, k^2/\varepsilon \qquad \ldots(4)$$

where C_μ is an empirical constant with a value of 0.09.

The standard k-ε turbulence model is satisfied only for high Reynolds number. The transport equation for turbulent kinetic energy is given as :

$$U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\frac{1}{Re} \left(\frac{\mu_t}{\sigma_k} + 1 \right) \frac{\partial k}{\partial x_j} \right] + \frac{\mu_t}{Re} \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \varepsilon \qquad \ldots(5)$$

The dissipation rate equation is given as :

$$U_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\frac{1}{Re} \left(\frac{\mu_t}{\sigma_\varepsilon} + 1 \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_1 \frac{\varepsilon}{k} \frac{\mu_t}{Re} \frac{\partial U_i}{\partial x_j} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{C_2 \varepsilon^2}{k} \qquad \ldots(6)$$

In these equations σ_k, C_1, C_2, and σ_ε are empirical constants and have the values of 1.0, 1.44, 1.92, and 1.3, respectively.

The discretisation of the governing equations into finite element form after breaking the tensor notation is given in Appendix - A.

BOUNDARY CONDITIONS

At the wake inlet i.e. near trailing edge, all the flow parameters are specified. Velocity components are taken from the experimental results. For cascade wake, turbulent kinetic energy is obtained from the experimental (Zierke and Deutsch, 1989), intensity profile. Assuming isotropic turbulence we can get

$$k = 1.5\ (Tu)^2 (u)^2 \ .$$

where Tu is local turbulence intensity,

For flat plate wake experimental data (Ramaprian et al., 1981) k is available and isotropic assumption is not necessary.

The rate of turbulent energy dissipation profile as given by Tulapurkara et al., (1991), is obtained from the following expression:

$$\varepsilon = (0.3k)^{1.5} / \, l$$

$$\text{where } l = 0.085\ \delta \tanh \left[\frac{\chi}{0.085} \frac{y}{\delta} \right], \qquad \chi = 0.41.$$

y is the distance from the wake centerline. It is zero on the centerline and positive on either side.

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At $y = 0$, ε is taken average of the nearest points on either side.

δ is boundary layer thickness at the wake inlet.

Experimental values are smoothed for calculation.

At the freestream edges, for flat plate wake, the streamwise velocities on either side of the wake centerline are specified from the wake inlet through out the streamwise direction as there is no streamwise pressure gradient. Neumann boundary condition i.e. no variation normal to the streamwise direction, is specified for the transverse velocity, turbulent kinetic energy and its rate of energy dissipation. For cascade on the periodic line, periodicity condition is specified. In the finite element code, periodicity is ensured by assigning the same number to the degrees of freedom that are to be identified. They are thus taken as identical by the assembly routine.

At the exit boundary, asymptotic boundary condition (i.e. second derivatives of the flow parameters along the stream wise direction vanish) is specified (Appendix B) . Theoretically, at infinity, the flow becomes uniform. But from the practical point of view with the limitation of computer memory and computing time, the flow domain must be as small as practical. Generally, one and a half axial chord downstream of the trailing edge is reasonable.

In incompressible flow, pressure does not appear in the continuity equation. Pressure is to specified at least at one point, as its first derivative is present in the governing equations and second derivative is not present. So pressure terms do not appear in the boundary terms of the Galerkin's residuals (Appendix - A). Hence pressure is specified only at one point. At the specification point, continuity equation gets lost and creates an imbalance in mass flow in that region. This point should be near the outlet boundary where the adjacent total area of the grids is higher and variation of the flow parameters is lower (Jackson, 1984). The grid system is shown in Fig. 1.

OUTLINE OF THE METHOD

(a) The governing equations are transformed into finite element equations, which are non-linear in nature.

(b) First grid is generated with 51 and 49 stations in x-direction and y-direction respectively.

(c) Initial values of all the flow parameters are provided at each node and the boundary nodes are identified.

(d) Integrations are carried out by Gaussian 5 x 5 points integration.

(e) The non-linear equations are solved by Newton-Raphson technique with relaxation.

(f) The set of linear equations are solved by frontal solver.

(g) Now the old values of the flow parameters are updated. Steps (d) to (f) are repeated till the convergence.

DIFFICULTIES FACED

Because of intense mixing in the near wake, the grid should be very fine.

Inlet profile should be smoothed properly otherwise numerical oscillations will be there causing divergence at times. This is because all the flow parameters change very fast in the near wake region.

Centreline portion of the grid should be finer because of the higher gradient of the flow parameters in that portion, normal to the streamwise direction.

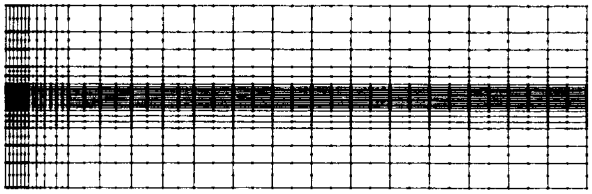

FIG. 1 GRID SYSTEM FOR PERIODIC AND NON-PERIODIC WAKE FLOWS

Pressure should be specified as mentioned earlier. Specification of pressure at the inlet might cause the system to diverge.

The system of equations are very sensitive as k and ε varying considerably over a very small distance. So some incorrect initial value or incorrect procedure to attack the problem leads to chaos.

The absence of the pressure term in the continuity equation causes the appearance of zero in the global matrix even after assembling the elemental matrices. So pivoting is necessary.

It takes a lot of memory and CPU time to solve the linear equations by band solver. Frontal solver is better than this in this respect though band solver is easier to program. But one has to be careful, because full pivoting of the assembled matrix leads to unwanted result, because of the elements of the assembled matrix vary significantly in magnitude. Scaling also does not help much sometimes. It is experienced that proper scaling and only row pivotisation yield good results.

There are two ways of solving the five governing equations. In the first method, all the five equations are solved simultaneously. In the second method, the momentum and continuity equations are solved together and then the turbulence equations (k-ε) are solved. This procedure is repeated till the convergence is attained. The latter method is more stable and requires lesser computer memory and hence adopted for the present investigation.

RESULTS AND DISCUSSION

The asymmetric wake is generated (Ramaprian et al., 1981) by roughening the upper surface of a flat plate of length 1.8 m. The boundary layer thickness on the upper and lower surfaces are 41 mm and 36 mm respectively at the trailing edge (TE). The TE freestream velocity is 19.53 m/s. The experimental data indicates that the freestream velocity at all stations starting from 12.7 mm from TE is nearly 22.25 m/s. In order to avoid steep velocity gradient 22.25 m/s is taken everywhere. The reference velocity and length taken for calculations are 22.25 m/s and 1.8 m respectively. The Reynolds number based on this velocity and the length of the plate is 2.7×10^6. In the downstream of the TE, 600 mm in streamwise direction and 80 mm normal to the streamwise direction on either side of the plate are considered to form the flow region for the present computation. The calculated results for the asymmetric wake, are shown along with experimental data and finite volume result (Tulapurkara et al., 1991) in Fig.2 & 3. It is seen that the agreement between calculated and experimental profiles is highly satisfactory in the near as well as far wake regions. Turbulent kinetic energy profiles are under predicted compared to the experimental data in the far wake (Fig. 3c). One thing is noted that, FV and FE calculations are in good agreement. So the

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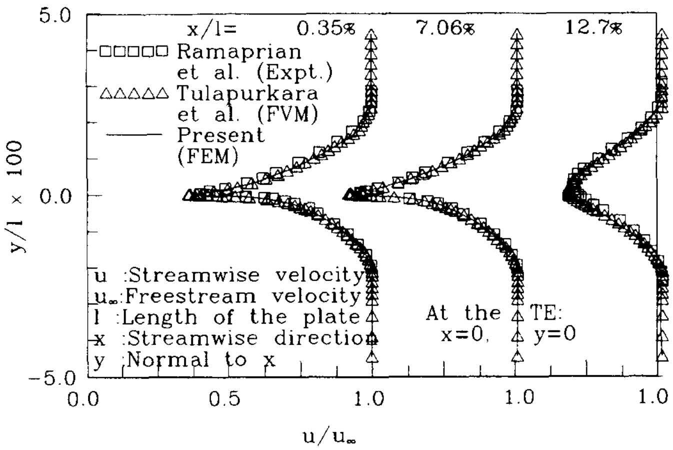

FIG. 2 TURBULENT NON-PERIODIC ASYMMETRIC WAKE : STREAMWISE VELOCITY PROFILE

disagreement between the calculated results and experimental data, in the case of turbulent kinetic energy, is not due to the inaccuracy of the code but due to the inherent incompleteness of the turbulence model itself. Comparison of wake centerline velocity is shown in Fig. 4, which shows very good agreement.

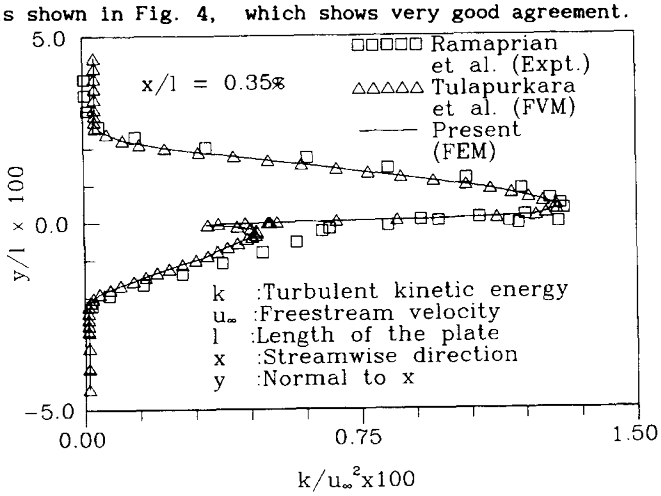

FIG. 3 (a)

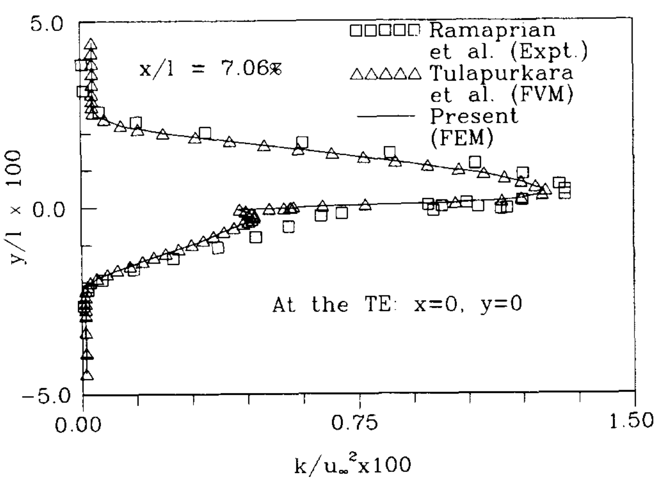

FIG. 3 (b)

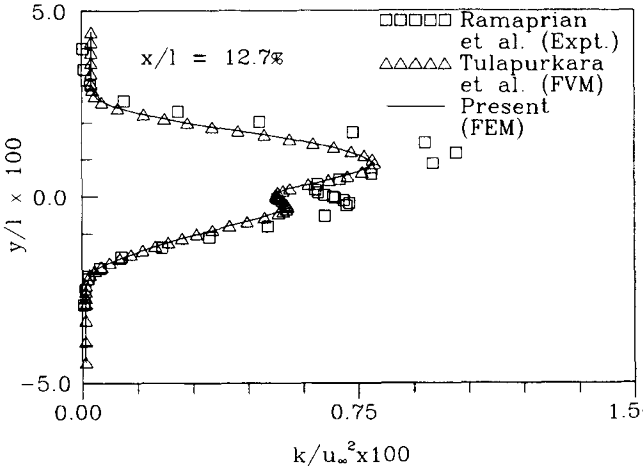

FIG. 3 (c)

FIG. 3 TURBULENT NON-PERIODIC ASYMMETRIC WAKE : TURBULENT KINETIC ENERGY

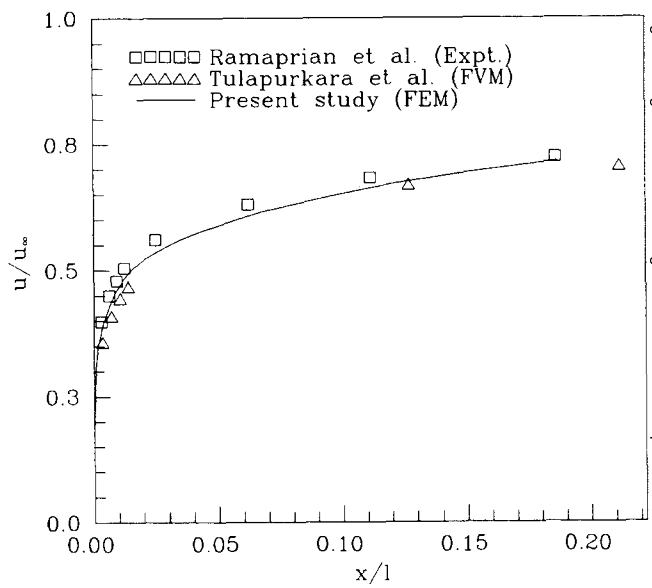

FIG. 4 WAKE CENTERLINE VELOCITY FOR NON-PERIODIC WAKE

Periodic asymmetric wakes are generated (Zierke and Deutsch, 1989) from a cascade of blades having the shape of a circular arc on both the surfaces. The blade chord length is 228.6 mm, blade spacing is 106.8 mm, maximum thickness 12.5 mm, stagger angle is 20.5°, inlet and outlet blade angles are 53.0° and -12.0° with the normal to the TE line of the cascade respectively. Anti-clockwise direction is chosen as positive. Radii of curvature of the suction surface, pressure surface and camberline of the blades are 189.1 mm, 212.8 mm and 246.8 mm respectively. Incidence angle is -8.5° and absolute inlet velocity is 33.28 m/s. Outlet flow angle is -0.6° with the normal to the TE line of the cascade. Reynolds number based on chord length and absolute inlet velocity is about 5.07×10^4. Results are shown in the Fig. 5 & 6. At the far downstream, the asymmetry reduced to a great extent, as the flow tries to forget its past history. The near wake velocity profiles and turbulent kinetic energy are

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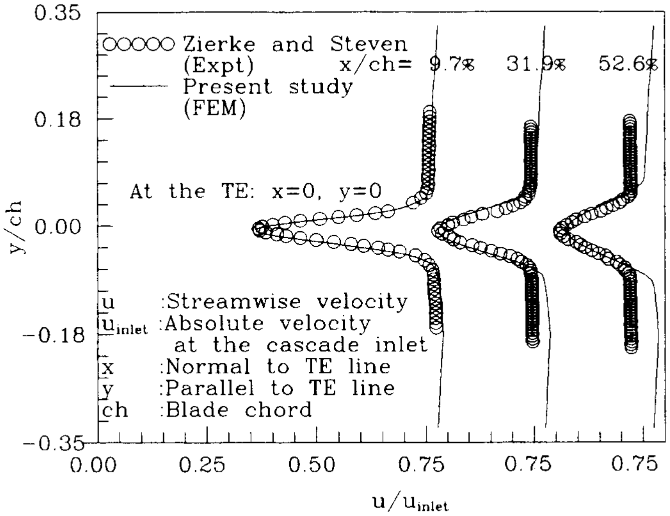

FIG. 5 TURBULENT PERIODIC ASYMMETRIC WAKE : STREAMWISE VELOCITY PROFILE

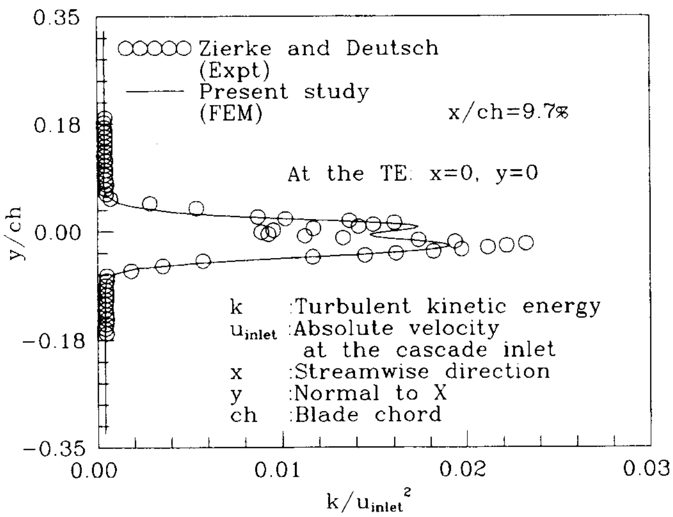

FIG. 6 TURBULENT PERIODIC ASYMMETRIC WAKE : TURBULENT KINETIC ENERGY

predicted satisfactorily. But the far wake freestream velocities are overpredicted (7.6%). The predicted wake centerline velocity is compared in the Fig. 7. The agreement for the wake centerline is fine but the edge velocity is over predicted. This may be due to the adverse pressure gradient the correction for which is not taken into account in the current model. Turbulent kinetic energy prediction in the near wake is also in good agreement.

CONCLUSIONS

Two dimensional time averaged, steady incompressible, adiabatic turbulent asymmetric near and far non-periodic and periodic wake flow problems are solved by Galerkin Finite Element Method. Though the present code is applied to wake flow in this paper this can be easily extended to full cascade flow.

The advantage of finite element method is that one can take any type of structured or unstructured grid and boundary condition can be incorporated easily compared to finite volume or finite difference method.

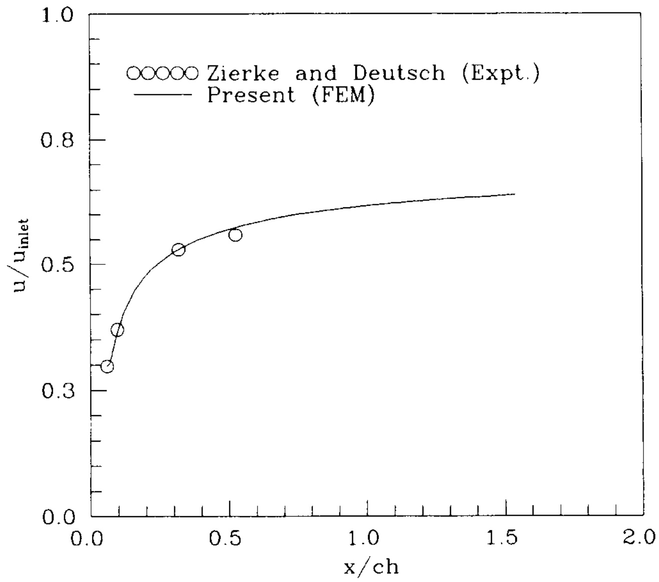

FIG. 7 WAKE CENTERLINE VELOCITY FOR PERIODIC WAKE

Though near wake is predicted well far wake prediction is not so well compared to the near wake. For periodic wake though wake centerline velocity is in good agreement, though edge velocity is overpredicted by 7-8% . Near wake turbulent kinetic energy is predicted reasonably well.

Most interesting feature is finite volume calculations are in extremely good agreement with the present method. So the under or over prediction is not due to the inaccuracy of the code but due to the inherent incompleteness of the turbulence model itself.

Curvature correction and pressure gradient correction might improve the prediction.

Some more research is necessary to acquire the required knowledge for specifying the inlet turbulent kinetic energy dissipation rate.

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APPENDIX - A

DISCRETISATION OF THE GOVERNING EQUATIONS INTO FINITE ELEMENT FORM

Now breaking tensor notation i.e. putting $i = 1$ & 2 and $j = 1$ & 2, and putting $U_1 = u$, $U_2 = v$, $x_1 = x$, and $x_2 = y$, five governing equations are obtained. So the Galerkin's elemental residuals are also of five types, (Segerlind, 1984) say $GU^{(e)}$, $GV^{(e)}$, $GK^{(e)}$, $GE^{(e)}$, $GP^{(e)}$. Shape function for velocity components, turbulence kinetic energy and its dissipation rate is denoted by N_1, where as for pressure it is denoted by M_k. In matrix form **N** and **M** respectively. So

$$\phi = \mathbf{N}\Phi^{(e)} = N_i \phi_i^{(e)}; \qquad \ldots (A1)$$

where $\phi = u, v, k, \varepsilon$ and p; $\Phi = \mathbf{u}, \mathbf{v}, \mathbf{k}, \mathbf{E}$ and $\mathbf{p}$; $i = 1$ to n and (subscript) $k = 1$ to m, where n is the number of node in the eth element and $m = n/2$ or $n/2$. (In this case $n = 8$ and $m = 4$).

It was shown by Kim and Chen (1988) that linear element is inferior to the quadratic element. Hence only quadratic element is considered in the present study.

Now the residuals for the governing equations can be written as follows:

$$Residuals = \int_A \mathbf{N}^T \left[LHS - RHS \right] dA \qquad \ldots (A2)$$

where LHS and RHS are left and right hand sides of the governing equation respectively. and A is elemental area.

Now applying Green's theorem we get,

$$\int_A \mathbf{N}^T \left[\frac{\partial}{\partial \varphi} \left\{ \frac{C}{Re} \left(\frac{\mu_t}{\sigma_\phi} + 1 \right) \left(\frac{\partial \phi}{\partial \varphi} \right) \right\} \right] dA =$$

$$- \int_A \frac{\partial \mathbf{N}^T}{\partial \varphi} \left[\frac{C}{Re} \left(\frac{\mu_t}{\sigma_\phi} + 1 \right) \left(\frac{\partial \phi}{\partial \varphi} \right) \right] dA$$

$$+ \oint_\Gamma \mathbf{N}^T \left[\frac{C}{Re} \left(\frac{\mu_t}{\sigma_\phi} + 1 \right) \left(\frac{\partial \phi}{\partial \varphi} \right) \right] n_\varphi \, d\Gamma \qquad \ldots (A3)$$

where $\phi = u, v, k$ and ε ; $\varphi = x$ and y; $\sigma_u = \sigma_v = 1$, $C = 1$ or 2.

n_φ is the cosine of the angle between φ-direction and the outward normal to the element boundary Γ. The cyclic integral term is called elemental boundary term.

The cyclic integral terms can be separated into two components, like. One component is integrated over

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Γ_{bc}, the side of element over which boundary condition is specified, and the other component is integrated over Γ_i, the side of element over which boundary condition is not specified. The second one leads to the interelement requirements which must be satisfied before Galerkin's residual is zero. Again if boundary conditions are of i) Neumann, ii) periodic, iii) asymptotic iv) and Dirichlet type for u, v, k and ε, we can take these boundary terms as zero in the element residual equations for simplicity since they contribute nothing in the global equations, after boundary conditions are incorporated. The boundary conditions for the flow regions under consideration are of the above four types only.

Now after a little manipulation (A1-A3) give,

$$GU^{(e)} = \int_A N^T \left[\left(Nu^{(e)}\right) \frac{\partial}{\partial x}\left(Nu^{(e)}\right) + \left(Nv^{(e)}\right) \frac{\partial}{\partial y}\left(Nu^{(e)}\right) + \frac{\partial}{\partial x}\left(Mp^{(e)}\right) \right] dA$$

$$+ \int_A \frac{\partial N^T}{\partial x} \left[\frac{2(\mu_t + 1)}{Re} \frac{\partial}{\partial x}\left(Nu^{(e)}\right) - \frac{2}{3}\left(Nk^{(e)}\right) \right] dA$$

$$+ \int_A \frac{\partial N^T}{\partial y} \left[\frac{(\mu_t + 1)}{Re} \left(\frac{\partial}{\partial y}\left(Nu^{(e)}\right) + \frac{\partial}{\partial x}\left(Nv^{(e)}\right) \right) \right] dA \quad \ldots (A4)$$

$$GV^{(e)} = \int_A N^T \left[\left(Nu^{(e)}\right) \frac{\partial}{\partial x}\left(Nv^{(e)}\right) + \left(Nv^{(e)}\right) \frac{\partial}{\partial y}\left(Nv^{(e)}\right) + \frac{\partial}{\partial y}\left(Mp^{(e)}\right) \right] dA$$

$$+ \int_A \frac{\partial N^T}{\partial y} \left[\frac{2(\mu_t + 1)}{Re} \frac{\partial}{\partial y}\left(Nv^{(e)}\right) - \frac{2}{3}\left(Nk^{(e)}\right) \right] dA$$

$$+ \int_A \frac{\partial N^T}{\partial x} \left[\frac{(\mu_t + 1)}{Re} \left(\frac{\partial}{\partial y}\left(Nu^{(e)}\right) + \frac{\partial}{\partial x}\left(Nv^{(e)}\right) \right) \right] dA \quad \ldots (A5)$$

$$GK^{(e)} = \int_A N^T \left[\left(Nu^{(e)}\right) \frac{\partial}{\partial x}\left(Nk^{(e)}\right) + \left(Nv^{(e)}\right) \frac{\partial}{\partial y}\left(Nk^{(e)}\right) \right] dA$$

$$+ \int_A \frac{1}{Re} \left(\frac{\mu_t}{\sigma_k} + 1 \right) \left(\frac{\partial}{\partial x}\left(Nk^{(e)}\right) \frac{\partial N^T}{\partial x} + \frac{\partial}{\partial y}\left(Nk^{(e)}\right) \frac{\partial N^T}{\partial y} \right) dA$$

$$- \int_A N^T \left[\frac{\mu_t}{Re} \left\{ 2\left(\frac{\partial}{\partial x}\left(Nu^{(e)}\right)\right)^2 + \left(\frac{\partial}{\partial y}\left(Nu^{(e)}\right) + \frac{\partial}{\partial x}\left(Nv^{(e)}\right)\right)^2 + 2\left(\frac{\partial}{\partial y}\left(Nv^{(e)}\right)\right)^2 \right\} \right] dA + \int_A N^T \left[\left(NE^{(e)}\right) \right] dA \quad \ldots (A6)$$

$$GE^{(e)} = \int_A N^T \left[\left(Nu^{(e)}\right) \frac{\partial}{\partial x}\left(NE^{(e)}\right) + \left(Nv^{(e)}\right) \frac{\partial}{\partial y}\left(NE^{(e)}\right) \right] dA$$

$$+ \int_A \frac{1}{Re} \left(\frac{\mu_t}{\sigma_\varepsilon} + 1 \right) \left(\frac{\partial}{\partial x}\left(NE^{(e)}\right) \frac{\partial N^T}{\partial x} + \frac{\partial}{\partial y}\left(NE^{(e)}\right) \frac{\partial N^T}{\partial y} \right) dA$$

$$- \int_A N^T C_1 \frac{\left(NE^{(e)}\right)}{\left(Nk^{(e)}\right)} \left[\frac{\mu_t}{Re} \left\{ 2\left(\frac{\partial}{\partial x}\left(Nu^{(e)}\right)\right)^2 + \left(\frac{\partial}{\partial y}\left(Nu^{(e)}\right) + \frac{\partial}{\partial x}\left(Nv^{(e)}\right)\right)^2 + 2\left(\frac{\partial}{\partial y}\left(Nv^{(e)}\right)\right)^2 \right\} \right] dA$$

$$+ \int_A N^T C_2 \frac{\left(NE^{(e)}\right)^2}{\left(Nk^{(e)}\right)} dA \quad \ldots (A7)$$

$$GP^{(e)} = \int_A M^T \left[\frac{\partial}{\partial x}\left(Nu^{(e)}\right) + \frac{\partial}{\partial y}\left(Nv^{(e)}\right) \right] dA \quad \ldots (A8)$$

The eddy viscosity or the turbulent viscosity is given by

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$$\mu_t = C_\mu Re \frac{\left(Nk^{(e)}\right)^2}{\left(NE^{(e)}\right)} \qquad \ldots(A9)$$

Here it may be noted that (A4)-(A7) are not perfect equations as boundary terms are removed, but when these equations are assembled to form global equations boundary conditions should be applied to these equations to get correct global equations.

Now (A4)-(A8) are assembled to form global equations and solved by Newton-Raphson method with relaxation as they are non-linear in nature.

APPENDIX - B

ASYMPTOTIC BOUNDARY CONDITION

Asymptotic boundary condition means second derivatives of all the dependent variables with respect to stream-wise direction are zero. Outlet boundary is chosen such that it is parallel to y-axis, so $n_y = 0$ on that boundary. Now our objective is to remove boundary term from the Galerkin residuals by applying asymptotic condition. As n_y is zero on the boundary we should replace

$$\frac{\partial^2}{\partial x^2} \text{ in terms of } \frac{\partial^2}{\partial y^2}, \ \frac{\partial^2}{\partial y \partial x}$$

and other quantities using the condition. Asymptotic boundary condition is applied only at far wake.

If x and y be the cartesian coordinate system and s and n be streamwise and cross streamwise direction and α be the angle between the streamwise direction and the x-direction , then we can write

$$\frac{\partial x}{\partial s} = \cos\alpha \ , \quad \frac{\partial x}{\partial n} = -\sin\alpha \ , \quad \frac{\partial y}{\partial s} = \sin\alpha \ , \quad \frac{\partial y}{\partial n} = \cos\alpha$$

and $\tan\alpha = v/u$.
Now

$$\frac{\partial \phi}{\partial s} = \frac{\partial \phi}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial \phi}{\partial y}\frac{\partial y}{\partial s} \ ; \qquad \text{where } \phi = u, \ v, k \ \& \ \varepsilon$$

$$\text{operator } \frac{\partial}{\partial s} = \frac{\partial x}{\partial s}\frac{\partial}{\partial x} + \frac{\partial y}{\partial s}\frac{\partial}{\partial y} \ ; \text{ and } \frac{\partial}{\partial s^2} = \frac{\partial}{\partial s}\frac{\partial}{\partial s}$$

So

$$\frac{\partial^2 \phi}{\partial s^2} = \cos^2\alpha \frac{\partial^2 \phi}{\partial x^2} + 2\sin\alpha \cos\alpha \frac{\partial^2 \phi}{\partial x \partial y} + \sin^2\alpha \frac{\partial^2 \phi}{\partial y^2}$$

since α is a function of u and v only.

$$\text{Now} \qquad \frac{\partial^2 \phi}{\partial s^2} = 0 \Rightarrow$$

$$\frac{\partial^2 \phi}{\partial x^2} = -\left(2\tan\alpha \frac{\partial^2 \phi}{\partial x \partial y} + \tan^2\alpha \frac{\partial^2 \phi}{\partial y^2}\right) \qquad \ldots(B1)$$

Now the value of $\frac{\partial^2 \phi}{\partial x^2}$ is subtituted in (A4)-(A7) and transformed to finite element form. Now the boundary terms will not be there in the Galerkin residuals and can simply be added to the form assembly matrix.

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