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Estimation of spatially varying heat transfer coefficient from a flat plate with flush mounted heat sources using Bayesian inference

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Abstract.

This paper employs the Bayesian based Metropolis Hasting - Markov Chain Monte Carlo algorithm to solve inverse heat transfer problem of determining the spatially varying heat transfer coefficient from a flat plate with flush mounted discrete heat sources with measured temperatures at the bottom of the plate. The Nusselt number is assumed to be of the form $Nu = aRe^b(x/l)^c$. To input reasonable values of 'a' and 'b' into the inverse problem, first limited two dimensional conjugate convection simulations were done with Comsol. Based on the guidance from this different values of 'a' and 'b' are input to a computationally less complex problem of conjugate conduction in the flat plate (15mm thickness) and temperature distributions at the bottom of the plate which is a more convenient location for measuring the temperatures without disturbing the flow were obtained. Since the goal of this work is to demonstrate the eficiacy of the Bayesian approach to accurately retrieve 'a' and 'b' , numerically generated temperatures with known values of 'a' and 'b' are treated as 'surrogate' experimental data. The inverse problem is then solved by repeatedly using the forward solutions together with the MH-MCMC aprroach. To speed up the estimation, the forward model is replaced by an artificial neural network. The mean, maximum-a-posteriori and standard deviation of the estimated parameters 'a' and 'b' are reported. The robustness of the proposed method is examined, by synthetically adding noise to the temperatures.

1. Introduction

In electronic cooling, a typical circuit board may contain several discrete components all dissipating heat at distinct rates on their surfaces. The convective heat transfer with non uniform thermal boundary conditions in the flow direction may lead to a non similarity in thermal boundary layer [1]. The corresponding local convective heat transfer coefficient is determined either from intrusive measurements in the experiments which may disturb the fluid flow or by computing fluid flow and the conduction equations in the circuit board which is expensive. In the present study, instead of a coupled CFD problem an inexpensive methodology is proposed to overcome the above problems by solving the inverse heat conduction problem of discrete heat sources flush mounted to a flat plate by using measured temperatures at the adiabatic bottom surface to estimate the local convective heat transfer coefficient using the Bayesian framework.

Ramadhyani et al. [2] and Ortega et al. [3] have reported heat transfer correlations that also take into account of heat conduction of the discrete heat sources on a plate. Yovanovich and Teertstra [4] proposed a composite model for the determination of area-average Nusselt number for forced flow parallel to a finite, isothermal rectangular plate for a wide range of Reynolds number. Gnanasekaran and Balaji [5] has reported the results for the simultaneous estimation of constants in a Nusselt number correlation by conducting transient heat transfer experiments coupled with Bayesian inference. Konda Reddy et al. [6] proposed an inverse methodology to estimate thermo-physical and transport properties individually and simultaneously from in-house experimental data obtained using transient Liquid Crystal Thermography (LCT) and Bayesian inference. Bhowmik [7] has reviewed topic on convection heat transfer in channel flow with discrete heater arrays for electronic cooling.

From the above literature review, it can be seen that the estimation of local convective heat transfer coefficient for a discrete heat sources flush mounted to a flat plate using Bayesian inference has not been explored adequately in literature. The variation of local convective heat transfer coefficient for a flat plate assembly is obtained from the Nusselt number correlation of the form $Nu = aRe^b(x/l)^c$. This correlation is developed by solving the conjugate conduction-convection equations. The goal is to retrieve a and b in the above equation using Bayesian framework, with a view to estimate the local convective heat transfer coefficient.

2. Forward model

Steady state two dimensional heat conduction of the flat plate assembly is simulated using COMSOL for driving the inverse problem. Fig.1 shows the geometry of the forward model with dimensions. The geometry considered for modeling consists of hylam plate with pockets and three identical embedded discrete aluminium heat sources. There is a uniform heat generation in each heat source. The heat input to the first, second and third heat source is 16 , 5 and 3 W respectively. The thermal conductivity of Hylam plate and aluminium heater source is 1.4 and $200\,W/m.K$. Initially heat generated in the three heat sources is conducted along and across the flat plate assembly, before getting dissipiated from the top surface of the plate by convection. The cooling medium is air at temperature $(T_\infty = 300K)$ and velocity $U_\infty = 1m/s$.
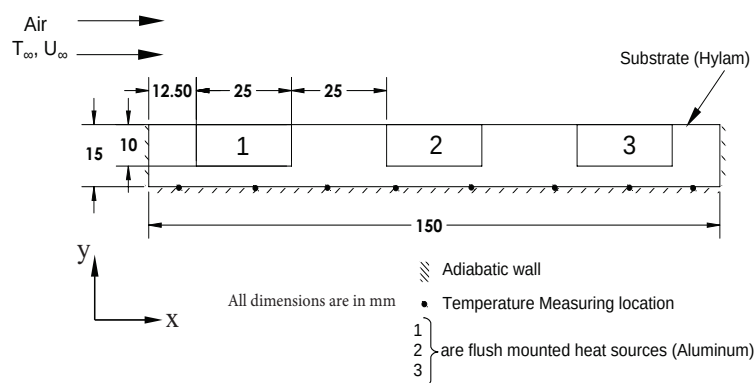

Figure 1. Problem geometry with dimensions

The governing equation for the problem under consideration is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q_v}{k} = 0 \tag{1}$$

In equation 1, $q_v = 0$ wherever there is no heat source and $q_v = q_1$ or q_2 or q_3 depending upon the location of (x, y)

The boundary conditions at the plate surfaces are given by

$$x = 0; \ \ 0 \le y \le H, \quad q_s = 0 \tag{2}$$

$$x = L; \ \ 0 \le y \le H, \quad q_s = 0 \tag{3}$$

$$y = 0; \ \ 0 \le y \le H, \quad q_s = 0 \tag{4}$$

$$y = H, 0 \le y \le H, \quad -k\frac{\partial T}{\partial y} = h_x(T - T_\infty), \tag{5}$$

where

$$h_x = \frac{Nu \times k}{x} \quad and \tag{6}$$

$$Nu = a \times Re^b \times (x/l)^c \tag{7}$$

The solution to the forward problem results in a temperature distribution throughout the domain including the adiabatic surface at the bottom. The inverse problem then is to get back 'a' and 'b' (c=0.3 in this study) given $T(x,0)$ from measurements. As already mentioned $T(x_8)$ obtained by solving the governing equations with known values of 'a' and 'b' is treated as 'experimental' data. It is clear that the inverse problem would involve repeated solution to the forward model such that $T(x,0)$ [simulated] match $T(x_8)$ [experimental]. A grid independence study was carried out and it was seen that 1000 elements was sufficient for simulation. In the problem under consideration, for the simultaneous estimation of two parameter i.e., constants $'a'$ and $'b'$ of the local convective heat transfer coefficient, a surrogate model which approximates the behavior of numerical model based on artificial neural networks is employed in the retrievals, in order to reduce the computational time and cost. The laminar fluid flow and conduction in the plate assembly were first solved using coupled CFD and energy equations for different values of Reynolds number and then a correlation of the form $Nu = aRe^b(x/l)^c$ was developed to obtain the values of $'a'$ and $'b'$ so that reasonable values of $'a'$ and $'b'$ are used in the problem. A total of three simulations was done for a velocity range of 1-1.5 m/s. For each velocity the spatially varying heat transfer coefficient was obtained which is then used to develop the correlation. Using Datafit software the constants a, b, and c are determined to be 0.18, 0.53 and 0.3 respectively. A typical temperature contours for $U_\infty = 1 \ m/s \ (Re = 8753)$ are shown in Fig. 2. In the forward model the value of $'c'$ is fixed in the equation 7, while estimating the values of $'a'$ and $'b'$.

3. Surrogate model
In order to reduce the computational time required for the inverse problem, the time consuming forward model is replaced by a surrogate model. In this work artificial neural network, a non linear regression tool is used to correlate the input and target data sets (i.e. temperatures at the adiabatic surface) to generate a network this returns the target values for a given set of inputs in a fractions of the time required for the inverse model . A neural network consists of an input layer, hidden layer and an output layer. The neural network configuration used in the present study is based on the feed forward back propagation network. The Levenberg Marquardt algorithm is used for training and tan sigmoid is the transfer function employed. For 100 input values (constants 'a' and 'b') corresponding target values (temperature distribution) are computed using the numerical model. A schematic of the neural network architecture is given in Fig. 3. Out of 100 data sets , 80 % of the randomly selected training data sets are used for training the network and the remaining 20 % are used to assess the network performance. The following parameters are considered to calculate the optimum number of neurons in the
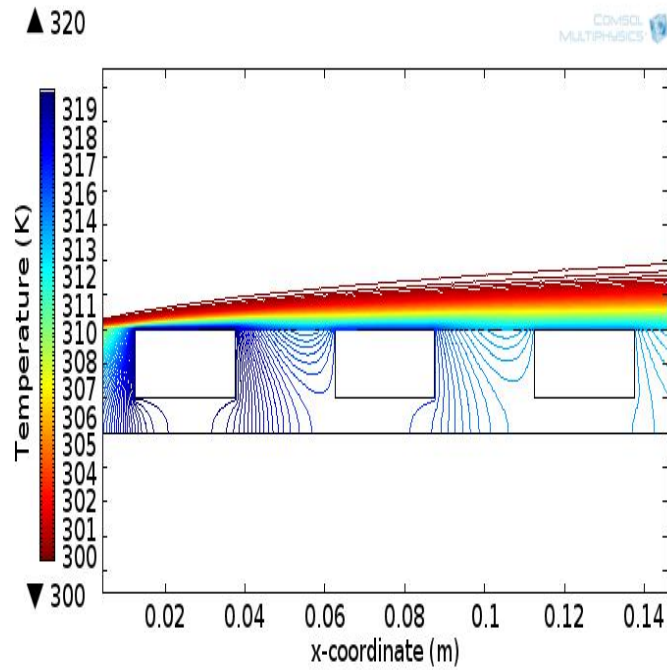
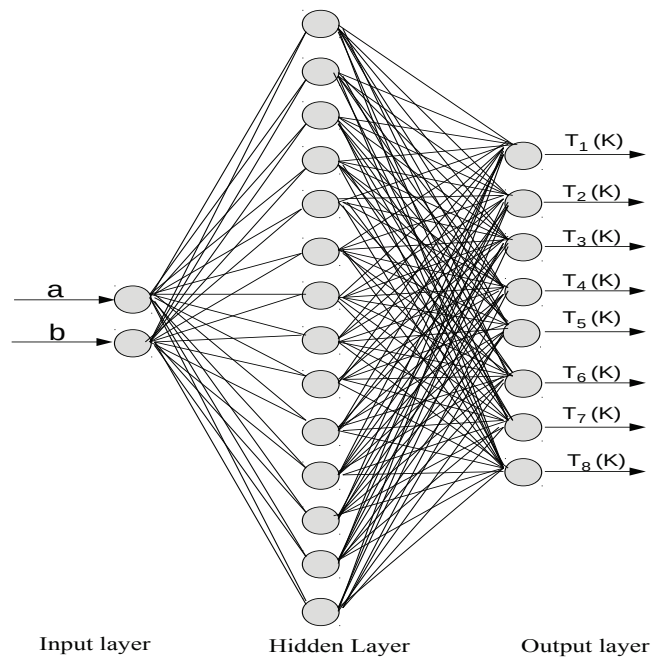**Figure 2.** Temperature contour at $U_\infty = 1\ m/s$

Figure 3. Neural network architecture

hidden layer [8].

$$Mean\ relative\ error = \frac{\sum_{i=1}^{N} \left| \frac{T_{ann} - T_{sim}}{T_{sim}} \right|}{N} \tag{8}$$

$$Mean \ square \ error = \frac{\sum_{i=1}^{N}(T_{ann} - T_{sim})^2}{N} \tag{9}$$

$$Absolute \ fraction \ of \ variance = 1 - \frac{\sum_{i=1}^{N}(T_{ann} - T_{sim})^2}{\sum_{i=1}^{N}(T_{sim})^2} \tag{10}$$

The optimum number of neurons required in the hidden layer for the network is determined to be 14.

4. Inverse model

Bayesian inference is a method of inference using Bayes theorem which states the relation between two conditional probabilities that are reverse of each other Eqn.11. i.e., The posterior probability of an event is directly proportional to the likelihood density function.

$$P\left(\frac{x}{y}\right) \propto P\left(\frac{y}{x}\right) \tag{11}$$

The probability density function of a vector x, given the experimental observations y is related using 'Bayes formula'

$$P\left(\frac{x}{y}\right) = \frac{P\left(\frac{y}{x}\right) \times P(x)}{P(y)} = \frac{P\left(\frac{y}{x}\right) \times P(x)}{\int P\left(\frac{y}{x}\right) \times P(x)} \tag{12}$$

where $P\left(\frac{x}{y}\right)$ is the posterior probability density function (PPDF), $P\left(\frac{y}{x}\right)$ is the likelihood density function, the $P(x)$ the prior density function. The likelihood density function, $P\left(\frac{y}{x}\right)$ is obtained by comparing the measured temperatures with the simulated temperatures for a given parameter

$$P\left(\frac{y}{x}\right) = \frac{1}{\left(\sqrt{2\pi\sigma^2}\right)^n} e^{-\left(\frac{R^2}{2\sigma^2}\right)} \tag{13}$$

the residue R^2 is given by $(Y_{measured} - Y_{simulated})^2$ and σ is the uncertainty in the measurements and the forward model. The prior density function $P(x)$ is 1.0 for the case of uniform prior and the corresponding PPDF becomes the likelihood density function. In the case of normal prior with mean μ_p and standard deviation σ_p of the parameters, $P(x)$ is represented as

$$P\left(x\right) = \frac{1}{\left(\sqrt{2\pi\sigma_p^2}\right)^n} e^{\frac{(x-\mu_p)^2}{2\sigma_p^2}} \tag{14}$$

substituting Eqn.13 and Eqn.14 in Eqn.12 the PPDF then becomes

$$P\left(\frac{x}{y}\right) = \frac{e^{-\left[\frac{\chi^2}{2} + \frac{(x-\mu_p)^2}{2\sigma_p^2}\right]}}{\int e^{-\left[\frac{\chi^2}{2} + \frac{(x-\mu_p)^2}{2\sigma_p^2}\right]} dx} \tag{15}$$

For a discrete data values of x with normal priors, the mean , maximum a posteriori(MAP) and the variance estimate of x are as follows.

$$\bar{x} = \frac{\sum_i x_i e^{-\left[\frac{\chi^2}{2} + \frac{(x-\mu_p)^2}{2\sigma_p^2}\right]}}{\sum_i e^{-\left[\frac{\chi^2}{2} + \frac{(x-\mu_p)^2}{2\sigma_p^2}\right]}} \tag{16}$$

$$x_{MAP} = max \left[\frac{x_i e^{-\left[\frac{\chi^2}{2} + \frac{(x-\mu_p)^2}{2\sigma_p^2}\right]}}{e^{-\left[\frac{\chi^2}{2} + \frac{(x-\mu_p)^2}{2\sigma_p^2}\right]}} \right] \tag{17}$$

$$\sigma_x^2 = \frac{\sum_i (x_i - \bar{x})^2 \; e^{-\left[\frac{\chi^2}{2} + \frac{(x-\mu_p)^2}{2\sigma_p^2}\right]}}{\sum_i e^{-\left[\frac{\chi^2}{2} + \frac{(x-\mu_p)^2}{2\sigma_p^2}\right]}} \tag{18}$$

For two parameter estimation the marginal PPDFs are calculated for each parameter. The data vector x in our case is $[x]=[a,b]$. The marginal PPDFs for the two parameters estimation are

$$P\left(\frac{x_1}{y}\right) = \frac{e^{-\left[\frac{\chi^2}{2} + \frac{(x_1-\mu_{x_1,p})^2}{2\sigma_{x_1,p}^2} + \frac{(x_2-\mu_{x_2,p})^2}{2\sigma_{x_2,p}^2}\right]}}{\int e^{-\left[\frac{\chi^2}{2} + \frac{(x_1-\mu_{x_1,p})^2}{2\sigma_{x_1,p}^2} + \frac{(x_2-\mu_{x_2,p})^2}{2\sigma_{x_2,p}^2}\right]} dx_2} \tag{19}$$

$$P\left(\frac{x_2}{y}\right) = \frac{e^{-\left[\frac{\chi^2}{2} + \frac{(x_1-\mu_{x_1,p})^2}{2\sigma_{x_1,p}^2} + \frac{(x_2-\mu_{x_2,p})^2}{2\sigma_{x_2,p}^2}\right]}}{\int e^{-\left[\frac{\chi^2}{2} + \frac{(x_1-\mu_{x_1,p})^2}{2\sigma_{x_1,p}^2} + \frac{(x_2-\mu_{x_2,p})^2}{2\sigma_{x_2,p}^2}\right]} dx_1} \tag{20}$$

The surrogate model is simulated to obtain the temperatures for a desired number of samples 'a' and 'b' to retrieve the unknown parameters.

In this study $x_1 = a$ and $x_2 = b$, y is the data vector of 'measured' temperatures and has temperature values at 8 locations along the adiabatic surface. Sample generation with Metropolis hasting-Markov Chain Monte Carlo algorithm is employed for generating the samples (a,b). The sampling procedure for two parameter estimation is as follows

(i) Initialize $x^1 = \{a^1, b^1\}$

(ii) for i=1,2, . . . , M
   for i=1,2, . . . , n

  (a) Draw a sample u $\sim$ U(0,1)
  (b) Calculate a next sample $x_j^* \sim N(x_{-j}^{i+1}, x_j^i, \sigma_{x_j^i}^2)$

  (c) If u<A(x_j^*, x_j^i), $x_j^{i+1}=x_j^*$
  (d) Else go to step 2 with $x_j^{i+1}=x_j^i$

In the above algorithm M is the number of samples, n is the number of parameters and $x_{-j}^{i+1}=\{x_1^{i+1}, ...x_n^i\}^T$.

Where A(x_j^*, x_j^i) is called acceptance ratio

$$A(x_j^*, x_j^i) = min\left(1, \frac{p(x_j^* \mid x_{-j}^{i+1}).q(x_j^i \mid x_j^*, x_{-j}^{i+1})}{p(x_j^i \mid x_{-j}^{i+1}).q(x_j^* \mid x_j^i, x_{-j}^{i+1})}\right) \tag{21}$$

The ratio $\frac{p(x_j^*|x_{-j}^{i+1})}{p(x_j^i|x_{-j}^{i+1})}$ is called as likelihood density ratio(with uniform prior) or PPDF density ratio (with normal prior) and can be calculated from Eqns.19 and 20 The ratio $\frac{q(x_j^i|x_j^*,x_{-j}^{i+1})}{q(x_j^*|x_j^i,x_{-j}^{i+1})}$, is called the proposal density ratio.

5. RESULTS AND DISCUSSION

5.1. Generation of prior

A novel method to generate priors is employed here where in an "offline" Bayesian approach is proposed. In this method samples are generated by dividing the original interval of uncertainties (upper bound –lower bound) into a certain number of pre-decided intervals ($0.1 \leq a \leq 0.4$ and $0.4 \leq b \leq 0.7$). Values thus chosen are used to run the ANN where output is used to generate the posterior densities function and from these approximate values of 'a' and 'b' can be estimated. This approach is often used to bracket the solution initially and the output of this can be used to generate priors for the actual Bayesian estimation. This approach is expected to drastically bring down the standard deviation or uncertainty in the final estimation of the quantities. Table 1 shows the mean, MAP and SD of the estimated parameters. The mean values obtained is further used as a Gaussian prior with 5 % of the mean as a standard deviation for the simultaneous estimation of constants 'a' and 'b' in the heat transfer coefficient equation using the regular MH-MCMC approach.

Table 1. Generation of prior using 'Offline' Bayesian method

	Estimation of 'a'			
no. of samples	Actual	mean	MAP	SD
1000	0.18	0.20	0.27	0.06
	Estimation of 'b'			
no. of samples	Actual	mean	MAP	SD
1000	0.53	0.52	0.48	0.03

5.2. The effect of number of samples

The effect of the number of samples on the estimation is similar to a grid independence study in numerical simulations. The number of samples need to be chosen such that the PPDF attains stationarity. Tables 2 , shows that as the number of samples increases the standard deviation decreases. It is observed that 10000 samples are adequate for the simultaneous estimation of parameters. In order to remove the influence of initial guess first 1000 samples are excluded to calculate the mean, MAP and SD respectively. This is frequently referred to as 'burn in' in Bayesian literature.

Table 2. Effect of number of samples on the results

	Estimation of 'a'				Estimation of 'b'			
no. of samples	Actual	mean	MAP	SD	Actual	mean	MAP	SD
5000	0.18	0.20	0.20	6.71×10^{-3}	0.53	0.53	0.53	3.97×10^{-3}
10000	0.18	0.20	0.20	6.62×10^{-3}	0.53	0.53	0.52	3.55×10^{-3}
12000	0.18	0.20	0.20	6.41×10^{-3}	0.53	0.53	0.52	3.65×10^{-3}

5.3. The effect of number of temperature data points

The optimum number of samples need to be chosen based on a sensitivity study. Table 3 shows that 8 data points on the adiabatic surface of the flat plate assembly are sufficient since it give a minimum standard deviation for the estimation of unknown parameters. Fig. 1 shows the flat plate with the locations of the 8 points at which the temperatures are 'measured'. Thermocouples or Thermochromic liquid crystal sheets can be used to measure the temperature.[6]

Table 3. Effect of number of temperature data points on parameter estimation

No. of temperature data points	Estimation of 'a'				Estimation of 'b'			
	Actual	mean	MAP	SD	Actual	mean	MAP	SD
4	0.18	0.22	0.23	8.01×10^{-3}	0.53	0.5	0.49	3.97×10^{-3}
8	0.18	0.20	0.20	6.21×10^{-3}	0.53	0.53	0.52	3.45×10^{-3}
12	0.18	0.20	0.20	5.84×10^{-3}	0.53	0.53	0.52	3.61×10^{-3}

5.4. Estimation with surrogate data

The constant 'a' and 'b' in the heat transfer coefficient are simultaneously retrieved using MH-MCMC based Bayesian approach for steady state two dimensional conduction of the three discrete heat sources flush mounted to the flat plate with surrogate temperature distributions with Gaussian prior for an air inlet velocity of $U_\infty = 1 \ m/s$ ($Re = 8753$) and free stream temperature $T_\infty = 300K$. Table 4 shows the mean, MAP, and SD. From the Table 1 and 4 it is clear that there is a drastic reduction of 89 % and 88 % in SD of retrieved parameters in comparison with the values estimated using 'Offline' Bayesian method. The convergence results and Marginal PPDFs for the parameter 'a' and 'b' are shown in Fig's. 4, 5, 6 and 7 respectively. The estimated parameters are used in the forward model to get back the simulated temperatures. These simulated temperatures is compared with surrogate temperatures data and results are shown in the form of a parity plot in Fig 8. From the Fig 8 it seem that the agreement between the two is quite good, thereby demonstrating the adequacy of the procedure employed in the study to estimate 'a' and 'b'.

Table 4. Retreived values of the constants 'a' and 'b' in local convective transfer coefficient for a discrete heat sources flush mounted to a flat plate

no. of samples	Estimation of 'a'			
	Actual	mean	MAP	SD
10000	0.18	0.20	0.20	6.54×10^{-3}
no. of samples	Estimation of 'b'			
	Actual	mean	MAP	SD
10000	0.53	0.52	0.48	3.51×10^{-3}

5.5. The effect of noise

To check the robustness of the method, Gaussian noise was added in the surrogate temperature data obtained from forward model for a given value of 'a' and 'b' with zero mean and $\frac{\sigma}{\mu}$ of 0, 1 and 2 % estimations were done. The results of this exercise are shown in Table 5. From the Table 5 it is clear that estimated parameters are close to the target values even with 2% noise.

6. CONCLUSIONS

The two dimensional steady state conduction equation of a flat plate assembly was solved for a precribed spatially varying local heat transfer coefficient on the top surface to obtain the temperature distribution in the domain and also at the bottom adiabatic surface using commercially available COMSOL. These were treated as 'measured' temperatures to solve the inverse problem of obtaining the spatially varying heat transfer coefficient from the temperatures. The variation of heat transfer coefficient was in the form of a Nusselt number correlation as $Nu = aRe^b(x/l)^c$ and 'c' was taken to be 0.3. A Markov Chain Monte Carlo based Metropolis
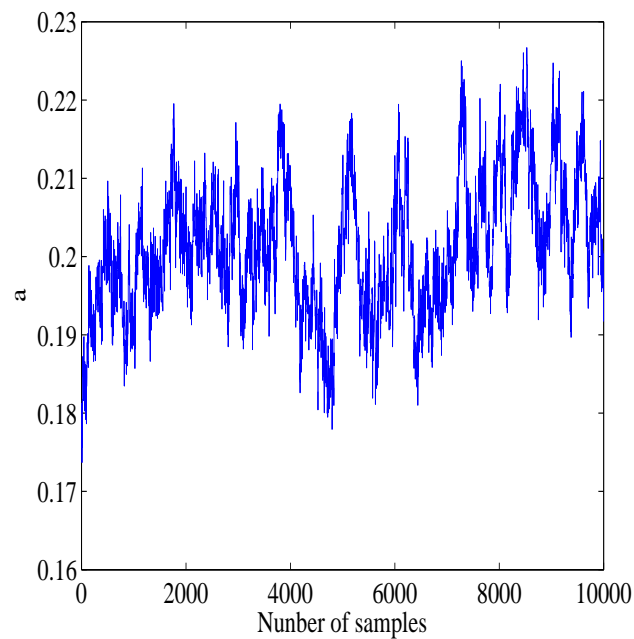
Figure 4. Convergence results for estimation of constant 'a' in the discrete heat sources flush mounted to a flat plate
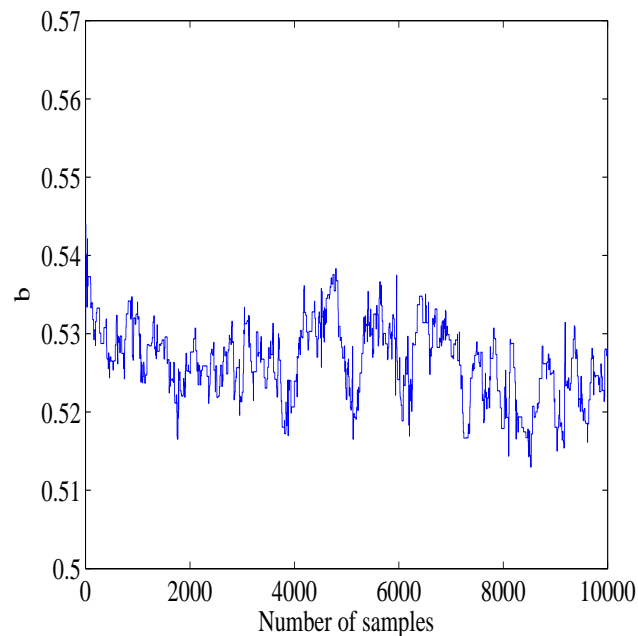

Figure 5. Convergence results for estimation of constant 'b' in the discrete heat sources flush mounted to a flat plate

Hastings algorithm was then used to generate the samples of 'a' and 'b' to the forward model (i.e. conduction problem) and Bayesian inference was adopted to estimate 'a' and 'b' with a
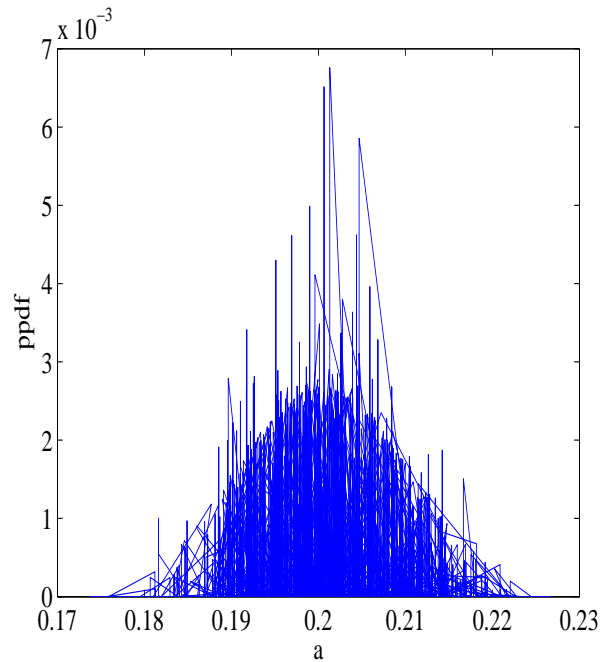
Figure 6. Marginal PPDF for estimation of constant 'a' in the discrete heat sources flush mounted to a flat plate
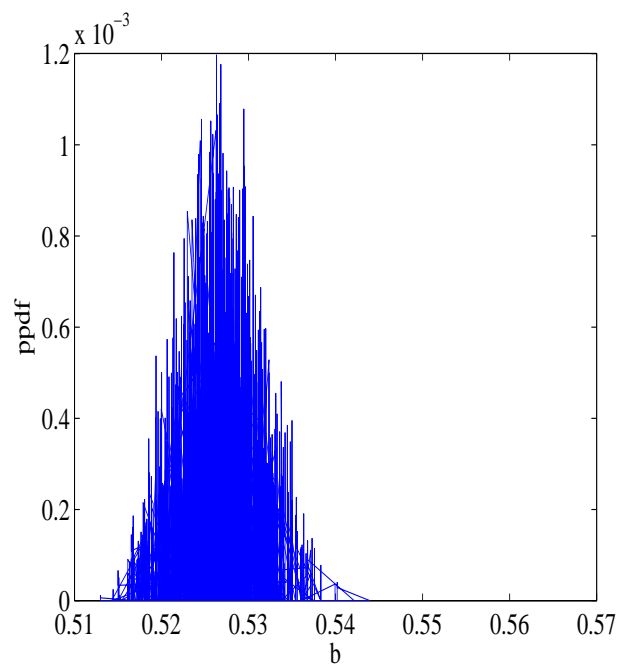

Figure 7. Marginal PPDF for estimation of constant 'b' in the discrete heat sources flush mounted to a flat plate

view to determine local convective heat transfer coefficients from the surrogate temperatures data. Point estimates like mean and maximum a posteriori along with standard deviation of the
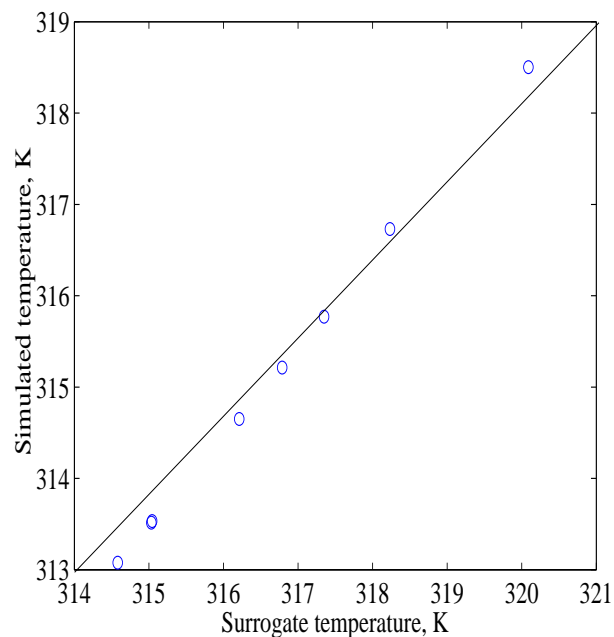
Figure 8. Parity plot between surrogate and simulated temperatures with the estimated value of 'a' and 'b'

Table 5. Effect of noise on estimation of 'a' and 'b'

	Estimation of 'a'			Estimation of 'b'		
noise %	mean	MAP	SD	mean	MAP	SD
0	0.20	0.20	6.54×10^{-3}	0.53	0.52	3.51×10^{-3}
1	0.20	0.19	6.71×10^{-3}	0.52	0.50	3.60×10^{-3}
2	0.2	0.19	6.02×10^{-3}	0.53	0.53	3.69×10^{-3}

parameters were reported. The estimated 'a' and 'b' were seem to be in good agreement with target values. A novel way of generating priors which is the hallmark of the Bayesian method was proposed. Additionally with the Bayesian method the standard deviation of the estimated parameters are directly obtained, which are not possible with other methods. It was seen that priors significantly reduce the uncertainties in the final estimation. The effect of noise on the estimation process was done to check the robustness of the proposed method.

In summary, this paper proposes a methodology by which temperatures measured at convenient places in a problem involving conduction and convection can be married with a simpler mathematical model and the Bayesian framework to obtain a spatially varying heat transfer coefficient on the surface over which the fluid flows. This approach thus avoids the need to measure temperatures and/or temperature gradients on the surface where fluid flow takes place, thereby disturbing the flow or making use of a fully coupled CFD model to obtain the heat transfer coefficient which will then not qualify to become an experimental procedure. The Bayesian framework additionally allows for a systematic injection of prior beliefs into the problem, thereby increasing the accuracy of the estimation process.

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NOMENCLATURE

h	convective heat transfer coefficient, W/m^2K.
H	height, m.
k	Thermal conductivity, W/mK.
L	length, m.
Nu	Nusselt number.
$P(x)$	prior density function
Re	Reynolds number.
T	Temperature, K.
U_∞	velocity, m/s

Abbreviation

ANN	artificial neural network
MRE	mean relative error
MSE	mean square error
MAP	maximum a posteriori
MH	Metropolis Hasting
$MCMC$	Markov Chain Monte Carlo
$PPDF$	posterior probability density function.