Sādhanā Vol. 36, Part 3, June 2011, pp. 393–410. © Indian Academy of Sciences

Evaluation of size dependent design shear strength of reinforced concrete beams without web reinforcement

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MS received 3 July 2009; revised 17 February 2011; accepted 23 April 2011

Abstract. Analytical studies on the effect of depth of beam and several parameters on the shear strength of reinforced concrete beams are reported. A large data base available has been segregated and a nonlinear regression analysis (NLRA) has been performed for developing the refined design models for both, the cracking and the ultimate shear strengths of reinforced concrete (RC) beams without web reinforcement. The shear strength of RC beams is size dependent, which needs to be evaluated and incorporated in the appropriate size effect models. The proposed models are functions of compressive strength of concrete, percentage of flexural reinforcement and depth of beam. The structural brittleness of large size beams seems to be severe compared with highly ductile small size beams at a given quantity of flexural reinforcement. The proposed models have been validated with the existing popular models as well as with the design code provisions.

Keywords. CE database; size effect; cracking strength; shear strength; code provisions; modelling.

1. Introduction

Many reinforced concrete structural elements such as slabs, footings and joists are constructed without shear reinforcement (ASCE-ACI 445 Committee 1998). Numerous experimental efforts made on reinforced concrete (RC) beams under concentrated loads showed that the shear strength decreases with increase in the beam depth. The reinforced concrete beams are classified into three types depending on the a/d ratios (maintaining the compressive strength of concrete, percentage of flexural reinforcement and depth of the beam constant) as (i) deep beams with $0 < a/d \leq 1$, (ii) short beams with $1 < a/d \leq 2.5$, and (iii) normal beams with $a/d > 2.5$ (Bresler &

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MacGregor 1967). For these a/d ratios, the failure mechanisms in RC beams are well-understood. Till today, though several models have been proposed to estimate the shear strength of RC beams, their applicability is limited to a particular class of beams only. The safety margins on the shear strength of beams vary with depth. In order to establish the uniform safety margins on the shear strength of RC beams, a large experimental data base needs to be made available or needs to be generated using a strong nonlinear FE analysis. In this study, a large number of reports have been reviewed for generating the reliable data on the shear strength of RC beams.

From the lessons learnt from the failure of Wilkins Air Force Department Warehouse in Shelby, Ohio in 1955, the design provisions existing at that time were verified. Subsequently, modifications to the ACI 318 provisions for the design of RC beams for shear based on the diagonal shear strength were proposed (ACI-ASCE 326 Committee 1962). The catastrophic shear failures of RC structures, during Hyogo-Ken Nanbu earth quake, Kobe in 1995 further attributed to the development of the size dependent models on the shear strength in the codes of practice (Maekawa *et al* 2003). The recent research efforts have revealed that the shear strength provisions for high strength concrete beams need to be re-established with size dependent forms.

2. Review of literature

As reported by the ACI–ASCE Committee, based on the review of several models, the shear strength of RC beams depends on the strength of concrete, the percentage of the longitudinal reinforcement and the span-to-depth ratio or stiffness of the beam. Clark (1951) reported that the nominal shear strength of RC beam is a function of the compressive strength of concrete, the percentage of the longitudinal reinforcement and the shear span-to-depth ratio. The extensive experimental investigations by Kani (1967) showed very strong size effect on the shear strength on RC beams without web reinforcement. It has been reported that a reduction of strength of beams up to about 40% was observed as the depth of the beam increased from 150 mm to 1200 mm. The size effect observed in Kani's experiments on the large size beams was assumed to be due to large beam depth-to-breadth ratios (Taylor 1972). The experimental studies on slender RC deep beams at different depth-to-thickness ratios varying between 25 and 67 demonstrate that the failure of the beams was strongly dependent on eccentricity of the loading (Kong *et al* 1986).

A size effect model was proposed for the ultimate shear strength of RC beams (Bazant & Kim 1984), which was later modified by incorporating the size of aggregate (Bazant & Sun 1987). It was reported that the size effect on the ultimate shear strength of RC beams was strong, while it is not-negligible on the diagonal cracking strength of geometrically similar RC beams in the size range of 1:16 (Bazant & Kazemi 1991). Similar observations have been reported on the shear strength of deep beams when the shear span-to-depth ratio was 1.0 (Tan & Lu 1999; Walraven & Lehwalter 1994). The complex stress distribution in the dowel splitting region of the beams without shear reinforcement still remains empirical (Chana 1987). The high strength concrete beams are sensitive to the size effect. The reduction of the ultimate shear strength is associated with the maximum spacing of the horizontal layers of the reinforcement rather than on the overall depth of the beam (Collins & Kuchma 1999). The analytical studies on the ultimate shear strength revealed the size effect, which is the product of the ratio of the neutral axis depth-to-effective depth of the beam, and the splitting tensile strength of concrete (Zararis & Papadakis 2001). In HSC deep beams, a strong size effect has been pronounced at small a/h ratios (Yang *et al* 2003).

3. Research significance

The shear strength of RC beams without web reinforcement appears to be affected by the beam depth and the shear span-to-depth ratio along with other influencing parameters. Despite a reasonable consensus on the existence of size effect on the shear strength of RC beams, there still seems some prejudices and lack of authenticity on the size effect to be proposed for the design in the codes of practice. A review and analysis of the selected reliable experimental data reported in the literature till date has been performed, which proves to be adequate enough to consider the size effect on the design shear strength of RC beams. Two simple models have been proposed to predict the diagonal cracking strength and the ultimate shear strength of RC beams incorporating all the influencing parameters including the beam depth.

4. Size effect in reinforced concrete

Two important models are widely recognised for estimating the size dependent shear strength of RC beams (Bazant & Sun 1987; Niwa *et al* 1987). It is understood that the shear strength decreases asymptotically as the beam depth increases, in geometrically similar RC beams. The causes of the size effect in RC beams could be attributed to the (i) material heterogeneity, and (ii) discontinuity of flow of stress (Karihaloo 1995). According to the Weibull's weak link theory, the strength of a structure is inversely proportional to its volume. This statistical strength theory cannot be applicable to concrete structures unless the failure occurs at the initiation of cracking (Bazant & Kazemi 1991). Hence, there is a need for an appropriate size effect law for predicting the design shear strength of practical range of sizes of RC structures. Such model should ensure prediction of actual shear strength and estimation of uniform safety margins on the ultimate shear strength with different sizes of beams.

5. Review of existing models

When the ultimate strength and the diagonal cracking strength of RC beams are equal, the size effect in such beams is non-existent. The fracture mechanics based size effect is related to the non-simultaneous or propagating type of failure as seen in the short and deep beams, which are associated with the ultimate shear strength (Bazant & Kazemi 1991). The ultimate shear strength of RC beams incorporating the beam depth by Bazant & Kim (1984) is as follows

$$v_u = \frac{10\rho^{1/3}}{\sqrt{1+\frac{\mathrm{d}}{25\mathrm{d_a}}}}\left(0.083\sqrt{f_c'} + 20.69\sqrt{\frac{\rho}{(\mathrm{a}/\mathrm{d})^5}}\right), \mathrm{MPa}. \tag{1}$$

The above model has been further modified by incorporating the size of coarse aggregate (Bazant & Sun 1987), as below.

$$v_u = \left(0.54\rho^{1/3}\right)\frac{1+\sqrt{\frac{5.08}{\mathrm{d_a}}}}{\sqrt{1+\frac{\mathrm{d}}{25\mathrm{d_a}}}}\left(\sqrt{f_c'} + 249.2\sqrt{\frac{\rho}{(\mathrm{a}/\mathrm{d})^5}}\right), \mathrm{MPa}. \tag{2}$$

In most of the cases, the onset of formation of first diagonal crack was by visual observations; while in some other cases it was the diagonal crack crossing the mid-height of the beam

(Pendyala & Mendis 2000). However, there appears some discrepancies in the observations while interpreting the diagonal cracking. The first empirical model for estimating the diagonal cracking strength of normal beams was by Zsutty (1968) based on the data base available without considering the beam depth into account. Subsequently, many models have been reported on the diagonal cracking strength of RC beams.

An important model on the diagonal strength of RC beams has been proposed by Niwa *et al* (1987), as below

$$v_{cr} = 1.125\,\frac{\rho^{1/3}}{d^{1/4}}\left(\mathrm{f}'_{\mathrm{c}}\right)^{1/3}\left(0.75 + \frac{1.4}{\mathrm{a/d}}\right), \mathrm{MPa} \tag{3}$$

where
d = depth of beam, mm, ρ = percentage of beam longitudinal reinforcement, and, $\mathrm{f_c}$ = compressive strength of concrete, MPa

Several codes (CSA 1994; BS 1997; JSCE 1986) adopted a depth factor for estimating the shear strength of RC beams. However, the current ACI code (ACI 318) base its designs on the diagonal cracking strength of the normal strength concrete (NSC) beams, with the depth less than about 400 mm, without considering the influence of beam size.

The design shear strength of RC beams as per ACI 318 M-05 (2005) is as follows,

$$\begin{aligned} v_c &= \frac{1}{7}\left(\sqrt{f'_c} + 120\,\rho\,\frac{\mathrm{V_u d}}{\mathrm{M_u}}\right) \le 0.3\sqrt{f'_c},\ \mathrm{MPa}, \\ \text{where}\ & \frac{\mathrm{V_u d}}{\mathrm{M_u}} \le 1.0\,. \end{aligned} \tag{4}$$

Equation 4 overestimates the shear strength of the concrete in the uncracked portion, while the contribution of the longitudinal reinforcement and the effect of the ratio $\mathrm{V_u/M_u d}$ are underestimated (ACI 318 2002).

According to the British code (BS 8110 1997), the beam depth has been included for a/d > 2. The nominal shear strength of the beam is as follows

$$v_\mathrm{C} = \frac{0.79}{\gamma_\mathrm{m}}\left(\frac{100A_s}{b_v d}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}\left(\frac{f_{cu}}{25}\right)^{1/3}\ \mathrm{MPa}, \tag{5}$$

$$v_c = (\mathrm{Eqn.8})\left(2\frac{\mathrm{d}}{\mathrm{a_v}}\right) \quad \text{for}\ {}^{\mathrm{a}}\!/\!_{d} < 2.0, \tag{5a}$$

where $\frac{100A_s}{b_v d} \le 3.0 \quad \frac{400}{d} \ge 1.0 \quad \gamma_\mathrm{m} = 1.25$ and $\mathrm{f_{cu}} \le 40.0$ MPa.

However, the drawback is that the depth of beam is limited to only 400 mm through the limit (400/d) $\ge$ 1.0 with compressive strength of concrete is less than or equal to 40 MPa and the percentage of the flexural reinforcement is 3.0%.

6. Present refined models

In order to develop the design models for predicting the diagonal cracking strength and the ultimate shear strength of the beams, the factors influencing the shear strength are identified in their practical ranges. The factors influencing the shear strength of reinforced concrete beams include; (i) compressive strength of concrete, $\mathrm{f'_c}$, (ii) percentage of the longitudinal reinforcement, ρ, (iii) shear span-to-depth ratio, a/d and (iv) depth of the beam, d. The influence of the size of the aggregate is ignored as its influence between 8 and 32 mm has no significant effect (Walraven & Lehwalter 1994).

For the development of models, nonlinear regression analysis was adopted. From the data available in the literature, 612 reliable data set points were selected for the ultimate shear strength, and 269 data sets for the diagonal cracking strength. The details of the data selected are shown in Appendix I. The effect of various influencing parameters has been understood by plotting the relationship between the shear strength and the individual parameter. From the observations, the exponentials with the individual parameters have been considered. For developing the final form of the model in the present study, the effect of individual influencing parameter i.e., the exponentials such as k_1, k_2, k_3, etc., to the individual parameters was maintained as obtained from the observed trend for determining the appropriate coefficients. When the data on high strength concrete beams is incorporated, the shear strength is proportional to the cubic root of the compressive strength of concrete, $\sqrt[3]{f'_c}$ rather than the square root, $\sqrt{f'_c}$. This is reflected in the ACI 318 (2005) provisions indicating that Eq. 4 overestimates the shear strength of concrete. The enhancement of shear strength of short beams beyond the cracking is about 5 times its diagonal cracking strength. The most important factor influencing the shear strength of the beams is the shear span-to-depth ratio. From the observations on the influence of the individual parameters, the forms of the shear strength models are as follows,

$$v_u = \left(A + \frac{B}{(a/d)^{k_1}}\right)\left[f_c'^{k_2}\,\rho^{k_3}\,d^{k_4}\right], \tag{6}$$

$$v_{cr} = \left(C\,(a/d)^{k_5} + \frac{D}{(a/d)^{k_6}}\right)\left[f_c'^{k_7}\,\rho^{k_8}\,d^{k_9}\right]. \tag{7}$$

The exponential coefficients from k_2 to k_4, and k_7 to k_9 were refined by suitably modulating them to match well with the experimental data, from the rigorous parametric study. The coefficients A, B, C, D, and the exponential coefficients k_1, k_5 and k_6 were obtained to best fit in the final forms of the models in the following equations,

$$v_u = \left(0.56 + \frac{4.0}{(a/d)^{3/2}}\right)\left[f_c'^{1/3}\rho^{1/2}\,d^{-1/4}\right], \text{MPa}, \tag{8}$$

$$v_{cr} = \left(0.28\sqrt[3]{(a/d)} + \frac{2.0}{(a/d)^{7/6}}\right)\left[f_c'^{1/3}\,\rho^{1/3}\,d^{-1/4}\right], \text{MPa}. \tag{9}$$

Alternatively, the diagonal cracking strength can be expressed in the following form

$$v_{cr} = v_u\frac{\sqrt[3]{a/d}}{2\rho^{1/6}}. \tag{10}$$

The a/d ratio is replaced with (M/Vd) ratio, in a beam subjected to uniformly distributed loading, where M and V are the bending moment and the shear force at the critical section. It is understood from the basic shear mechanism that the shear strength of RC beams is contributed from the uncracked concrete in compression, aggregate friction along the diagonal cracked plane and dowel action of the longitudinal reinforcement. As soon as the diagonal cracking is formed, with further loading, the additional strength up to the ultimate stage could be ensured by the dowel action of the longitudinal reinforcement embedded in concrete in the tension face of the beam.

Table 1. Summary of NLRA for ultimate and diagonal cracking strength.

Source	Ultimate strength			Diagonal cracking strength		
	Deg. of freedom	Sum of squares	Mean squares	Degrees of freedom	Sum of squares	Mean squares
Regression	2	4658.62	2329.31	2	970.76	485.38
Residual	610	475.97	0.78	267	65.39	0.245
Uncorrected total	612	5134.60		269	1036.15	
(Corrected total)	611	2441.90		268	198.02	
R^2		0.805			0.67	
F-static		2986.3			1981.1	
Prob > F		0.0001			0.0001	

It may be assumed that in Eqs. 8 and 9, the dowel action of the longitudinal reinforcement may cause the overestimation of the ultimate shear strength. Before diagonal cracking, the dowel action of the longitudinal reinforcement is not expected to be significant. The cubic root variation of the flexural reinforcement ratio seems to be appropriate on the diagonal cracking strength.

The summary of the analysis is reported in table 1. The values of 'F' and 'Prob (F)' test the overall significance of the regression model. A low value of 0.0001 for 'Prob (F)' indicates that the independent variables are not purely random with respect to the dependent variable. Also, the coefficient of determination is 0.80 for the ultimate shear strength, and 0.67 for the diagonal cracking strength. The scatter of the results for the ultimate shear strength is better than that of the diagonal cracking strength. Therefore, it looks more appropriate to base the design of RC beams on the ultimate strength rather than on the diagonal cracking strength. Further, uniform safety margins can be set for different sizes on the ultimate strength.

7. Results and discussion

Figures 1a–i show the variation of the normalised ultimate shear strength by the compressive strength of concrete with the beam depth, as per the earlier models, and also by the codes of practice. The appropriate partial safety factors were imposed on the shear strength proposed in the codes of practice. The comparison of the calculated shear strength of the beams is shown in table 2. The RMSE for the beams with $a/d = 1$, by Kani (1966) was 0.633* (*over estimation), estimated from Eq. 2 against 0.528 from the present model, Eq. 8. Similarly, for $a/d < 1.0$, Eq. 2 overestimates the ultimate shear strength, whereas Eq. 3 predicts reasonably well the diagonal cracking strength of the normal beams. However, the shear strength of the short and deep beams is underestimated. The proposed models in Eqs. 8 and 9, show a fair degree of prediction of both the ultimate and diagonal cracking strengths. The low RMSE with the present models show better correlation with the data in table 2, and in figures 1a–i.

The ACI code underestimates the shear strength on the small size beams with the a/d ratios between 1.0 and 2.5 as shown in figures 1b and c, and overestimates on the large size HSC beams as shown in figure 1(i). The measured ultimate shear strength and the estimated strengths as per the refined model, in Eqs. 8 and 2, and also predicted by the ACI code are shown in figures 2b and c. The shear strength of the beams evaluated by the ACI code provisions are shown in figures 2a and d respectively. The correlation coefficient (r) evaluated from the ACI code, Eq. 2, is 0.794, and on the proposed model is 0.91.

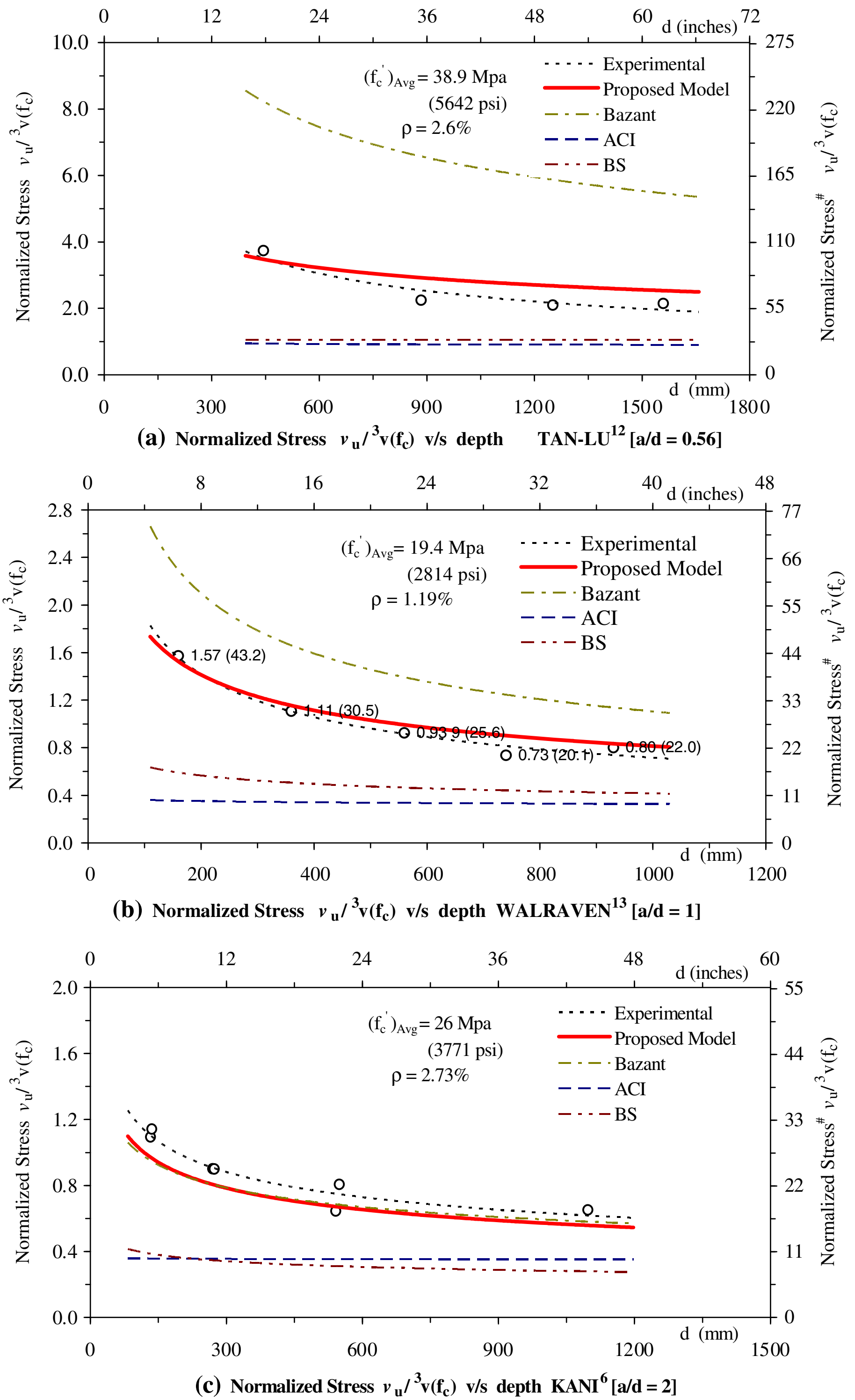
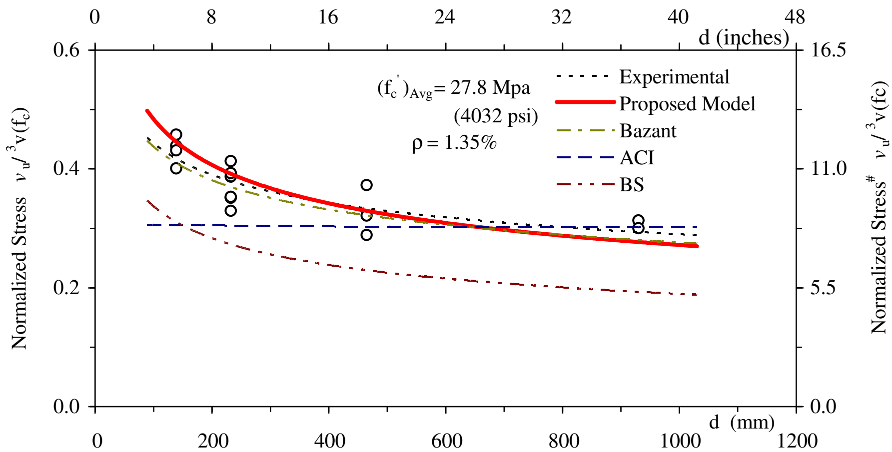
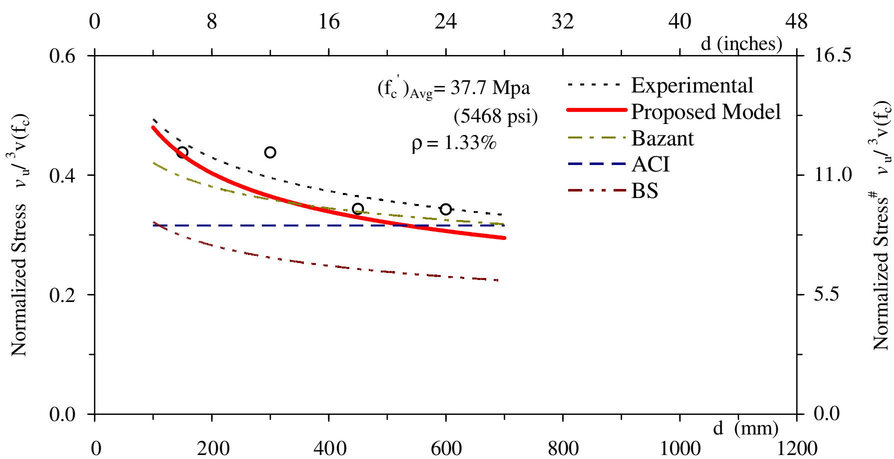


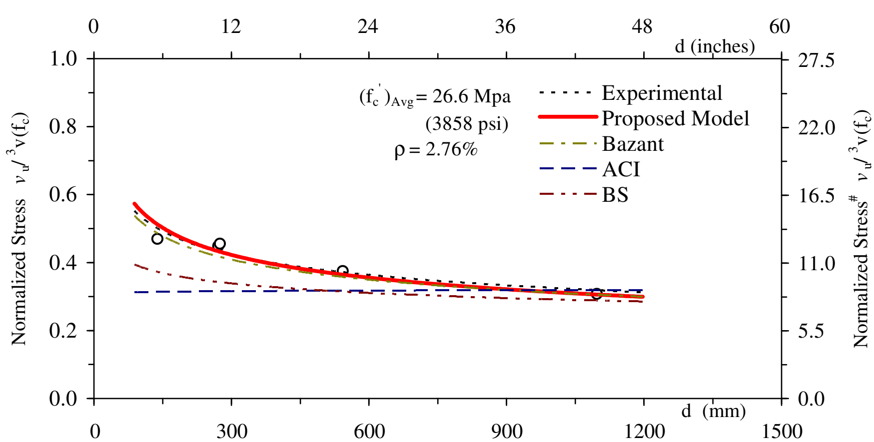
Figure 1. Comparison of $\upsilon_u/\sqrt[3]{f_c}$ with depth.



(d) **Normalized Stress $v_u/\sqrt[3]{(f_c)}$ v/s depth TAYLOR[7] [a/d = 3]**

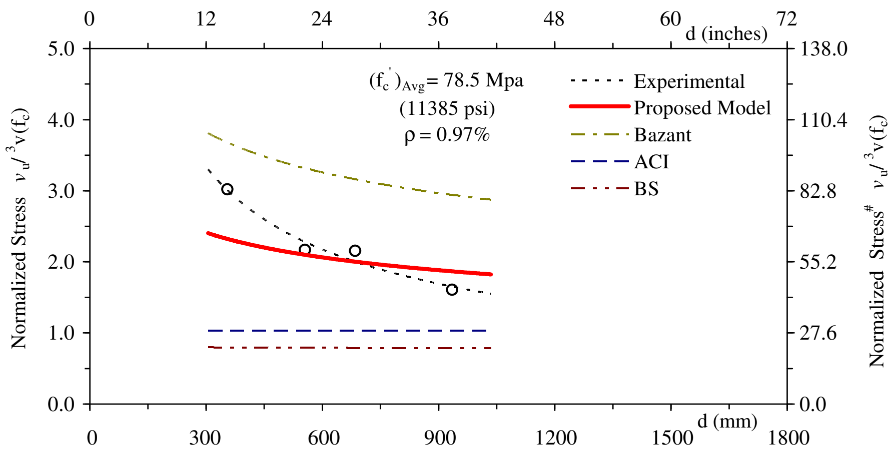


(e) **Normalized Stress $v_u/\sqrt[3]{(f_c)}$ v/s depth LEONHARDT[+] [a/d = 3]**

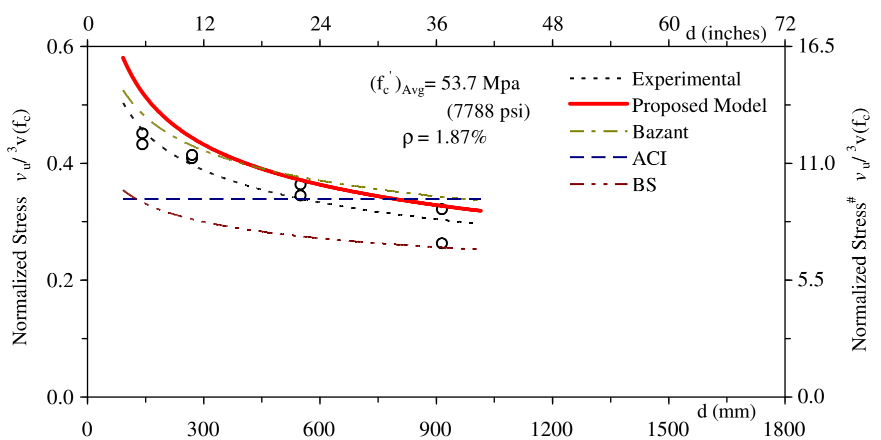


(f) **Normalized Stress $v_u/\sqrt[3]{(f_c)}$ v/s depth KANI[6] [a/d = 4]**

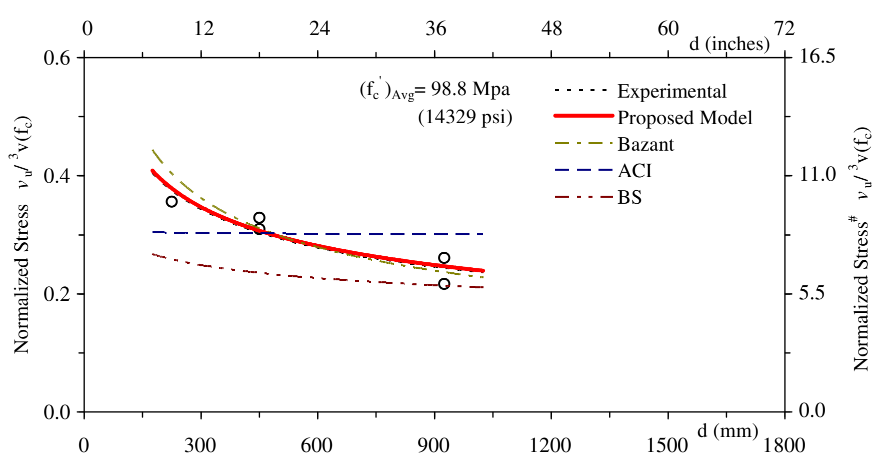
Figure 1. (continued).



(g) **Normalized Stress $v_u / \sqrt[3]{(f_c)}$ v/s depth YANG et.al[17] [a/d = 0.54]**



(h) **Normalized Stress $v_u / \sqrt[3]{(f_c)}$ v/s depth KIM-PARK[29] [a/d = 3.0]**



(i) **Normalized Stress $v_u / \sqrt[3]{(f_c)}$ v/s depth COLLINS-KUCHMA[15] [a/d = 2.97]**

Figure 1. (continued).

Table 2. Root mean square error for ultimate and diagonal cracking strength.

Authors	a/d	Ultimate strength			Cracking strength	
		Proposed model	Bazant–Sun	ACI	Proposed model	Niwa *et al*
Kani 1967	1.00	0.53	0.66*	1.99	–	–
	2.00	0.11	0.12	0.55	–	–
	2.50	0.06*	0.07*	0.26	–	–
	3.00	0.05*	0.03*	0.17	–	–
	4.00	0.02	0.02	0.12	–	–
	5.00	0.03	0.04	0.14	–	–
	6.00	0.07	0.04	0.14	–	–
	7.00	0.07	0.04	0.05	-	
	8.00	0.06	0.04	0.02	–	–
Taylor 1972	3.00	0.03	0.03	0.08	–	–
Bazant & Kazemi 1991	3.00	0.11*	0.07*	0.15	–	–
Tan & Lu 1999	0.56	0.52*	4.04*	1.80	0.05	0.10
	0.84	0.62	0.67*	1.60	0.09*	0.09*
	1.13	0.72	0.54	1.39	0.11	0.12
Walraven & Lehwalter 1994	1.0	0.09*	0.54*	0.75	0.06	0.09
	3.00	0.02	0.03	0.08	–	–
Chana 1987	3.00	0.05*	0.03*	0.13	–	–
Collins & Kuchma 1999	3.00	0.02	0.02	0.06	–	–
	2.92	0.03	0.03	0.05	–	–
	3.00	0.07	0.10	0.10	–	–
	3.00	0.07*	0.07*	0.12	–	–
Yang *et al* 2003	0.54	0.40*	2.56*	1.17	0.07	0.16
	1.1	0.44	0.37	0.90	0.21	0.17
	0.54	0.38	1.08*	1.52	0.08	0.19
	1.1	0.65	0.68	1.07	0.34	0.30
Kim & Park 1994	3.00	0.05*	0.04*	0.07	–	–
	4.56	0.16	0.17	0.44	–	–
Kotosovos 2001	3.00	0.03	0.08	0.18	–	–
	3.00	0.03	0.04	0.08	–	–
Bhal (Bazant *et al* 1987)	3.00	0.03	0.03	0.07	–	–
	3.00	0.03*	0.04*	0.06	–	–
Leonhardt (Bazant & Sun 1987)	3.00	0.04	0.05	0.09	–	–
	3.00	0.03	0.08	0.20	–	–

* Indicates that the experimental values are overestimated by a model. The blank space indicates that the tests were performed only for the limited parameters

The size effect on the diagonal cracking strength of RC beams observed by Walraven & Lehwalter (1994) and Yang *et al* (2003) are shown in figures 3a, b and d. About 43% decrease on the diagonal cracking strength is observed in figure 3d against 49% on the ultimate shear strength, as shown in figure 1b. The ratio $(v_{cr}/\sqrt[3]{f'_c})_{\text{test}}$ varies with $d^{-0.3}$ and the ratio $(v_u/\sqrt[3]{f'_c})_{\text{test}}$ varies with $d^{-0.42}$. This shows that the size effect on the diagonal cracking strength is not negligible. In the ACI code, the comment about the size effect on the shear strength of RC beams is made, but not incorporated in the design models. According to the ACI code, the shear strength of RC beams is overestimated on the large size beams at small percentages of the

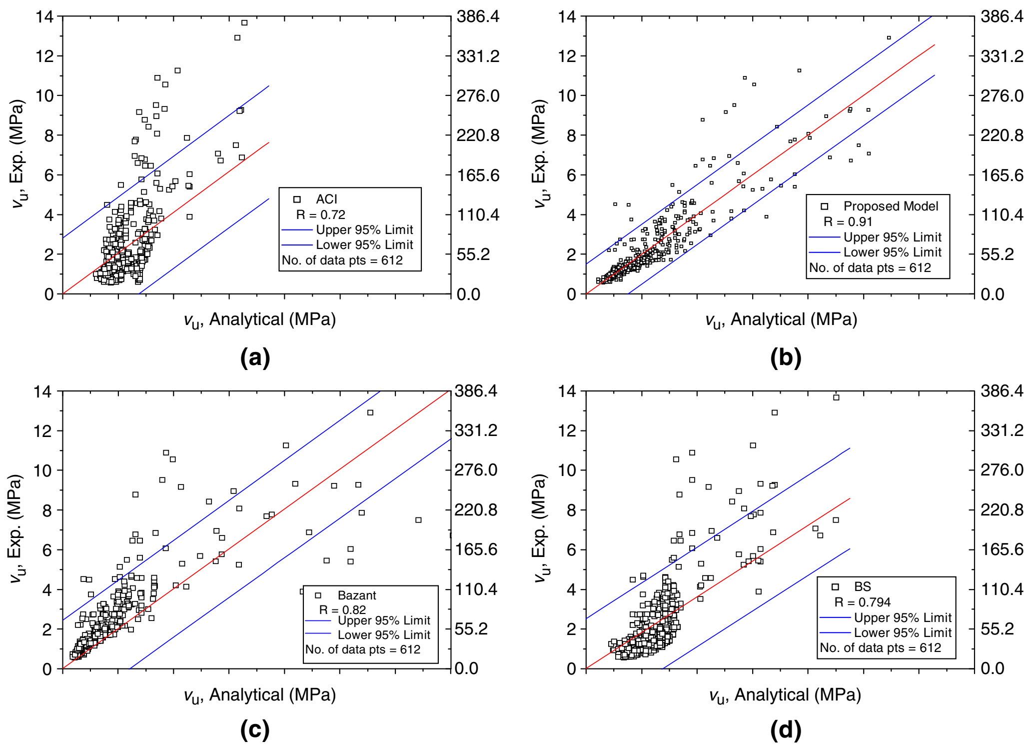
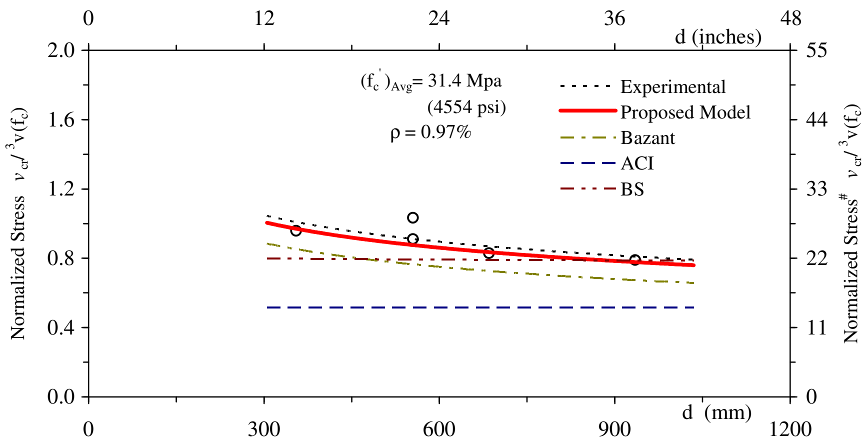


Figure 2. Ultimate shear strength – measured versus calculated values.

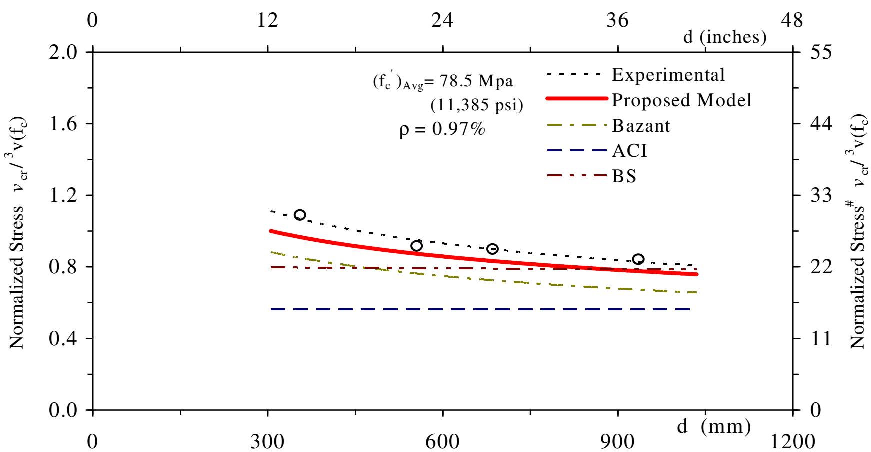
longitudinal reinforcement as shown in figures 1d, e and i. The shear strength predicted by the ACI code for the a/d ratios, $1 < a/d < 2.5$, are very conservative. The beam depths beyond which the ultimate strengths are overestimated by the ACI code, Eq. 6 are shown in table 3. The safety margins estimated by the ACI code without the size effect factor are non-uniform on different sizes with the same concrete, longitudinal reinforcement and a/d ratio.

The diagonal cracking strength with the beam depth estimated from Eq. 3 correlates well with the experimental observations on the normal beams. However, the ultimate strength of the beams with $a/d < 2.5$ is underestimated. Thus, Eq. 3 is in good agreement with the experimental observations on the normal beams ($a/d > 2.5$). The RMSE is small on the proposed model than that of Eq. 3. The measured cracking strength is compared with the calculated values as per the refined model, in Eq. 9, and Eq. 3, also showed in figures 4a–b. The correlation coefficient for the refined model is 0.85, which is slightly higher than 0.82 for Eq. 3 with better correlation. The data on large size beams is very limited. Due to this there exists a slight deviation of predicted strength from the experimental findings from the reported results.

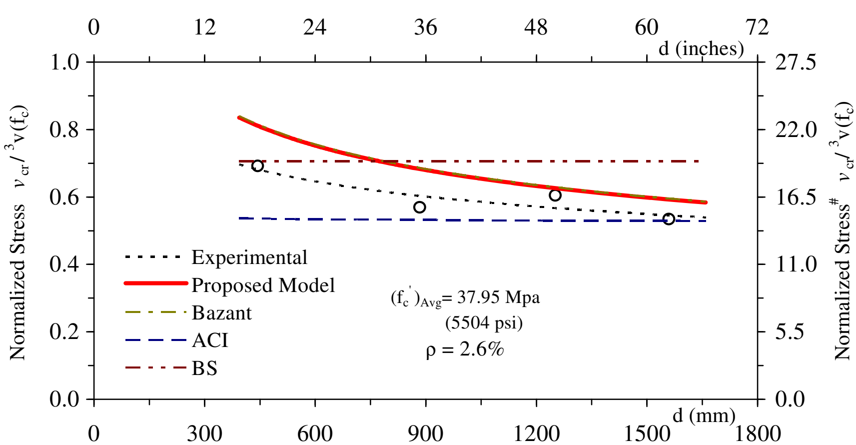
The century-old conventional strength theory does not explain the reasons for the size effect and the catastrophic mechanism associated with the diagonal failure. Though it is very complex to explain, several theories have been proposed for explaining such mechanisms through fracture mechanics of concrete. It was reported (So & Karihaloo 1993) that the ultimate shear strength could be calculated by adding the contributions from the tension-softening effect of cracked concrete and the bond of reinforcement. However, the complex shear mechanism is influenced by



(a) **Normalized Stress $v_{cr}/\sqrt[3]{f_c}$ v/s depth YANG et al.[17] [a/d = 0.54]**

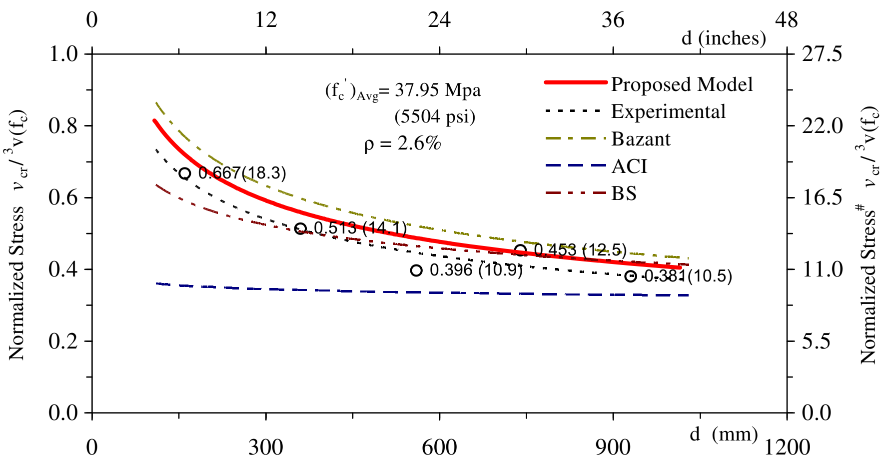


(b) **Normalized Stress $v_{cr}/\sqrt[3]{f_c}$ v/s depth YANG et al.[17] [a/d = 0.54]**



(c) **Normalized Stress $v_{cr}/\sqrt[3]{f_c}$ v/s depth TAN-LU[12] [a/d = 0.84]**

Figure 3. Comparison of $v_{cr}/\sqrt[3]{f_c}$ with depth.



(d) Normalized Stress $v_{cr}/\sqrt[3]{f_c}$ v/s depth WALRAVEN[13] [a/d = 1.0]

Figure 3. (continued).

the type of concrete, maximum size of coarse aggregate, surface characteristics of aggregate, fracture energy, brittleness and tensile strength of concrete. Fracture mechanics of concrete can explain the variation of brittleness with strength and size of the member. The brittleness of concrete increases with increase in strength and the size of the member (Appa Rao 2001, Appa Rao & Raghu Prasad 2002). Therefore, the large size beams made up of high strength concrete exhibit high brittleness. The fracture properties of concrete such as stress-crack opening displacement response, fracture energy, tensile strength and hence the brittleness number can vary the behaviour and strength of RC deep beams. The size of fracture process zone associated with the tension-softening response as discussed by So & Karihaloo (1993) could explain the reasons for the size effect and decrease in the shear strength with increase in the beam depth. In the large size beams, the fracture process zone is small relative to its size, whereas in small size beams, the fracture process zone is relatively large. A significant amount of nonlinear fracture energy is dissipated in the fracture process zone, due to which the small size beams fail at high stress

Table 3. Limiting depth beyond which the shear strength is overestimated by ACI.

Authors	a/d	ρ %	f'_c, Mpa	Depth, mm
Kani 1967	4.0	2.76	26.7	1134
Kani 1967	5.0	2.8	25.9	912
Taylor 1972	3.0	1.35	27.8	773
Collins & Kuchma 1999	2.97	1.13	98.8	460
Kim & Park 1994	3.0	1.87	53.7	545
Kotosovos 2001	3.0	1.62	38	490
		1.34	40	585
Leonhardt	3.0	1.66	35.7	555
(Bazant & Sun 1987)		1.33	37.7	915

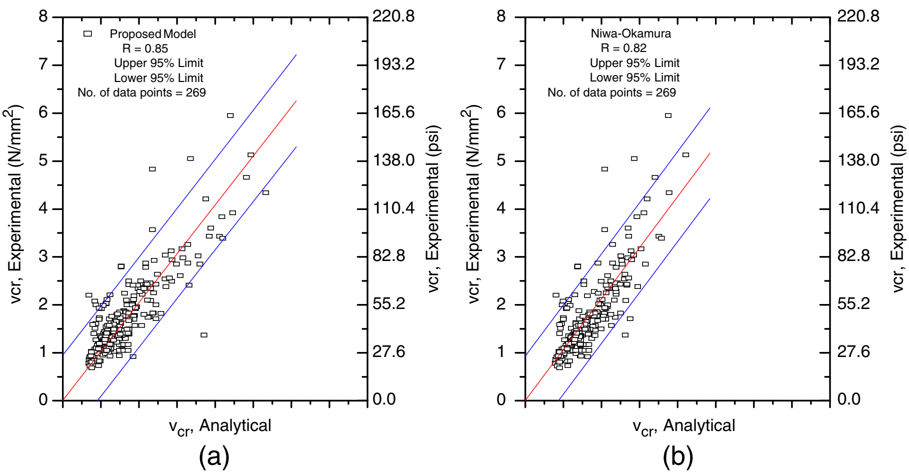


Figure 4. Diagonal cracking strength – measured versus calculated values.

levels. In the large size beams, the brittleness increases with relatively less energy dissipation in the process zone. Hence, the failure stress decreases with increase in the beam size. The shear span-to-depth ratio and the boundary conditions play a significant role on the shear strength of RC deep beams. In addition to the above, the interaction of adjacent reinforcement bars, the quality of bond due to type of interface formed in concrete and the ability of reinforcing bars to resist slippage would be reduced due to interaction of the cylindrical failure surfaces due to bond mechanisms, effect of casting depth of concrete in large size beams, bar size, embedment length, clear concrete cover, number of reinforcing bars and the type of anchorage provided influence the ultimate shear strength of RC deep beams (Appa Rao *et al* 2010). Further, variation of shear mechanisms and extent of their contribution are very important in explaining the size effect on the shear strength of RC beams. The shear strength of uncracked concrete and aggregate internal friction between the aggregate interfaces, and the dowel action of longitudinal reinforcement are also influencing the shear strength. Improved shear strength prediction may be ensured by considering dowel action and aggregate interlock in the calculation.

8. Conclusions

Based on the present investigations, the following conclusions have been drawn:

(i) The model proposed to predict the ultimate shear strength is simple and predicts the shear strength of RC beams with a fair degree of accuracy on the deep, short and normal beams for a wide range of values of the influencing parameters.

(ii) It is appropriate to design the RC beams in shear based on the ultimate shear strength to achieve uniform safety margins on different beam depths rather than on the diagonal cracking strength.

(iii) The size effect is more pronounced on the ultimate shear strength. However, the size effect on the diagonal cracking strength is not negligible.

(iv) The estimated shear strength of RC beams by the ACI code provisions seems to be conservative, which does not incorporate the size effect on the design of RC beams.

Notation

A_s = area of tension reinforcement
f'_c = cylindrical compressive strength in MPa
f_{cu} = characteristic strength of concrete cube, MPa
ρ = longitudinal reinforcement ratio
a = shear span
d = effective depth in mm
a/d = shear span-to-depth ratio
e = eccentricity of load
h = overall depth of member
d_a = maximum aggregate size
M_u = factored moment occurring simultaneously with V_u at critical section
V_u = factored shear force at the critical section
v_u = ultimate shear strength
v_{cr} = diagonal cracking strength
γ_m = partial safety factor

Appendix I. Details of test data from literature.

Authors	a/d	Depth, mm	ρ (%)	f_c, N/mm^2	No of beams: Ultimate strength	No of beams: Diagonal cracking
Clark 1951	1.16–2.32	393.7	0.98	21.5–26.2	12	12
Kani 1967,1966	1.0–8.0	132–1097	2.55–2.87	24.8–30.8	44	
	0.98–5.0	264–287	0.5, 0.76, 1.8	15.4–36.4	77	
Taylor 1972	3.0	139–930	1.35	20.8–32.1	15	–
Bazant & Kim 1984	3.0	40.6–165.1	1.64	46.5	18	–
Tan & Lu 1999	0.56–1.13	444–1559	2.6	30.8–49.1	12	12
Walraven &	1.0	160–930	1.08–1.52	18.1–20	5	5
Lehwalter 1994	3.0	125–720	0.74–0.83	34.2–34.8	3	–
Chana 1987	3.0	106–356	1.73–1.78	24.7–39.5	19	–
Collins &	2.92–3.07	110–925	0.76–0.91	37.2	4	–
Kuchma 1999	2.92–3.0	225–925	0.76–1.01	98.0–98.8	5	–
	2.92	225.0–925	1.05–1.31	36.0–39.0	5	–
	2.92	225–925	0.5–1.31	36.0–94.0	10	–
Yang *et al* 2003	0.53–1.13	355–935	0.9–1.0	31.4–78.5	21	21
Kim & Park 1994	3.0	142–915	1.01–1.88	53.7	10	–
	4.5–6.0	270	1.87	53.7	10	–
Kotosovos 2001	3.0	70–600	1.34 & 1.62	36.1–40	8	–
Ahmad *et al* 1984	1.0–4.0	184–208	1.77–6.64	60.8–67	35	35
Bresler & Scrodelis 1963	3.97–6.93	254–466	1.03–3.1	16.8–37.6	3	3
Cossio & Siess 1960	2.01–6.04	252.5–448.1	1.72–3.61	19.1–36.7	6	6

Appendix I. (continued).

Authors	a/d	Depth, mm	ρ (%)	f_c, N/mm^2	No of beams Ultimate strength	Diagonal cracking
Elzanty *et al* 1986	2,4 & 6	266.7–271.8	0.6–3.27	20.7–79.3	15	–
Kim *et al* 1999	2.5,3 & 4	250	1.08 & 1.94	19.6	8	8
Krefield & Thurston 1966	2.34–9.74	237.7–482.6	0.8–5.01	12.2–39	78	62
Mathey & Watstein 1963	1.51	403	0.75–3.05	21.9–27	16	16
Mattock 1969	2.74	254	1.03–3.1	17.1–46.9	7	7
Moody *et al* 1954	1.52	533	2.7,3.4, 4.25	21.9	12	12
Mphonde *et al* 1984	1.5,2.5, 3.6	298	3.36	20.8–93.7	19	19
Pendyala & Mendis 2000	2.0 & 5.0	140	2.0	34, 63, 87	6	6
Mohan Rao *et al* 2004	2.35	175	0.25–3.0	74.5	7	7
Rajagopalan & Ferguson 1968	3.93–4.22	258.6–268.4	0.25–1.73	23.7–36.5	10	–
Shin *et al* 1999	1.5,2.0, 2.5	215	3.77	52.4, 73.1	6	6
Taylor & Brewer 1963	3.8	220.7–222.3	1.24 & 1.94	22.4–30.3	12	–
Taylor 1960	1.64–4.91	279.4	2	17.9–18.8	5	–
Van Den Berg 1962	2.7–4.2	228.6–300	1.72–4.32	14.2–46.9	32	26
Xie *et al* 1994	1–3.0	215.9	2.07	37.7 –98.9	6	6
Leonhardt	3.0	70–600	1.33–2.07	28.4 –37.7	14	–
(Bazant & Sun 1987)	4–8	270–278	2.01–2.07	28.4 –31.5	9	–
Bhal (Bazant 1987)	3.0	297–1200	0.59–1.29	22.8–29.1	8	–
Zararis & Papadakis 2001)	1.5,2.0, 2.5	200	0.8–1.2	24–25.3	20	–
			Total		612	269

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